ACMS 60850: Applied Probability, Midterm Test
Oct 7, 2015.

Instructor: Bei Hu **Name:**______________________________________

There is a total of 100 points on this exam plus a possible 5 bonus points. This is a 75 minutes test.

1. (10 points) State precisely

(a) The definition of mass function $f(x)$ of a discrete random variable.

(b) The definition of covariance of random variables X and Y.

2. (20 points) Determine whether each of the following statements is true or false. No justification is needed.

(a) If the events A and B are disjoint, then the events A and B must also be independent.

True ___ False ___

(b) For any random variables X and Y, we always have $E(XY) = E(X) \cdot E(Y)$.

True ___ False ___

(c) A random variable X must either be continuous or discrete.

True ___ False ___

(d) Suppose that $F(x) = P(X \leq x)$ is the distribution of the random variable X. Then it is always true that $F(x)$ is right-continuous, i.e., $F(x+h) \to F(x)$ as $h \searrow 0$ $(h > 0)$.

True ___ False ___

3. (15 points) Factories A and B produce toy drones. Factory A produces 3 times as many drones as factory B (i.e, for every 4 drones, 3 from A and 1 from B). A drone is considered defective if it cannot stay in the air for specified amount of time. The probability that a drone produced by factory A is defective is 0.1 and the probability that a drone produced by factory B is defective is 0.05. Show all your work.

(a) A drone is selected at random. What is the probability that it is defective?

(b) A drone is selected at random and is found to be defective. What is the probability it came from factory A?

4. (15 points) A daycare center with 100 children.

(a) Let's make the assumptions mathematically rigorous. Assume $P(B) = P(G) = \frac{1}{2}$, assume also that conceiving a baby of any gender is independent of any other conceptions, assume further each conception resulted exactly one child either a boy or a girl. What is the expected number of boys in this daycare center? Show all your work.

(b) Continued from (a). What is the probability that this daycare center has exactly 50 girls? (Please give your answers. Do not simplify). Show all your work.

5. (15 points)

Let X and Y be independent discrete random variables with mass function $f_X(x)$ and $f_Y(x)$. Find the following in terms of f_X and f_Y.

(a) $P(\min\{X, Y\} \leq x)$.

(b) $P(X = Y)$.

(c) $P(X + Y = x)$.

6. (15 points) (Single sided absorbing barrier). Consider the simple random walk: $S_n = S_0 + \sum_{i=1}^{n} X_i$, where X_i are independent, and takes the value -1 and 1. Assume that $P(X_i = 1) = p$, $\quad P(X_i = -1) = 1 - p = q$, $p > 0, 1 - p = q > 0$. The stopping rule is the site 0 is hit.

(a) Let J_k be the probability of hitting 0 at the stopping time while starting at site k ($S_0 = k$). Find the equation for J_k for $k \geq 1$. Obviously $J_0 = 1$.

(b) Assuming $p \neq q$. Find the general solution for J_k containing two constants.

(c) We only have one boundary condition $J_0 = 1$. In the case $p > q$, determine two constants found in the general solution in (b).

(d) (Extra 5 bonus points - complete only if you have time) In the case $p < q$, determine two constants found in the general solution in (b).

7. (10 points) Find the constant C so that $f(x) = \dfrac{C}{1+x^2}$ is a density function.

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