Entropy as a Unified Measure To Evaluate Autonomous Functionality of Hierarchical Systems

Dr. Ing. Kimon P. Valavanis
John Evans Professor, Director, Research and Innovation
D. F. Ritchie School of Engineering & Computer Science
kimon.valavanis@du.edu

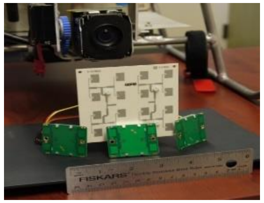
October 27, 2018

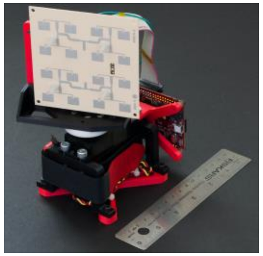
The ‘History’ – Intelligent Control

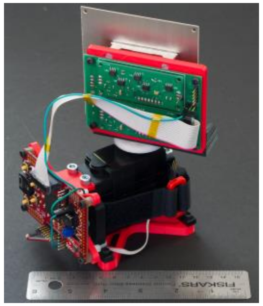
- **Foundations of classical control – 1950’s**
- **Adaptive and learning control – 1960’s**
- **Self-organizing control – 1970’s**
- **Intelligent control -1980’s**
- **K. S. Fu (Purdue) - 1970’s coins the term ‘intelligent control’**
- **G. N. Saridis (Purdue) introduces ‘hierarchically intelligent control systems’ (PhDs: J. Graham, H. Stephanou, S. Lee)**
- **The 1980’s**
  - **J. Albus (NBS, then NIST)**
  - **Antsaklis – Passino**
  - **Meystel**
  - **Ozguner – Acar**
  - **Saridis – Valavanis**

Common theme: multi-level/layer architectures; time-based and event-based considerations; mathematical approaches

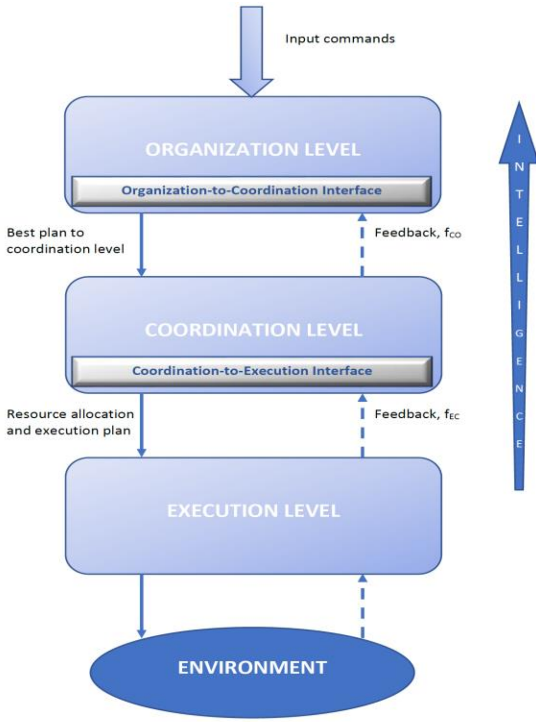
Common limitation: lack of computational power (very crucial)

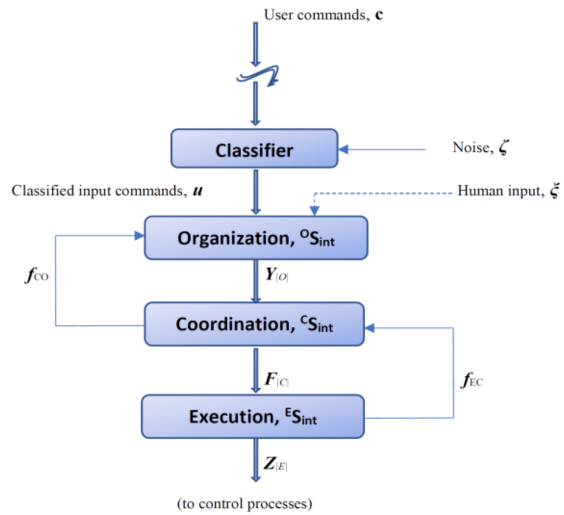


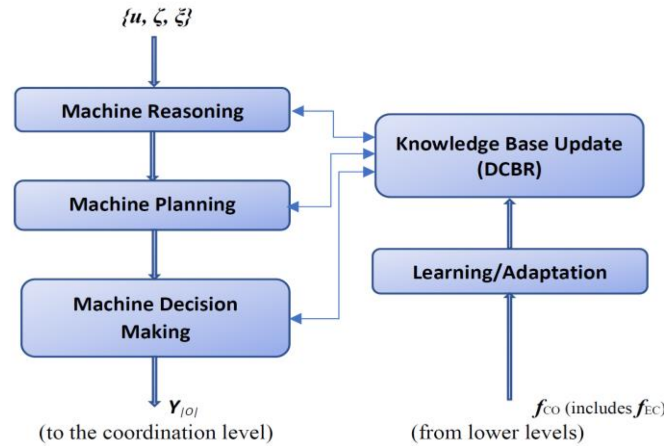




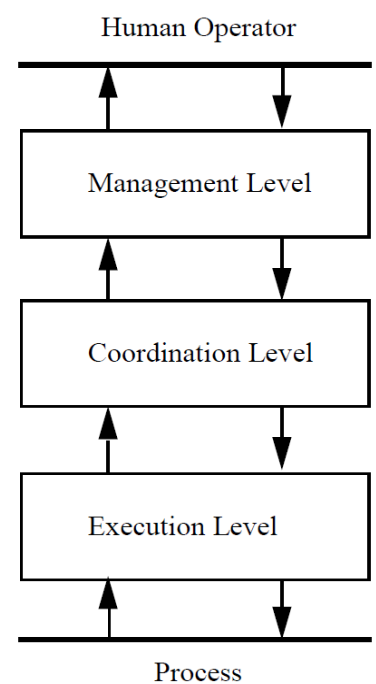
Hierarchical Architecture (Saridis – Valavanis)







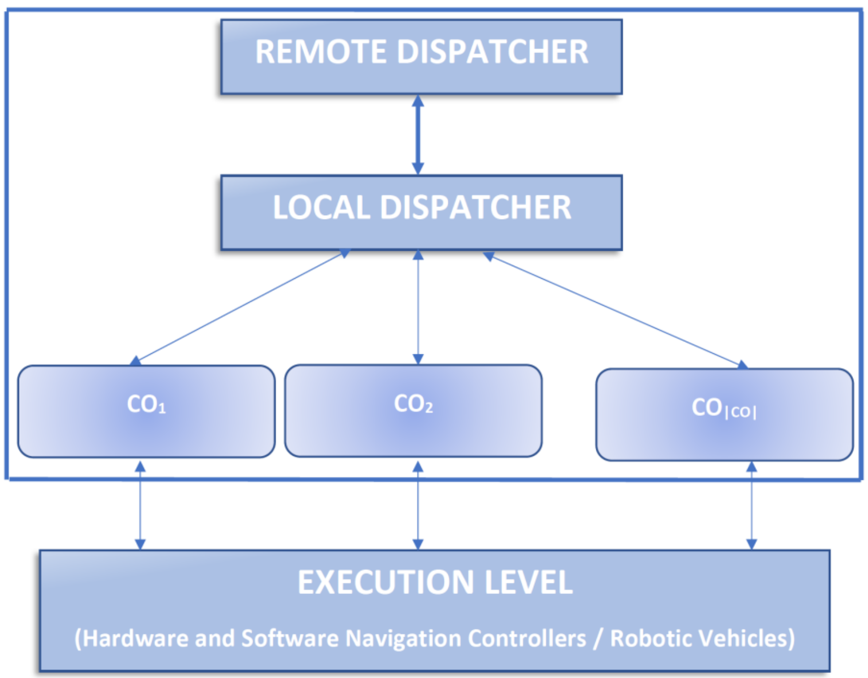
Antsaklis - Passino

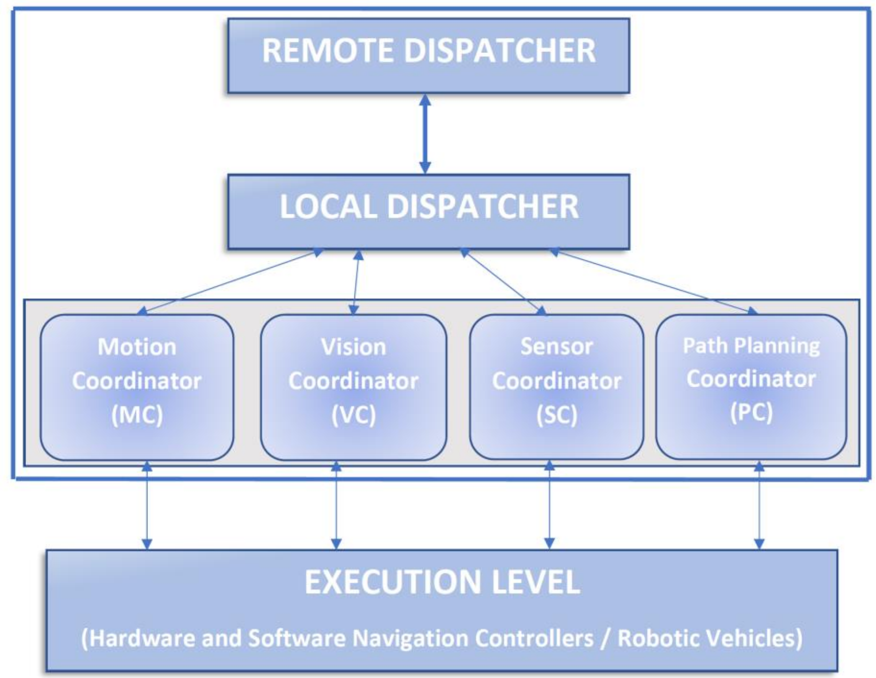


Functionality – One Framework

- Modular
- Spatio-temporal
- Explicit human interaction modeling
- Event-based and Time-based
- On-line / Off-line components
- Vertical/horizontal functionality
- Independent of specific methodologies used for implementation

Coordination Level



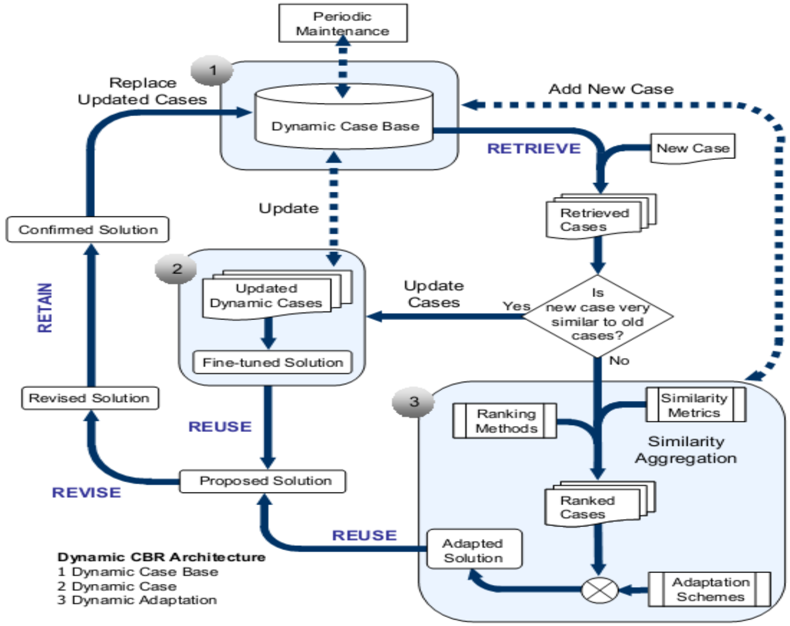


$$p(k+1/u_i) = p(k/u_i) + \beta_{i+1}[\xi - p(t/u_i)]$$

$$J(k+1/u_i) = J(k/u_i) + \gamma_{i+1}[J_{obs}(k+1/u_i) - J(k/u_i)]$$

← **Learning**

Adaptation/Learning (Vachtsevanos et al, 30 years later….)



Ent_e is a new case, Ent_j represents previous cases; El_i is a feature; $n_{i,pert}$ is a pertinence weighted variable associated with the description element El_i; $n_{i,pred}$ is a predictive weighted variable associated with each case in memory, which is increased as the corresponding element (feature) is favorably selecting a case, and decreased as this selection leads to a failure; α is an adjustable parameter. Incremental learning will occur whenever a new case is processed, and its results are identified.

$$sim(Ent_e, Ent_j) = \frac{\sum_{k=1}^{n} \alpha \times sim(El_{i,k}, El_{l,k}) + \sum_{k=1}^{n} n_{k_i,pred} \times n_{i,pert} \times sim(El_{i,k}, El_{l,k})}{\alpha \times n + \sum_{k=1}^{n} n_{k_i,pred} \times n_{i,pert}}$$

Incremental learning will be pursued using Q-Learning, a popular reinforcement learning scheme for agents learning to behave in a game-like environment. Q-Learning is highly adaptive for on-line learning since it can easily incorporate new data as part of its stored database.

Advantage: COMPUTATIONAL POWER!!!

…35 years later
(Lin–Antsaklis–Valavanis– Rutherford)

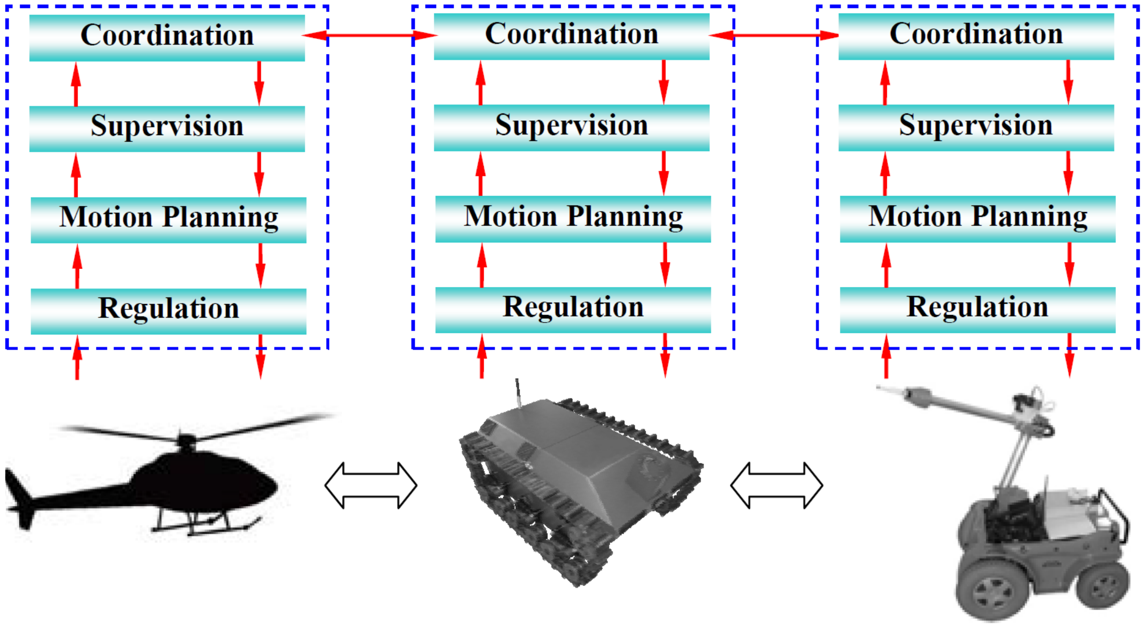


Figure 1: Hybrid hierarchical control architecture for multi-robot systems. The red arrows between layers represent the information flow and feedback between layers. The red arrows between coordination layers of two robots stand for communication between robots, while the physical interactions and passive reactions are denoted as arrows between physical robots.

Advantage: COMPUTATIONAL POWER!!!

2012: Challenge of Autonomy

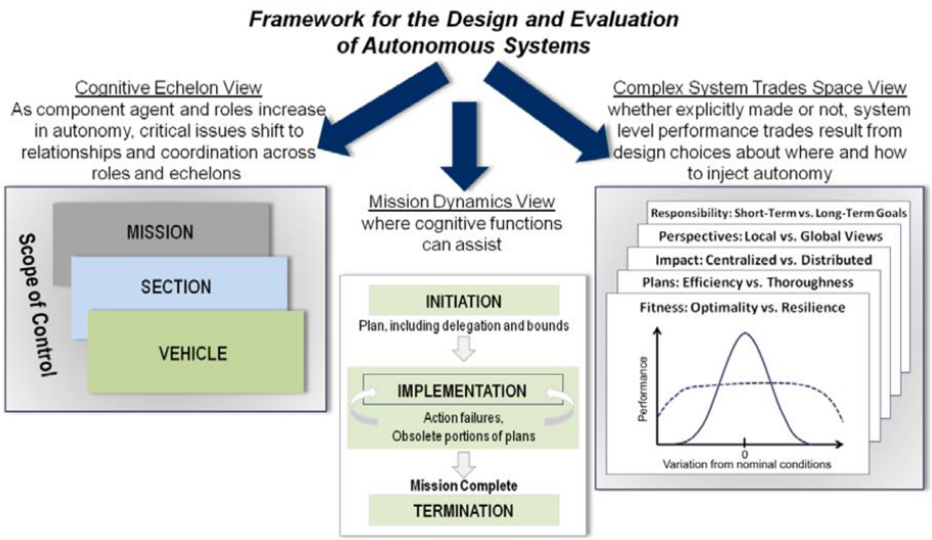


Figure 1-1 Framework for the Design and Evaluation of Autonomous Systems

U.S. DoD

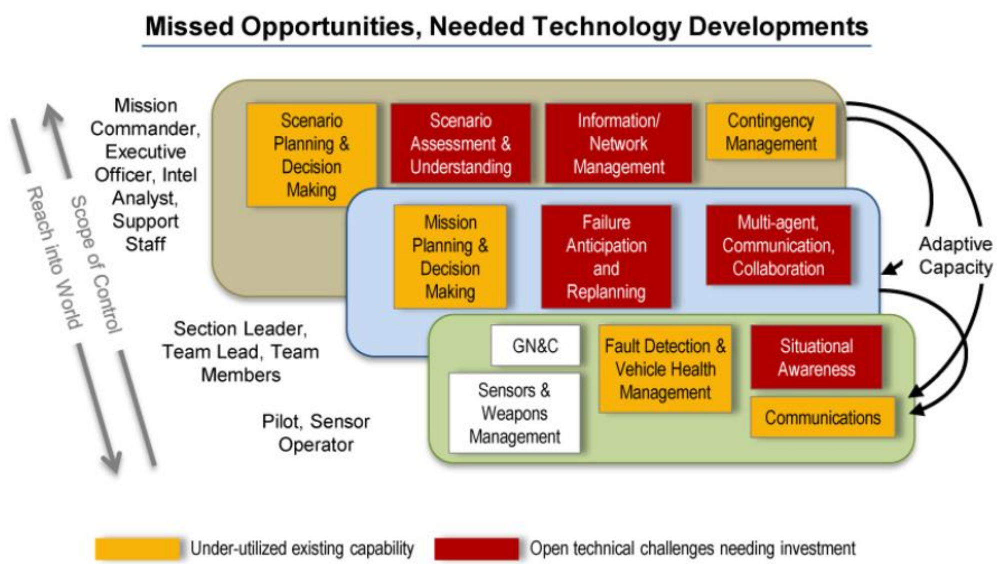


Figure 1-3 Status of Technology Deployment and Remaining Challenges

Why Entropy?

- Duality of the concept of Entropy
  - Measure of uncertainty as defined in Information Theory (Shannon). Measures throughput, blockage, internal decision making, coordination, noise, human involvement etc., of data / information flow in any (unmanned) system. Minimization of uncertainty corresponds to maximization of autonomy / intelligence.
  - Control performance measure, suitable to measure and evaluate precision of task execution (optimal control, stochastic optimal control, adaptive control formulations)
  - Entropy measure is INVARIANT to transformations – major plus

- Deviation from 'optimal' is expressed as cross-Entropy and shows autonomy robustness / resilience
- Additive properties
- Accounting for event-based and time-based functionality
- Horizontal and vertical measure
- Suitable for component, individual layer, overall system evaluation
- Independent of specific methodologies used for implementation
- One measure fits all!

Metrics to evaluate Autonomy/Intelligence (Vachtsevanos – Valavanis – Antsaklis)

- **Performance and Effectiveness metrics**
  - **Confidence (expressed as reliability measure, probabilistic metric)**
  - **Risk is interpreted via a 'value at risk level', which is indicative of not nominal situation, i.e., fault, failure, etc.**
  - **Trust and trust consensus are evaluated through Entropic measures indicating precision of execution, deviation from optimal, information propagation, etc.**
  - **Remaining Useful Life (RUL) of system components, sub-systems**
  - **Probabilistic measure of resilience (PMR) - to quantify the probability of a given system being resilient to forecasted environmental conditions, denoting the ratio of integrated real performance over the targeted one – thus, expressed as Entropy, too**

$$R(T) = \int_0^T P_R(t)dt \Big/ \int_0^T P_T(t)dt$$

Entropy for control (Saridis – Valavanis)

Boltzmann (*theory of statistical thermodynamics)* defined the Entropy, S, of a perfect gas changing states isothermally at temperature T in terms of the Gibbs energy ψ, the total energy of the system H and Boltzmann's universal constant k, as

$$S = -k\int_x \{(\psi - H)/kT\}\, e^{(\psi-H)/kT} dx \qquad S = -k\int_x p(x) \ln p(x) dx \qquad p(x) = e^{(\psi-H)/kT}$$

When applying dynamical theory of thermodynamics on the aggregate of the molecules of a perfect gas, an average Langangian, I, may be defined to describe the performance over time of the state x of the gas

$$I = \int L(x, t) dt$$

$S = -k\int_x \{(\psi-H)/kT\} e^{(\psi-H)/kT} dx$, $I = \int L(x, t) dt$ are equivalent, which leads to $S = I/T$

with T the constant temperature of the isothermal process of a perfect gas.

Entropy for control, cont…

Express performance measure of a control problem in terms of Entropy: for example, consider the optimal feedback deterministic control problem with accessible states for the *n*-dimensional dynamic system with state vector $x(t)$, $dx/dt = f(x, u, t)$, with initial conditions $x(t_o)=x_o$, and cost function $V(u, x_o, t_o) = \int L(x, u, t)dt$, where the integral is defined over $[t_o, T]$, and with $u(x, t)$ the *m*-dimensional control law. An optimal control $u^*(x, t)$ minimizes the cost $V(u^*; x_o, t_o) = \min_u \int L(x, u, t)dt$ with the integral defined over $[t_o, T]$. Saridis proposed to define the differential Entropy for some $u(x, t)$ as

$$H(x_o, u(x, t), p(u)) = H(u) = - \int_{\Omega u}\int_{\Omega x} p(x_o, u) \ln p(x_o, u) dx_o du$$

where the integrals are defined over Ω_u and Ω_x, and found necessary and sufficient conditions to minimize $V(u(x, t), x_o, t_o)$ by minimizing the differential Entropy $H(u, p(u))$ where $p(u)$ is the worst Entropy density as defined by Jayne's Maximum Entropy Principle [104, 105].

By selecting the worst-case distribution satisfying Jaynes' Maximum Entropy Principle, the performance criterion of the control is associated with the Entropy of selecting a certain control law." Minimization of the differential Entropy results in the optimal control solution.

Entropy in general - duality

$H(X) = -\sum_x p(x) log p(x)$ *or* $H(X) = \int f(x) ln f(x) dx$

Conditional Entropies

$$H_Y(X) = -\sum_{x,y} p(x, y) log p(x/y) = -\sum_y p(y) \sum_x p(x/y) log p(x/y) \qquad (9)$$

Transmission of information

$$T(X : Y) = H(X) + H(Y) - H(X, Y) = H(X) - H_Y(X) = H(Y) - H_X(Y)$$

Entropy – Intelligence and Robust Intelligence

Entropy Interval = $H_{max} - H_{min}$ Kullback-Leibler (K-L) measure of cross-Entropy (1951) and Kullback's (1959) minimum directed divergence or minimum cross-Entropy principle, MinxEnt

Human intervention introduced mathematically via additional probabilistic constraints, for example p_i, $i=1, 2, 3\ldots, n$, $\sum p_i=1$, and $\sum c_i p_i=c$, c_i*'s* are weights and c a bound, which are imposed on (unconstraint) probability distributions and influence/alter the $H_{max} - H_{min}$ interval.

$\boldsymbol{p} = (p_1, p_2\ldots, p_n)$ and $\boldsymbol{q} = (q_1, q_2, \ldots, q_n)$ may be measured (and evaluated) via the K-L measure $D(\boldsymbol{p}:\boldsymbol{q}) = \sum p_i ln(p_i/q_i)$. For example, when $\boldsymbol{q}$ is the uniform distribution (indicating maximum uncertainty), then $D(\boldsymbol{p}:\boldsymbol{q}) = lnn\text{-}H(\boldsymbol{p})$ where $H(\boldsymbol{p})$ is Shannon's Entropy.

Under this information theory related approach, which connects Entropy with the event-based attributes of multi-level systems, the system starts from a state of maximum uncertainty and through adaptation and learning, uncertainty is reduced as a function of accumulated and acquired knowledge and information over time.

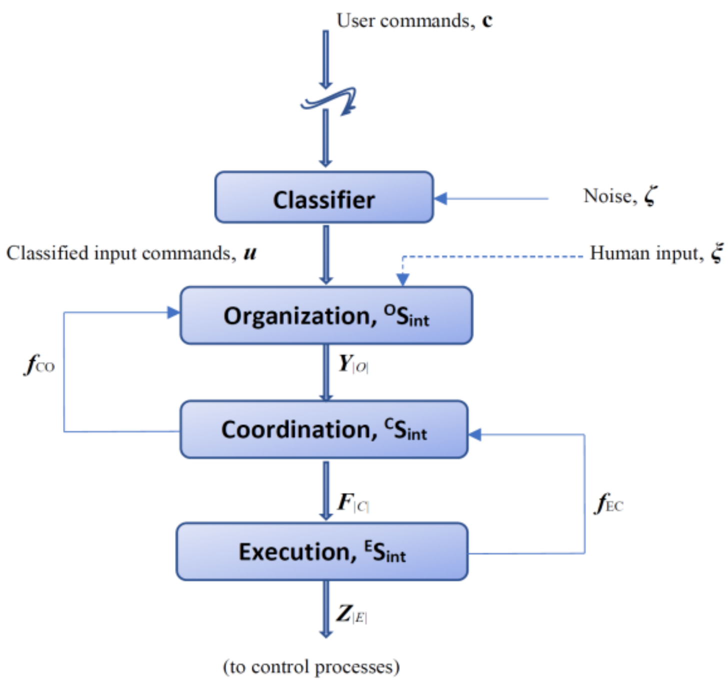
Entropy for control, cont….

$DS = \{S_O, S_C, S_E\}$ - $S_O = \{\boldsymbol{u}, \zeta, \xi, \boldsymbol{f}_{\mathrm{CO}}, {}^{\mathbf{O}}\mathbf{S}_{\mathbf{int}}, \boldsymbol{Y}_{|O|}\}$ - $S_C = \{\boldsymbol{Y}_{|O|}, \boldsymbol{f}_{\mathrm{EC}}, {}^{\mathbf{C}}\mathbf{S}_{\mathbf{int}}, \boldsymbol{F}_{|C|}\}$

$S_E = \{\boldsymbol{F}_{|C|}, {}^{\mathbf{E}}\mathbf{S}_{\mathbf{int}}, \boldsymbol{Z}_{|E|}\}$

$DS = \{S_O, S_C, S_E\} = \{\boldsymbol{u}, \zeta, \xi, \boldsymbol{f}_{\mathrm{CO}}, \boldsymbol{f}_{\mathrm{EC}}, {}^{\mathbf{O}}\mathbf{S}_{\mathbf{int}}, {}^{\mathbf{C}}\mathbf{S}_{\mathbf{int}}, {}^{\mathbf{E}}\mathbf{S}_{\mathbf{int}}, \boldsymbol{Z}_{|E|}\}$

Augmented input is $\boldsymbol{U} = \{\boldsymbol{u}, \zeta, \xi\}$, internal variables are $S_i = \{\boldsymbol{f}_{\mathrm{CO}}, \boldsymbol{f}_{\mathrm{EC}}, {}^{\mathbf{O}}\mathbf{S}_{\mathbf{int}}, {}^{\mathbf{C}}\mathbf{S}_{\mathbf{int}}, {}^{\mathbf{E}}\mathbf{S}_{\mathbf{int}}\}$ and the output is $\boldsymbol{Z}_{|E|}$.



GPLIR considers external and internal noise; internal control strategies and internal coordination of the levels and between the levels to execute the requested mission

GPLIR may be derived for each top-down and bottom-up function of the organizer

GPLIR is also derived for the coordination and execution levels.

THANK YOU

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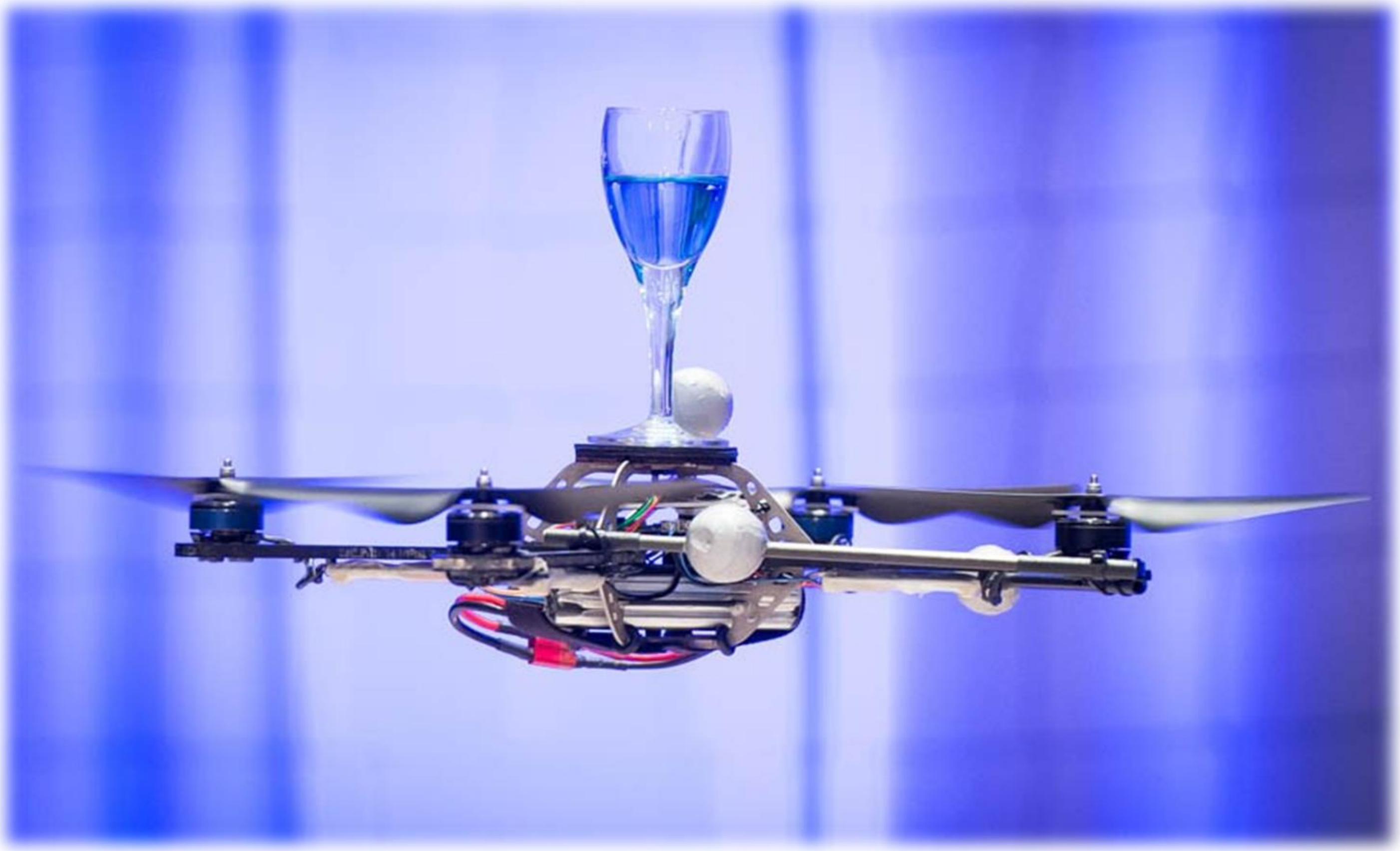

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