CMND Thematic Program on Rationality and Hyperbolicity

Conference Week - Summer 2023

June 26 – June 30, 2023

TALK ABSTRACTS

Kenneth Ascher (University of California - Irvine)

"Wall-crossing and moduli spaces of higher dimensional varieties"

Let X be a smooth, complex Fano 4-fold, and b_2 its second Betti number. We will discuss the following result: if b_2>12, then X is a product of del Pezzo surfaces. The proof relies on a careful study of divisorial elementary contractions f: X ->Y such that the image S of the exceptional divisor is a surface, together with my previous work on Fano 4-folds. In particular, given f: X->Y as above, under suitable assumptions we show that S is a smooth del Pezzo surface with -K_S given by the restriction of -K_Y.

Cinzia Casagrande (*Università degli studi di Torino)*

"Fano 4-folds with b_2>12 are products of surfaces"

Let X be a smooth, complex Fano 4-fold, and b_2 its second Betti number. We will discuss the following result: if b_2>12, then X is a product of del Pezzo surfaces. The proof relies on a careful study of divisorial elementary contractions f: X ->Y such that the image S of the exceptional divisor is a surface, together with my previous work on Fano 4-folds. In particular, given f: X->Y as above, under suitable assumptions we show that S is a smooth del Pezzo surface with -K_S given by the restriction of -K_Y.

Nathan Chen (Columbia University)

"Minimal degree curves on very general hypersurfaces"

We will explore the possible degrees of curves on very general complex hypersurfaces. Later on, we will see a few related results and applications to measures of irrationality.
Some of this will be joint with David Yang.

Laure Flapan (Michigan State University)

"Fixed loci of anti-symplectic involutions on hyperkähler manifolds"

We study the fixed loci of involutions induced by an ample class of degree 2 on hyperkähler manifolds arising as deformations of Hilbert schemes on a K3 surface.
This is joint work with Emanuele Macrì, Kieran O'Grady, and Giulia Saccà.

Lena Ji (University of Michigan)

"Intermediate Jacobian torsors and rationality over non-closed fields"

The intermediate Jacobian obstruction to rationality for complex threefolds was introduced by Clemens–Griffiths in their proof of the irrationality of the cubic threefold. For conic bundles over P^2, this obstruction characterizes rationality over the complex numbers. Recently, over non-closed fields k, Hassett–Tschinkel and Benoist–Wittenberg refined this obstruction using *torsors* over the intermediate Jacobian. Their results, together with work of Kuznetsov--Prokhorov, showed that this refined obstruction can be used to characterize k-rationality for Fano threefolds of Picard rank 1. In this talk, we study the IJ torsor obstruction for conic bundles and explain why it does *not* characterize k-rationality in this higher Picard rank setting.
This talk is based on joint work with S. Frei–S. Sankar–B. Viray–I. Vogt and joint work with M. Ji.

Sándor Kovács (University of Washington)

"Arakelov inequalities in higher dimensions: boundedness, and hyperbolicity"

In 1963 Shafarevich conjectured that there are only finitely many non-isotrivial families of smooth projective curves of fixed genus over a fixed curve and that there are no such families unless the curve is hyperbolic. Parshin (1968) and Arakelov (1971) reinterpreted this conjecture in three parts: boundedness, rigidity, and hyperbolicity. These adjectives describe properties of the moduli maps of those families, so in particular, hyperbolicity refers to hyperbolicity of the moduli space of the fibers. Arakelov's proof included an equality that compares the number of singular fibers in a family parametrized by a curve to other quantities that are either known explicitly or have known bounds. Interestingly, this inequality proved boundedness and hyperbolicity at the same time. Beyond the fact that this inequality proved the original conjectures, it actually did more by giving an explicit bound on these qualities that have been used by subsequent authors.
Almost 40 years later Viehweg generalized Shafarevich's conjecture to higher dimensional families over higher dimensional bases. The rigidity part almost trivially fails as soon as the fiber dimension is at least 2, but the boundedness and hyperbolicity parts are still interesting. Viehweg's conjecture was proved as a result of the cumulative effort of several authors. Along the way some of the results contained an Arakelov type inequality for families parametrized by a curve.
In a survey article, reviewing the advance on his conjecture, Viehweg analyzed the importance of Arakelov type inequalities and demonstrated its connection to stability, variations of Hodge structures, geodecity of curves in the moduli space of abelian varieties, Higgs fields, and the Milnor-Wood inequality. The survey ends with a section on "Open ends" and its main focus is on the absence, at the time, of an Arakelov type inequality over higher dimensional bases.
In this talk I will report on joint work with Behrouz Taji, in which we establish an Arakelov type inequality for families over higher dimensional bases that recovers the known Arakelov inequalities over one dimensional bases. This inequality includes an invariant that might explain the reason it took such a long time to go from one dimensional bases to higher dimensional ones: This invariant appears in the estimate as an exponent of the dimension of the base, thus it disappears for one dimensional bases. In other words, it seems, the key to fulfilling Viehweg's dream of an Arakelov type inequality for families over higher dimensional bases hinged on realizing the importance of this invariant, which we call "Viehweg number".

Steven Lu (Université du Québec à Montréal)

"On varieties dominable by C^n"

A variety is said to be dominable by C^n if it admits a dominable meromorphic map from C^n, in partial analogy with being unirational in the algebraic setting. In this talk, I will discuss some higher dimensional extensions of my dominability results with Greg Buzzard in the 90's. There, we proved that a complex projective surface which is neither a non-elliptic nor a non-Kummer K3 is dominable by C^2 if and only if the surface is special. Also, meromorphic dominability turns out to be equivalent to holomorphic dominability in this case. These results can be generalized to compact Kähler surfaces without additional complications but the problem for the special case of K3 surfaces has remained to this day. In this talk, I will focus mainly on hyperKähler varieties, which are higher dimensional generalizations of K3 surfaces, and discuss strategies to bridge the remaining case of K3 surfaces.
This is joint work with Ljudmila Kamenova.

Jackson Morrow (University of California Berkeley)

"Boundedness of hyperbolic varieties"

Let C_1, C_2 be smooth projective curves over an algebraically closed field K of characteristic zero. What is the behavior of the set of non-constant maps $C_1 \to C_2$? Is it infinite, finite, or empty? It turns out that the answer to this question is determined by an invariant of curves called the genus. In particular, if C_2 has genus $g(C_2)\geq 2$ (i.e., C_2 is hyperbolic), then there are only finitely many non-constant morphisms $C_1 \to C_2$ where C_1 is any curve, and moreover, the degree of any map $C_1 \to C_2$ is bounded linearly in $g(C_1)$ by the Riemann--Hurwitz formula.
In this talk, I will explain the above story and discuss a higher dimensional generalization of this result. To this end, I will describe the conjectures of Demailly and Lang which predict a relationship between the geometry of varieties and topological properties of Hom-schemes. To conclude, I will sketch a proof of a variant of these conjectures, which roughly says that if X/K is a hyperbolic variety, then for every smooth projective curve C/K of genus $g(C)\geq 0$, the degree of any map $C\to X$ is bounded uniformly in the genus of C. Time permitting, I will discuss the ideas behind the proof which include the construction of a non-Archimedean Kobayashi pseudo-metric and new results concerning uniform limits of analytic morphisms in the non-Archimedean setting.

Deepam Patel (Purdue University)

"Enriched Hodge Structures"

An important application of Hodge theory is in the study of algebraic cycles or K-theory (for example via the Hodge or Bloch-Beilinson conjectures) of smooth projective varieties. On the other hand, mixed hodge theory is often insufficient for the study of cycles/K-theory in the open or singular settings. For example, hodge theory does not capture Pic^0(X), where X is a cuspidal rational curve. I will discuss an enrichment of the category of mixed Hodge structures and some results on the existence of natural objects in this category which allow for the possibility of understanding cycles/K-theory in more general settings (for example, on complex analytic links).
This is based on joint work with M. Nori and V. Srinivas.

Erwan Rousseau (Université de Bretagne Occidentale)

"An Albanese construction for Campana's C-pairs"

We will explain a construction of Albanese maps for orbifolds (or C-pairs) with applications to hyperbolicity such as a generalization of the Bloch-Ochiai theorem.
This is joint work with Stefan Kebekus.

David Stapleton (University of Michigan)

"Birational geometry of complex Fano hypersurfaces via characteristic p"

In 1995, Kollár showed that birational geometry of Fano hypersurfaces can be studied by reduction modulo p using the exotic behaviour of differential forms in positive characteristic. This was used by Kollár (resp. Totaro) to study the rationality problem (resp. the stable rationality problem) for these hypersurfaces. In several recent papers, joint with Nathan Chen and Lena Ji, we show that reduction modulo p can also be used to study and refine our understanding of the birational geometry of these Fano hypersurfaces. For example, the forms help to control their degrees of irrationality, their rational endomorphisms, and the prime to p torsion in their birational automorphism groups. The guiding principle is that these differential forms cause the mod p reductions to behave like general type varieties in characteristic p.

Zhiyu Tian (Peking University)

"Deformation of stable maps and algebraic equivalence"

In this talk, I will explain a theorem that turns algebraic equivalence of one cycles into deformations of stable maps, and discuss its applications in some arithmetic and geometric problems.
This is based on the joint work with J. Kollár.

Amos Turchet (Università degli studi Roma Tre)

"Arithmetic and Algebraic Hyperbolicity of pairs"

We will give a survey on arithmetic and algebraic hyperbolicity of pairs, focusing on the case of complements of divisors in projective space.
We will then present some new results obtained in joint works with C. Gasbarri, E. Rousseau and J. Wang.

Julie Wang (Academia Sinica)

"GCD theorems and applications toward Vojta's abc conjecture for function fields"

We will first introduce the GCD theorems in various situations. As applications, we derive Vojta's general abc conjecture for 2-dimensional algebraic tori for function fields and state some results toward the higher dimensional situation. We will also mention some applications in the complex case, including the proof of a case of Campana's orbifold conjecture.
The talk includes a work joint with Guo, Nguyen, and Sun, as well as another one with Guo.

Sai-Kee Yeung (Purdue University)

"Hyperbolicity from the perspective of Carathèodory geometry"

The goal of the talk is to explain some geometric results on quasi-projective manifolds from the perspective of Carathèodory metrics and distances. We will study some conjectures of Lang on manifolds which satisfied some Carathèodory conditions. The results are also used to study hyperbolicity of suitable compactifications of the non-compact manifolds involved.
Most of the results to be presented are joint work with Kwok-Kin Wong