Reliably Accurate State Estimation for Connected and Autonomous Highway Vehicles

A Symposium to Honor Panos Antsaklis

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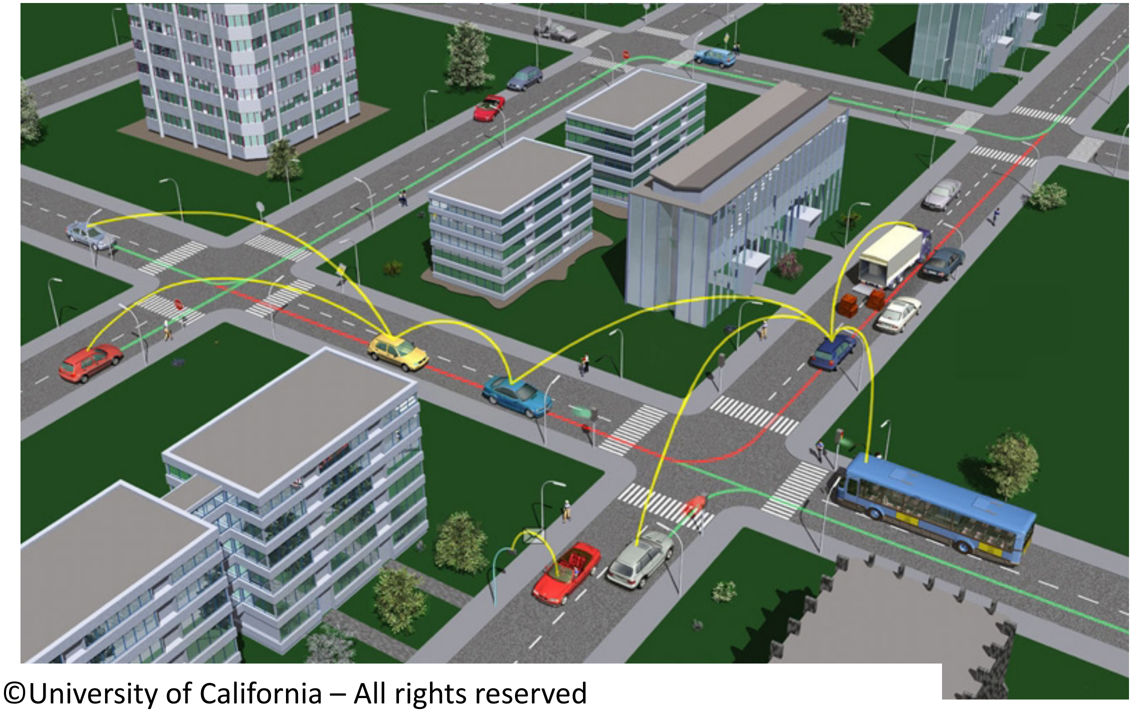
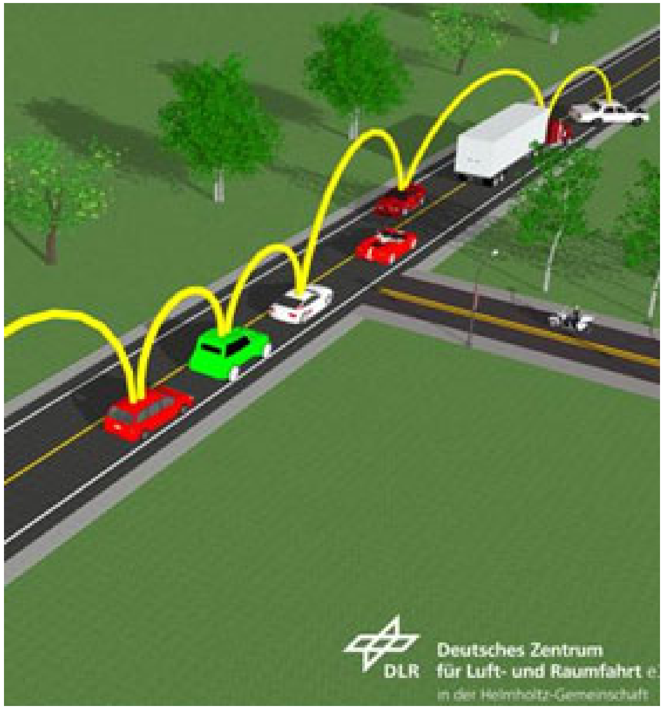
Connected Vehicles

Goals: Enhanced throughput & safety with decreased emissions.

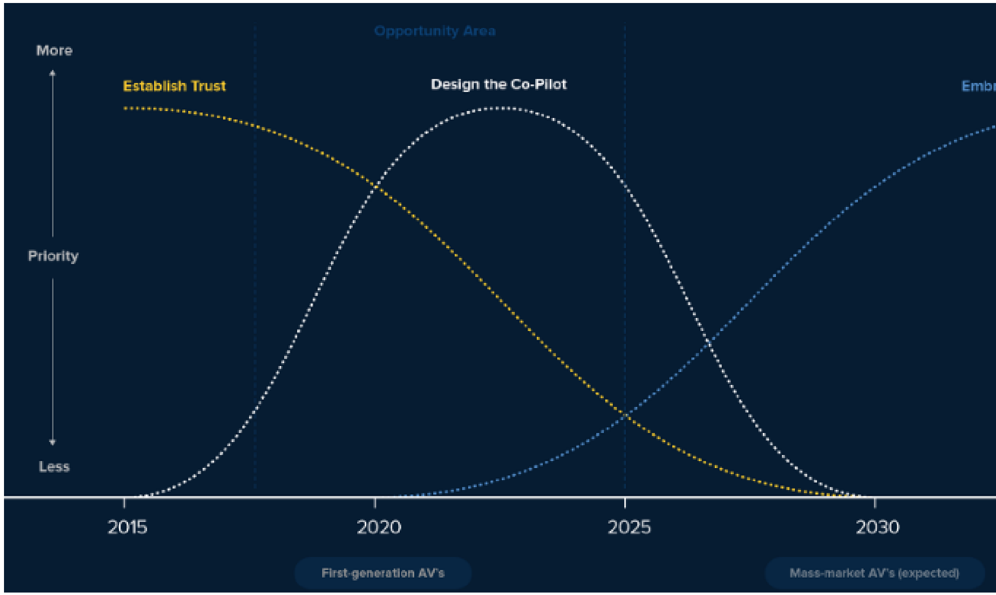
- V2V: Vehicle to vehicle
- V2I: Vehicle to Infrastructure





Reliable Precise Positioning: AV & CV

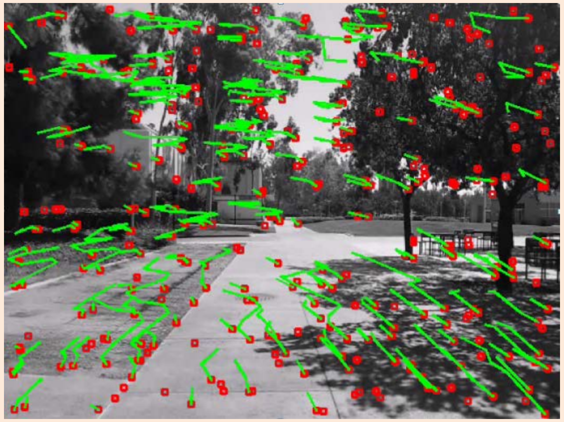
- Autonomous & Connected Vehicles are in our future.
  - Early Phase: Commercial
  - Later: Consumer
- Vehicle Position Accuracy
  - Routing (10.0 m)
  - Coordination (1.0 m)
    - Infrastructure
    - Other vehicles
  - Control (0.1 m)
- Sensor Fusion
- Reliability = Trust



Signal Rich Environments

Significantly more measurements are available than are required to achieve observability or to meet the accuracy specification: $P_x < P_u$ or $J_x > J_l$ where $P_u = (J_l)^{-1}$

Camera-based navigation



- Hundreds of features per frame
- Each requires tracking and association between frames.

Global Navigation Satellite Systems

- GPS (original): 7-10 sv available per epoch w/ 1 signal/sv ➔ 7-10 measurements/epoch
- GPS (modified): 7-10 sv available per epoch w/ 3 signal/sv ➔ 21-30 measurements/epoch
- Now also have GLONASS, Galileo, Beidou …

On the order of 50 measurements soon to be available per epoch

- Four sufficiently diverse measurements are needed for observability
- As the number of used measurements increases, both accuracy and risk increase

What is the most appropriate risk-reward tradeoff, given high probability of outliers?
Which measurements should be selected to achieve a stated accuracy specification?

Sensor Fusion: Inertial Navigation

- **Inertial Navigation:** *Full state, acceleration, angular rate*
  - Pro: High bandwidth, high sampling rate, high reliability
  - Pro & Con: Well modeled slow, but unbounded, error growth
  - Pro: Used in military & commerce since 1960's

IMU price point now appropriate for commercial applications

Given:

- IC: $\boldsymbol{x}(0) \sim N(\hat{\boldsymbol{x}}(0), \boldsymbol{P}(0))$
- IMU data: $\tilde{\boldsymbol{u}} = \begin{cases} \tilde{\boldsymbol{u}}_a = \mathbf{f} + \boldsymbol{b}_a + \boldsymbol{n}_a, \\ \tilde{\boldsymbol{u}}_g = \boldsymbol{\omega} + \boldsymbol{b}_g + \boldsymbol{n}_g, \end{cases}$
- Kinematic model: $\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t))$

The INS numerically solves:

$$\hat{\boldsymbol{x}}(\tau_{i+1}) = \hat{\boldsymbol{x}}(\tau_i) + \int_{\tau_i}^{\tau_{i+1}} \mathbf{f}(\hat{\mathbf{x}}(\tau), \hat{\mathbf{u}}(\tau))\mathbf{d}\tau$$

where τ_i represents the i-th IMU sample time and $\boldsymbol{\phi}(\hat{\boldsymbol{x}}(\tau_i), \hat{\mathbf{u}}(\tau_i)) \doteq \hat{\boldsymbol{x}}(\tau_i) + \int_{\tau_i}^{\tau_{i+1}} \mathbf{f}(\hat{\mathbf{x}}(\tau), \hat{\mathbf{u}}(\tau))\mathbf{d}\tau$

State: $\boldsymbol{x}(t) = [\boldsymbol{p}^\intercal(t), \boldsymbol{v}^\intercal(t), \boldsymbol{q}^\intercal(t), \boldsymbol{b}_a^\intercal(t), \boldsymbol{b}_g^\intercal(t)]^\intercal \in \mathbb{R}^{n_s}, \quad n_s \geq 15$

Sensor Fusion: Aiding Measurements

Aiding Measurement Model: $\tilde{\boldsymbol{y}}_k = \boldsymbol{h}(\boldsymbol{x}_k) + \boldsymbol{\eta}_k + \boldsymbol{e}_k$

Residual Computation: $\delta \boldsymbol{y}_k \doteq \tilde{\boldsymbol{y}}_k - \boldsymbol{h}(\hat{\boldsymbol{x}}_k)$

Residual Model: $\delta \boldsymbol{y}_k = \boldsymbol{H}_k \delta \boldsymbol{x}_k + \boldsymbol{\eta}_k + \boldsymbol{e}_k$

$$\hat{\boldsymbol{x}}_k = \hat{\boldsymbol{x}}(t_k) = \hat{\boldsymbol{x}}(kT)$$
$$T \gg \tau$$

- **GNSS:** *Position, attitude, velocity*
  - Pro: Bounded absolute position error
  - Pro: Bound is dependent on processing and signals used
  - Con: Reliability is dependent on environment
- **Feature sensors:** *Relative position*
  - Pro: Bounded feature relative accuracy. Abundant in urban areas.
  - Con (?): No absolute accuracy (without EDM).
  - Con: Not robust to environmental conditions (e.g. lighting)

Outlier Accommodation:

NP-KF: Discard measurements for which the residual: $|\delta \mathbf{y}_k| > \gamma$

Linear KF Problem Statement

Given:

- $\boldsymbol{x}_{k-1} \sim N(\hat{\boldsymbol{x}}_{k-1}^{+}, \boldsymbol{P}_{k-1}^{+})$
- $\boldsymbol{x}_k = \boldsymbol{\Phi}\boldsymbol{x}_{k-1} + \boldsymbol{B}\mathbf{u}_{k-1} + \boldsymbol{\omega}_{k-1}$
- $\boldsymbol{y}_k = \boldsymbol{H}\boldsymbol{x}_k + \boldsymbol{\eta}_k$

Find: $\hat{\boldsymbol{x}}_k^{+} = \arg\max_{\boldsymbol{x}_k} \left(p_{\boldsymbol{\eta}_k}(\boldsymbol{y}_k - \boldsymbol{H}\boldsymbol{x}_k) p_{\boldsymbol{x}_k}(\boldsymbol{x}_k) \right)$

where

$$
\begin{aligned}
p_{\boldsymbol{x}_k}(\boldsymbol{x}_k) &= N(\hat{\boldsymbol{x}}_k^{-}, \boldsymbol{P}_k^{-}) \\
\boldsymbol{P}_k^{-} &= \boldsymbol{\Phi}_{k-1}\ \boldsymbol{P}_{k-1}^{+}\ \boldsymbol{\Phi}_{k-1}^{\top} + \mathbf{Q}_{k-1} \\
\hat{\boldsymbol{x}}_k^{-} &= \boldsymbol{\Phi}\hat{\boldsymbol{x}}_{k-1}^{+} + \boldsymbol{B}\mathbf{u}_{k-1}
\end{aligned}
$$

$$
\begin{aligned}
\hat{\boldsymbol{x}}_k^{+} &= \arg\min_{\boldsymbol{x}_k} \left(\|\boldsymbol{x}_k - \hat{\boldsymbol{x}}_k^{-}\|_{\boldsymbol{P}_k^{-}}^2 + \|\boldsymbol{y}_k - \boldsymbol{H}\boldsymbol{x}_k\|_{\boldsymbol{R}_k}^2 \right) \\
\hat{\boldsymbol{x}}_k^{+} &= \left(\boldsymbol{P}_k^{-1} + \boldsymbol{H}^{\top}\boldsymbol{R}_k^{-1}\boldsymbol{H}\right)^{-1} \left(\boldsymbol{P}_k^{-1}\hat{\boldsymbol{x}}_k^{-} + \boldsymbol{H}^{\top}\boldsymbol{R}_k^{-1}\boldsymbol{y}_k\right)
\end{aligned}
$$

Standard Kalman Filter in Information Form

New Perspectives: Signal Rich Environment

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- **Sensor rich Environments:** many more signals available than required
  - GNSS: GPS + GLONASS + Galileo + ….
  - Images with numerous features
  - Sliding Window of measurements
- **Important Points:**
  - The specified accuracy can be achieved with a subset of measurements
  - Using all measurements, exposes the estimate to unnecessary risk

New Perspectives:

- Maximal consistent set: Choose a maximal subset of self-consistent measurements
  - ➢ L. Carlone, A. Censi, and F. Dellaert. "Selecting good measurements via ℓ_1 relaxation: A convex approach for robust estimation over graphs." Intelligent Robots and Systems (IROS), 2014.
  - ➢ N. Sünderhauf and P. Protzel, "Towards a robust back-end for pose graph SLAM," in Robotics and Automation (ICRA). IEEE, 2012, pp.1254–1261.

- Risk-Averse Performance-Specified (RAPS): Choose a subset of measurements with minimum risk that achieves specified accuracy.
  - ➢ E. Aghapour and J. A. Farrell, "Performance specified state estimation with minimum risk." American Control Conference (ACC), 2018

MAP Cost: Risk Quantification

$$x_k^+ = \underset{x}{\operatorname{argmax}}\, p(x, x_{k-1}, u_{k-1}, z_k)$$
$$= \underset{x}{\operatorname{argmax}}\, p(x_{k-1})p(x|x_{k-1}, u_{k-1})p(z_k|x)$$

$$x_k^+ = \underset{x}{\operatorname{argmin}}\, \Big(\|x_{k-1} - x_{k-1}^+\|^2_{P_{k-1}^+} + \|f(x_{k-1}, u_{k-1}) - x\|^2_{Q_k} + \|h(x) - z_k\|^2_{R_k}\Big)$$

$$x_k^+ = \underset{x}{\operatorname{argmin}}\, \left[\|x - x_k^-\|^2_{P_k^-} + \|h(x) - z_k\|^2_{R_k}\right]$$

Assumptions:

- State transition and prior are trusted.
- Measurements may have outliers.

$$NP1 : \min_{x,\, b} \left[\|x - x_k^-\|^2_{P_k^-} + \|\sum_{i=1}^{m} \frac{b_i}{\sigma_i} e_i e_i^\top (h(x) - z_k)\|^2\right]$$

$$\text{subject to: } \Big(\sum_{i=1}^{m} \frac{b_i}{\sigma_i^2} h_i^\top h_i + J_k^-\Big) \geq J_l$$

$$b_i \in \{0, 1\} \text{ for } i = 1, \ldots, m.$$

Solution Approach: Relaxation

Nonbinary RAPS: *NP2*

1) *Selecting the measurements:* Given b^ℓ and x_k^ℓ, find the optimal $b^{\ell+1}$:

$$NP2_b : \min_b \|\sum_{i=1}^m \frac{b_i}{\sigma_i} e_i e_i^\top (z_k - h(x_k^\ell))\|^2 + \lambda \|b - b^\ell\|^2$$

$$\text{subject to: } J_l - (\sum_{i=1}^m \frac{b_i}{\sigma_i^2} h_i^\top h_i + J_k^-) \leq 0$$

$$b_i \in [0,1] \text{ for } i = 1, \ldots, m.$$

where $\lambda > 0$ is a user-defined proximal parameter, $H_k^\ell = \nabla h(x)|_{x=x_k^\ell} \in \mathbb{R}^{m \times n}$, and h_i is the i-th row of H_k^ℓ. This is a standard semidefinite programming (SDP) problem.

2) *State update:* Given b^ℓ and x_k^ℓ, find the optimal $x_k^{\ell+1}$:

$$NP2_x : \min_x \left[\|x - x_k^-\|_{P_k^-}^2 + \|\sum_{i=1}^m \frac{b_i^\ell}{\sigma_i} e_i e_i^\top (h(x) - z_k)\|^2 + \beta \|x - x^\ell\|^2 \right]$$

Problem $NP2_x$ can be solved over multiple linearized iterations.

Summary

- Multiple iterations to achieve convergence
- Interior point methods solve the SDP.

Drawbacks:

- User-selected proximal parameters λ and β affect the rate of convergence.
- Final solution only converges to a local minimum, even when *h(x)* is convex.

H. Attouch, J. Bolte, P. Redont, and A. Soubeyran, "Proximal alternating minimization and projection methods for nonconvex problems: An approach based on the Kurdyka-Łojasiewicz inequality," *Mathematics of Operations Research*, vol. 35, no. 2, pp. 438–457, 2010.

Solution Approach: Binary Search

Binary RAPS

Find Minimum Risk Feasible Measurement Subset:

$$C(x,b) = \|x - x_k^-\|^2_{P_k^-} + \|\Phi(b)(h(x) - z_k)\|^2_{R_k}.$$

1) For each $b \in \mathscr{M}_s$, keep the zero elements of b unchanged and deactivate exactly one of the active elements of b. Each $b \in \mathscr{M}_s$ will produce the $(m-s)$ permutations denoted as b^ℓ for $\ell = 1, \ldots, (m-s)$. Each of the resulting b^ℓ vectors will have: $n_z(b^\ell) = (n_z(b)+1) = (s+1)$.
   (a) For each b^ℓ, solve the least square optimization:
   $$\hat{x}^\ell = \arg\min_x C(x, b^\ell).$$
   Define $c^\ell = C(\hat{x}^\ell, b^\ell)$.
   (b) Check whether b^ℓ satisfies the performance constraint. If it does, then add b^ℓ to $\mathscr{M}$ and $\mathscr{M}_{s+1}$. If $c^\ell < c^\star$, then set $c^\star = c^\ell$, $\hat{x}^+ = \hat{x}^\ell$, and $b^\star = b^\ell$.
2) If $\mathscr{M}_{s+1}$ is empty the algorithm terminates; otherwise set $s = s+1$ and go to Step 1.

Since the algorithm considers all b's for which $J_b^+ \geq J_l$, the final $b^\star$ and $\hat{x}^+$ achieve the global minimum.

Summary
- Expands the full feasible set
- Finds the globally minimum risk feasible subset of measurements

Drawbacks:
- The number of feasible vectors is $O(2^s)$ where s < m
- Computationally prohibitive for large m.

Computationally Feasible Implementation

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Select measurements based on prior:

$$\min_b \left[\| \sum_{i=1}^{m} \frac{b_i}{\sigma_i} e_i e_i^\top (h(x_k^-) - z_k) \|^2 \right]$$

$$\text{subject to: } \left(\sum_{i=1}^{m} \frac{b_i}{\sigma_i^2} h_i^\top h_i + J_k^- \right) \geq J_l$$

$$b_i \in \{0,1\} \text{ for } i = 1, \dots, m,$$

where h_i is the i^{th} row of $H = \nabla h(x)|_{x=x_k^-}$

Diagonal Performance Specification:

$$NPD: \min_{x,\,b} \| \sum_{i=1}^{m} \frac{b_i}{\sigma_i} e_i e_i^\top (h(x_k^-) - z_k) \|^2$$

$$\text{subject to: } \sum_{i=1}^{m} \frac{b_i}{\sigma_i^2} \begin{bmatrix} h_{i1}^2 \\ \vdots \\ h_{in}^2 \end{bmatrix} + diag(J_k^-) \geq J_d$$

$$b_i \in \{0,1\} \text{ for } i = 1, \dots, m.$$

Algorithm 1: Greedy Search for b_0

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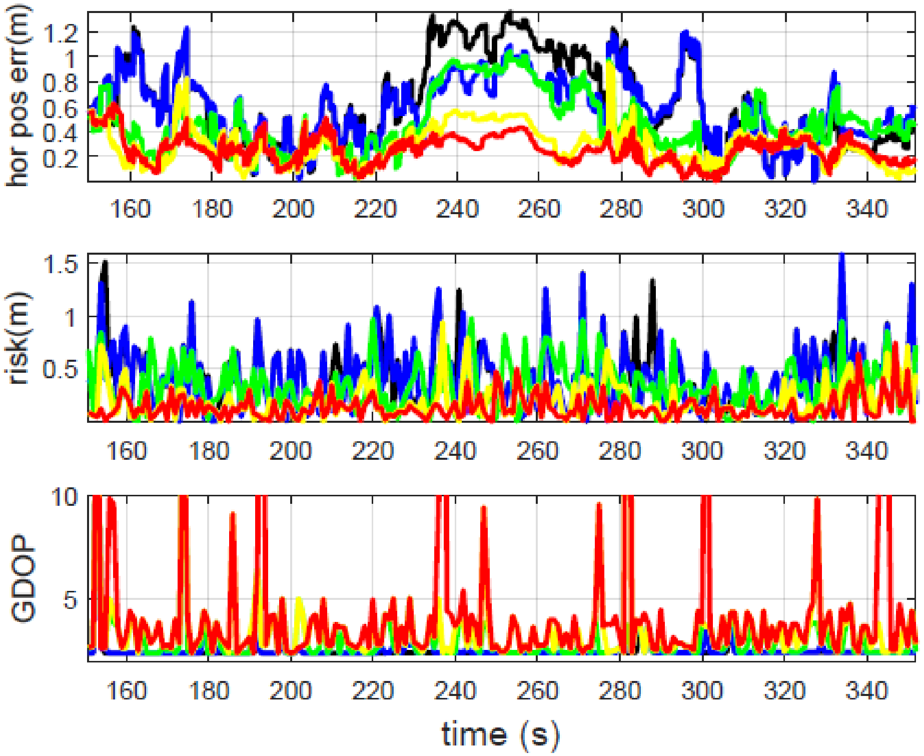
1) *Definitions.*
Let ℓ be the number of selected measurements. The set $\mathbb{S}$ contains the indices of deactivated bits in b_0 (i.e., $i \in \mathbb{S}$ means the i^{th} element of b_0 is 0). Let $H = \nabla h(x)|_{x=x_k^-}$.
2) *Initialization.*
Initialize $\ell = 0$, $\mathbb{S} = \{1, \cdots, m\}$, $b_0 = 0 \in \mathbb{R}^m$, and vector $J_p = diag(J_k^-) - J_d$.
3) *Choose the next measurement.*
Let $[J_p]_j$ be the minimum element in J_p. Activate a non-active bit of b_0 to maximize the added information to $[J_p]_j$:

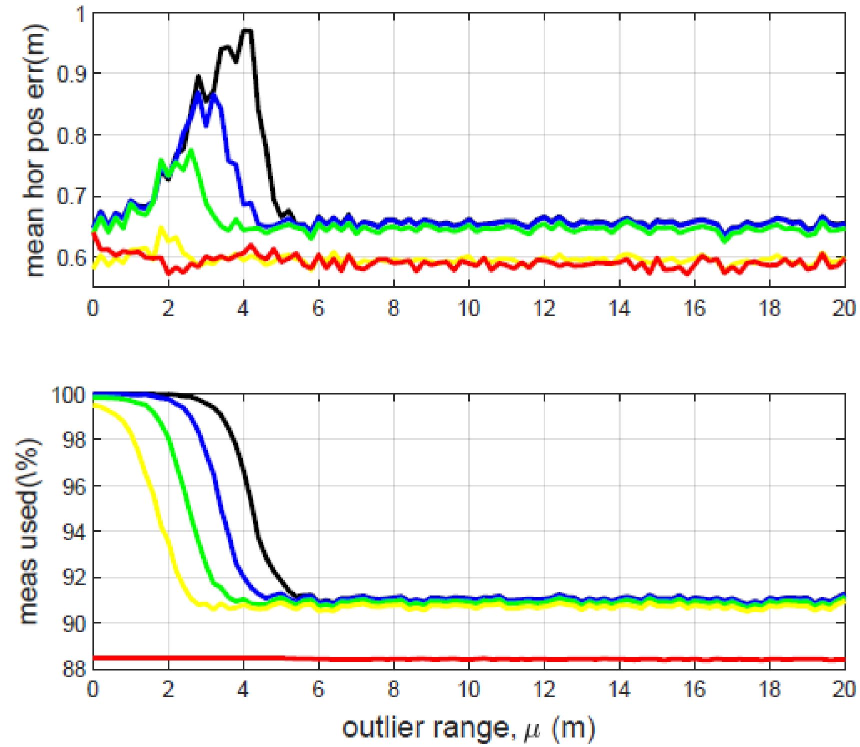
$$i = \operatorname*{argmax}_{i \in \mathbb{S}} \left(\frac{h_{ij}}{\sigma_i} \right)^2. \qquad (47)$$

4) *Update.*
$J_p = J_p + \frac{1}{\sigma_i^2} diag(h_i^\top h_i)$, $S^{\ell+1} = S^\ell \setminus i$, $b_0 = b_0 \oplus e_i$, and $\ell = \ell + 1$. If $J_p < 0_n$, go to Step 3. Otherwise stop.

GNSS Aided INS Experimental Results



Horizontal error, risk, and information diversity (i.e., GDOP) for μ = 2



Mean horizontal position error and the percentage of selected measurements versus mean outlier magnitude.

The yellow, green, blue and black curves display the results for NP-EKF with γ =2, 3, 4, and 5, respectively.

The red curve shows the results for RAPS.

GNSS Aided INS Experimental Results

Algorithm Comparison for $\mu = 2$ (top) and $\mu = 7$ (bottom)

Methods	Mean of error (m)	Std. of error (m)	Sub-meter accuracy	Maximum error (m)
NP-EKF1 $\gamma = 5$	0.63	0.35	0.77	1.35
NP-EKF1 $\gamma = 4$	0.58	0.26	0.92	1.23
NP-EKF1 $\gamma = 3$	0.45	0.23	0.99	1.04
NP-EKF1 $\gamma = 2$	0.28	0.15	1	0.94
RAPS1	0.24	0.10	1	0.62
NP-EKF2 $\gamma = 5$	0.35	0.19	0.99	1.34
NP-EKF2 $\gamma = 4$	0.35	0.19	0.99	1.34
NP-EKF2 $\gamma = 3$	0.34	0.18	1	0.90
NP-EKF2 $\gamma = 2$	0.30	0.15	1	0.89
RAPS2	0.27	0.10	1	0.59

%

: GNSS-INS Vertical Performance Statistics For $\mu = 2$ (top) and $\mu = 7$ (bottom).

Methods	Mean of error (m)	Std. of error (m)	Error < 2 m	Maximum error (m)
NP-EKF1 $\gamma = 5$	1.89	0.52	0.62	3.46
NP-EKF1 $\gamma = 4$	1.92	0.51	0.61	3.46
NP-EKF1 $\gamma = 3$	1.42	0.61	0.83	2.90
NP-EKF1 $\gamma = 2$	0.59	0.47	1	1.85
RAPS1	0.37	0.39	1	1.63
NP-EKF2 $\gamma = 5$	0.39	0.36	1	1.86
NP-EKF2 $\gamma = 4$	0.39	0.36	1	1.86
NP-EKF2 $\gamma = 3$	0.38	0.35	1	1.74
NP-EKF2 $\gamma = 2$	0.36	0.37	1	1.70
RAPS2	0.39	0.40	1	1.82

%

- All results are post-processed filters (not smoothers)
- Error computed relative to “ground truth”, which is a full trajectory smoothed estimate based on integer-resolved carrier phase measurements.
- Real-time implementations are being investigated.

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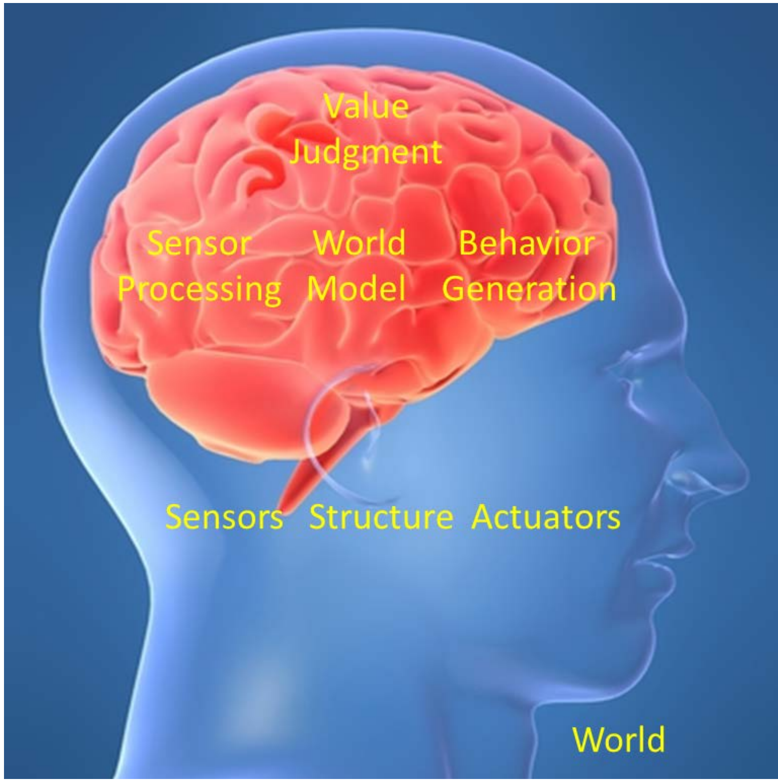

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