Terahertz laser frequency combs Supplementary information

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Design of dispersion-compensating corrugations

In order to design the corrugations used for dispersion compensation, one-dimensional simulations were first performed that captured the essential behavior of the structures. The key parameters, shown in Supp. Fig. 1a, are the corrugation length, its start period, its stop period, how quickly its amplitude tapers on (e.g., linearly), and its largest amplitude. To first order, the start and stop periods determine the maximum and minimum frequencies at which dispersion is compensated, while the corrugation length determines the amount of compensation. How quickly the amplitude tapers on determines how strong ripples in the final group delay are, while the largest amplitude determines the bandwidth over which the group delay can remain linear. Full-wave finite element (FEM) simulations were then performed to verify design efficacy, and the compensators' group delay versus frequency were plotted. Supp. Fig. 1b shows two such designs. Even though they differ in their compensation by nearly an order of magnitude, linearity is maintained over the whole design range of 3 THz to 4 THz. Though sidewall-based corrugations were used here, any perturbation that introduces a refractive index change into the waveguide (such as an etched trench or a region of removed metal) can also be used to construct compensators.

To demonstrate the necessity of proper compensation, we plot in Supp. Fig. 1c the RF power generated by three QCLs differing in their dispersion compensation by steps of only 6.7%. When the compensation is detuned from the correct compensation of 1.25 ps^2/mm (+13.3%) even slightly, no RF beatnote is generated and no comb is formed. (At high biases in the devices' negative differential resistance regimes, RF radiation spanning several GHz can be generated, though this is merely due to electrical instability.) Indeed, this highly sensitive dependence on proper dispersion compensation explains why no spontaneous broadband comb formation has ever been reported in THz QCLs to date.

Setup for frequency locking and SWIFTS

Supplementary Figure 2a shows both the setup used for stabilizing the QCL's repetition rate against mechanical vibration of the cryocooler and for homodyne interferometry (SWIFTS). For repetition rate stabilization, the free-running beatnote emanating from the QCL is first observed on a spectrum analyzer, which is typically near 6.8 GHz. An external frequency synthesizer (HP 8673E) is tuned to be 10 MHz away from the free-running signal. The QCL beatnote is then downconverted twice, first to 10 MHz and then to DC, and is used as the error signal for a PI controller. The

output of the PI controller is added to the QCL bias with a 3 $k\Omega$ resistor, and since the QCL's bias affects the refractive index and



Supplementary Figure 1 | Dispersion compensation design. a, Schematic showing the key parameters of the dispersion compensators. **b**, Simulated group delay versus frequency plots for the smallest and the largest compensators developed, respectively. A linear response compensates for second-order dispersion. **c**, RF beatnote emanating from three QCLs with compensations of 1.17 ps²/mm (+6.6%), 1.25 ps²/mm (+13.3%), and 1.32 ps²/mm (+20%).

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therefore repetition rate, this locks the QCL's repetition rate to the frequency of the synthesizer plus 10 MHz. Note that as we are not performing injection locking, the added current only has frequency components up to about 100 kHz, and has an amplitude of less than a milliamp (much smaller than the DC bias of ~ 1 A). Also note that while the HEB has alternatively been used to perform repetition rate stabilization, doing so prevents it from being used in homodyne interferometry. Supp. Figure 2b shows the effect of locking the repetition rate on the beatnote, as measured on the HEB. The inset shows an RF spectrum of an unstabilized laser. Even though the signal-to-noise ratio of the measurement is quite high, 45 dB, its center frequency fluctuates over long time scales as a result of the laser's unstable environment, giving an apparent linewidth of almost 100 kHz. In contrast, when the beatnote is locked to an external local oscillator, its linewidth can be reduced substantially, to only a few Hz or less.

During SWIFT measurements, the signal from the HEB is amplified and downconverted by the reference synthesizer (near 6.8 GHz) to 10 MHz. It is then demodulated using a 10 MHz I/Q demodulator, producing the two components of the RF signal, and



Supplementary Figure 2 | Locking the repetition rate and performing SWIFTS. a, Block diagram showing the repetition rate stabilization and the homodyne interferogram measurement. b, Phase error generated by mixing the locked HEB beatnote with the local oscillator. The FWHM linewidth is 1.95 Hz, and the only major sidebands are 60 Hz harmonics likely arising from clock error. Inset: Unstabilized HEB beatnote measured on an RF spectrum analyzer. Because the beatnote is dominated by long-term frequency fluctuations, a large video bandwidth of 3 MHz was used. The FWHM linewidth is 98 kHz.

measured with a pair of lock-in amplifiers. (The quasi-DC component of the HEB signal is also measured.) All three lock-ins use the same time constant, amplitude, and phase settings, and as the Michelson interferometer's stage is scanned, their signals are simultaneously recorded.

SWIFT spectroscopy analysis

If E(t) is the electric field produced by a light source, the instantaneous intensity signal impinging on a detector after passing through a Michelson interferometer can be expressed as

$$S(t) = \frac{1}{2} (E(t) + E(t - \tau))^2$$

where S(t) is the instantaneous intensity, E(t) is the electric field, and τ is the stage delay. If the electric field is expanded in terms of its Fourier components using the convention that

$$E(t) = \sum_{\omega} E(\omega) e^{i\omega}$$

then the usual interferogram measured at DC can be expressed as the zero-frequency component of S(t), or

$$S_{0}(\tau) = \left\langle (E(t) + E(t - \tau))^{2} \right\rangle = \sum_{\omega > 0} |E(\omega)|^{2} e^{i\omega\tau} + |E(\omega)|^{2} + c.c.$$

To measure a SWIFT spectrum, the signal is instead mixed with a local oscillator, defined here using the convention that $V(t)=2\cos(\Delta\omega t+\phi)$ (where $\Delta\omega$ is its frequency and ϕ is its phase). One can then show that the resulting signal is

$$S_{\Delta\omega,\phi}(\tau) = \sum_{\omega>0} \left[E^*(\omega) E(\omega - \Delta\omega) e^{i\phi} + E^*(\omega) E(\omega + \Delta\omega) e^{-i\phi} \right] e^{i\omega\tau} + \left[E^*(\omega) E(\omega - \Delta\omega) e^{i\phi} \right] (1 + e^{i\Delta\omega\tau}) + c.c.$$

Some comparisons between the homodyne and normal interferograms are in order. Both of course contain terms oscillating at the frequencies the laser produces, and both contain constant terms as well. Interestingly enough, the homodyne interferogram also contains a term oscillating at $e^{i\Delta\omega r}$, the frequency of the laser's repetition rate. This term is clearly visible in the zoomed-out interferograms of Figure 3a. The constant term corresponds to the non-interferometric contribution of the interferometer's fixed arm, while the term oscillating at $\Delta\omega$ corresponds to the non-interferometric contribution of the interferometer's variable arm. In addition, while the components of the normal interferogram are strictly positive (and have zero phase), the homodyne interferograms obey no such constraint. As a result, they can be asymmetric about the zero-path delay.

Since the global group delay is arbitrary, without loss of generality one can assume that $\phi=0$ for the in-phase signal and $\phi=-\pi/2$ for the in-quadrature signal. The positive frequency components are then found to be the following:

$$S_{I}(\omega) = E^{*}(\omega)E(\omega - \Delta\omega) + E^{*}(\omega)E(\omega + \Delta\omega)$$

 $S_{\mathcal{Q}}(\omega) = i(-E^{*}(\omega)E(\omega - \Delta\omega) + E^{*}(\omega)E(\omega + \Delta\omega))$

This in turn implies that

$$E^{*}(\omega)E(\omega \pm \Delta \omega) = \frac{1}{2}(S_{I}(\omega) \mp iS_{Q}(\omega)) \equiv X_{\pm}(\omega)$$

as stated in the text. One way to validate this approach is to note that $X_{+}(\omega)$ and $X_{-}(\omega)$ would ideally not be independent of each other; in fact they should be trivially related by $X_{-}(\omega+\Delta\omega)=X_{+}^{*}(\omega)$. By plotting $X_{+}(\omega)$'s and $X_{-}(\omega)$'s magnitudes and shifting one of them by $\Delta\omega$, we can see if this is the case. Supp. Figure 3 shows the result of this process. The top panel shows the Fourier transforms of the raw I and Q interferograms, which differ quite a bit since there's no reason for them to be the same. In contrast, the computed correlations in the bottom panel would be identical if not for the presence of noise. In fact, they can even be averaged to marginally improve the signal-tonoise ratio of the measurement.

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Supplementary Figure 3 | Consistency of SWIFT data. a, Top panel: raw Fourier transforms of the I and Q interferograms of the data in Fig. 3a. Bottom panel: calculated SWIFT correlation magnitudes, with X. shifted by the repetition rate of the laser. b, The zoomed-in region is plotted on a linear scale.

Finally, note that the phase of the correlation functions represents the phase difference between adjacent comb lines, $\Delta \phi$. In principle, this makes SWIFTS a rival to full-field pulse characterization techniques like FROG¹ and SPIDER². In practice, however, this is difficult since only phase differences are measured and therefore requires a cumulative sum to get the actual phases. The resulting inference is highly subject to noise. However, since group delay can be estimated as $\tau_g \approx \Delta \phi / \Delta \omega$, this means that SWIFTS can still be used to determine the frequency-dependent group delay (modulo the cavity round-trip time).

References

- Trebino, R. et al. Measuring ultrashort laser pulses in the time-frequency domain using frequency-resolved optical gating. Review of Scientific Instruments 68, 3277–3295 (1997).
- Iaconis, C. & Walmsley, I. A. Spectral phase interferometry for direct electric-field reconstruction of ultrashort optical pulses. Opt. Lett. 23, 792–794 (1998).