Frequency comb ptychoscopy: folded spectroscopy of arbitrary sources using combs

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ABSTRACT

The unique structure of frequency combs, light sources whose lines are perfectly evenly-spaced, allows for new avenues in the measurement of optical signals. I will discuss our work on frequency comb ptychoscopy, a radiometric technique that allows the spectrum of passive sources to be measured by many comb lines at once.[1] Because the heterodyne signals from many signals are folded on top of one another at intermediate frequencies, an inversion algorithm is demonstrated that is able to unravel these beatings. This technique combines the resolution and speed of heterodyne spectroscopy with the bandwidth of comb spectroscopy, and has particular promise in the domain of remote sensing.

Keywords: terahertz, mid-infrared, frequency combs, quantum cascade lasers, nonlinear optics

1. INTRODUCTION

Frequency combs are light sources whose lines are perfectly evenly-spaced. Beating the many lines of a comb with a signal to be measured allows one to measure broadband spectra with high sensitivity, combining the features of multiplexed and laser spectroscopy into a single system. For example, dual comb spectroscopy[2–6] can be used to measure the spectrum of another comb, comb-referenced approaches can measure the spectrum of a laser,[7,8] and vernier spectroscopies[9–11] can be used to measure the spectra of multi-line lasers. However, all of these techniques require that the resulting spectra do not overlap at intermediate frequencies (IFs), as overlapping IFs create an unavoidable ambiguity in the spectrum. This restricts their use to coherent spectra, such as lasers. This is a major limitation—most spectra found in nature are incoherent! In applications where the optical signal is broadband and may even be broader than the comb spacing (remote sensing, astronomy, biological systems, etc.), combs can only be used for system calibration. Multiheterodyne techniques are well-suited for measuring the *transmission* of an optical system, but not the *emission* spectrum of a remote source.

This problem is ultimately one of bandwidth. When detecting a complex signal beating with a comb, avoiding ambiguities requires that the total bandwidth of the signal be less than the comb spacing. Any broader, and the beatings will fold onto one another. At the same time, the comb spacing must also be smaller than half the optical detection bandwidth, typically a few gigahertz. Most incoherent optical signals are significantly broader than this limit. Of course, similar bandwidth limitations exist in optical imaging systems, which have limited space-bandwidth products. One particular solution to this problem is to utilize a class of computational methods known as ptychography, which can reconstruct a high-bandwidth image by iteratively stitching together multiple low-bandwidth images.[12–16]

We recently introduced *ptychoscopy*, a high-resolution multiheterodyne technique that is able to construct the spectrum of arbitrary light sources, even broadband sources whose linewidths are *much greater* than the comb spacing.[1] Inspired by ptychography and by the interferometer-based techniques that disambiguate individual comb lines,[17–21] we show that a signal mixing with a comb can be fully disambiguated using a pair of combs, even when multiple signals overlap at the same IF. The essential idea is shown in Figure 1. Two spectrograms are produced, and by processing them with an appropriate inversion algorithm, high-resolution mini-spectra are produced. Each mini-spectrum represent's the source's spectrum around a comb line, but is limited in bandwidth to the detector's bandwidth. These are then composited to produce a high-resolution, broad bandwidth spectrum.

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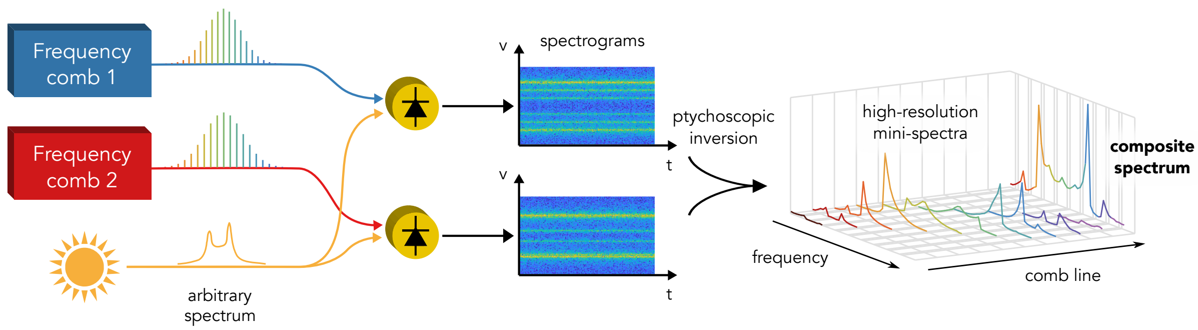

Figure 1. Basic idea of frequency comb ptychoscopy. An arbitrary spectrum is independently beat with two combs, resulting in a pair of spectrograms. By using a ptychoscopic inversion procedure, high-resolution mini-spectra are produced, representing the beating of the source with each comb line. These spectra are limited in bandwidth by photodetection, but their resolution is Fourier-limited. By compositing these spectra, a high-resolution broadband spectrum is produced. Modified from Ref. 1.

As in ptychography,[12–16] all of the spectral information is encoded into a narrow bandwidth by the combs. Each version of this measurement has analogues to Fourier spectroscopy and preserves many of its features, such as the throughput and multiplex advantages.[22,23] Though the approach relies on the comb structure, it does *not* require that the combs have a particular phase profile—the combs can be pulsed[9,11,24,25] or not.[20,26–32] Not only do we present the inversion algorithm and the methods needed to perform ptychoscopy, but we also present a proof-of-concept measurement at microwave frequencies. Provided suitable detectors and combs are available, this concept can in principle be done at any frequency range. It is particularly valuable for longer wavelengths, where heterodyne techniques[3,33,34]are more attractive than direct techniques.

2. RESULTS

2.1 Theory

Ptychoscopy has two implementations, shown in Figure 2. In both versions, the source to be measured is split and is mixed with two combs. In the dual comb version, these two combs come from separate sources. They may or may not be mutually coherent. In the delayed comb version, the second comb is generated from the first by using a variable delay element. Essentially, it is a Doppler-shifted comb.[17] In each case, the detector signals are digitized and processed into complex spectrograms. (This is done by dividing the data into batches and computing a short-time Fourier transform.) The product of the two spectrograms is computed, and the result is then Fourier transformed again to achieve the final result. Even for a fully incoherent source this correlation function is proportional to the power of the signal (offset from nth comb line), which allows it to reconstruct essentially any source. The details of the two versions differ only slightly.

In the dual comb version, the complex spectrograms $F_i(\omega, T)$ are functions of the IF frequency ω and the time of each spectrogram T. The respective position of the nth comb lines as $\omega_n^{(c_1)}$ and $\omega_n^{(c_2)} \equiv \omega_n^{(c_1)} + \Delta_n$, where Δ_n is the separation between corresponding lines, and the respective complex amplitudes are $E_n^{(c_1)}$ and $E_n^{(c_2)}$. By correlating the two spectrograms and computing $C_n(\omega)$, the Fourier transform of the spectrogram correlation along T,

$$C_n(\omega) \equiv \mathcal{F}_T[F_2(\omega - \Delta_n, T)F_1^*(\omega, T)](-\Delta_n). \tag{1}$$

one can show[1] that this function is statistically-related to the spectrum of the source P_s by

$$\langle C_n(\omega)\rangle = E_n^{(c_2)*}E_n^{(c_1)}P_s(\omega_n^{(c_1)} + \omega). \tag{2}$$

In other words, the spectrum near every comb line can be determined simply by dividing out the amplitude of the dual comb beat signal $E_n^{(c_2)*}E_n^{(c_1)}$ and stitching the results together. Even though ω is small when compared

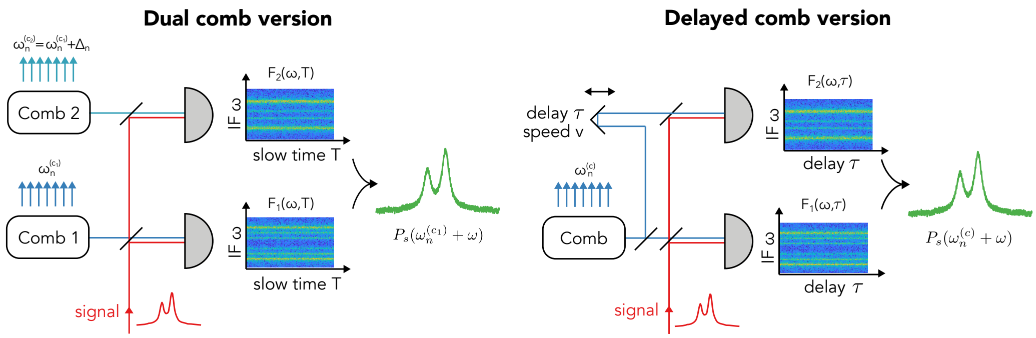

Figure 2. Two approaches to ptychoscopy. In the dual comb version two combs are independently beat with the signal; in the delayed comb version one comb is split and delayed. Both detectors' spectrograms are computed and are correlated to reproduce the original signal's spectrum to high resolution (the measurement time divided by the number of comb lines). Modified from Ref. 1.

with the total bandwidth of the signal, the spectrum can be measured anywhere comb lines are present. This result holds for both positive and negative IF frequencies as well as overlapping IF frequencies, allowing for complete disambiguation of the signal. The delayed comb version is similar.

These approaches are extremely general, reconstructing the source in practically all cases. The sole situation in which they will not correctly reproduce the spectrum of the signal is when there exist frequencies for which $\langle E_s(\omega)E_s^*(\omega + n\omega_r)\rangle \neq 0$ over the duration of the measurement (where ω_r is the repetition rate of a comb and n is an integer). Over sufficiently long timescales, this will only fail when the source under consideration is deliberately chosen to match the combs' repetition rates, for example by attempting to measure another comb with the same spacing. The approach is also general for all types of combs, irrespective of the phase of the comb lines.

2.2 Experimental results

To prove our concept, we present ptychoscopic reconstruction of microwave signals, using the dual comb version outlined in Figure 2. Note that nothing in our approach relies on the spectral range of interest or on the type of source or detector. In order to correct for systemic experimental errors, several calibrations and corrections must be conducted. First, an IF calibration is done to account for the detector's response. A broad flat spectrum spanning multiple comb lines is used to generate a flat IF spectrum, and any deviation from flatness is attributed to the detector response and is corrected. Secondly, the signals are phase-corrected using the method described in Ref. 1. Lastly, the signal was normalized to correct for comb line power variations.

For the first measurement, we use combs of repetition rates 5 MHz and 4.997 MHz, respectively, and detect them with a bandwidth of 2.5 MHz. We generate our spectrum by combining 4 different single-frequency synthesizers, and the result is a simulacrum of the previous terahertz simulations. The lines are located at, 1006.5, 1229.7, 1772.8, and 2021.5 MHz, and our spectrum spans 203 comb lines and is octave-spanning. In addition, the first and fourth comb line were chosen to both be offset from their nearest comb line by 1.5 MHz. Their successful resolution illustrates the ability of our technique to differentiate signals that overlap in IF. The results of this measurement are reported in Figure 3. The signal was generated using a 40 ms acquisition time with a resolution of 6.8 kHz. Both the reconstructed signal and the ground truth measured via an RF spectrum analyzer are shown. While there are some discrepancies in the exact height and shape of the peaks due to differences in the resolution of the two measurements, the power of each line is correctly recovered, giving respective errors of 2.1, 6.0, 3.6, and 13.8 %. (The higher error of the last measurement is due to the weakness of the comb sources at 2 GHz.)

While the previous reconstruction was correct, because it consisted of a few discrete lines it could have been inverted using vernier techniques.[11] It does not display the full power of ptychoscopy. To do this, we consider a highly degenerate spectrum where the intermediate frequencies overlap multiple times. For this measurement,

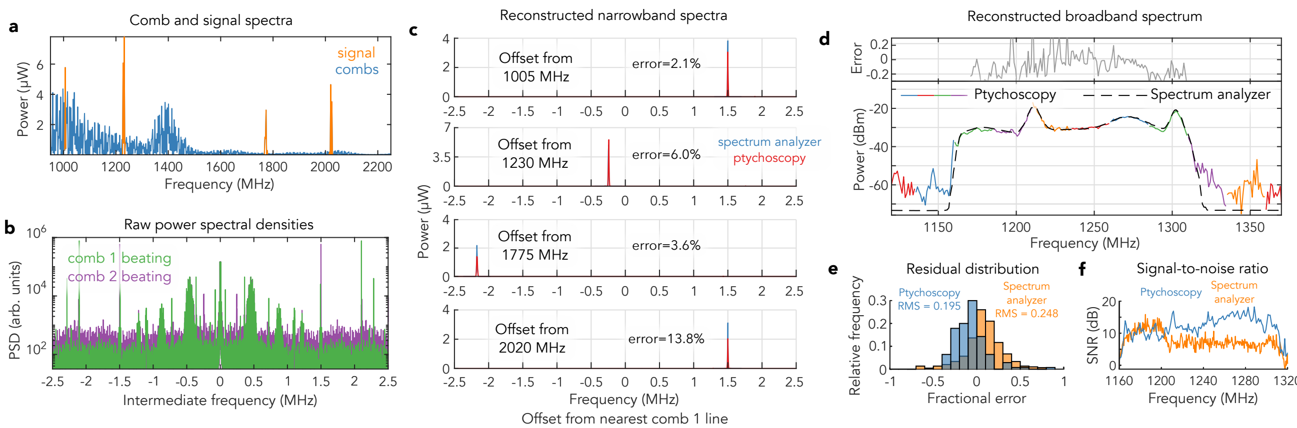

Figure 3. a. Spectrum of the signal that is to be measured, along with the spectrum of combs' lines. b. Raw power spectral density recorded on the two detectors, showing a complex mix of spectra. c. Ptychoscopic reconstruction of data and of real signals, along with the error of the integrated power. Note that the two methods have different instrument lineshapes and resolution bandwidths—this results in different peak heights, but the integrated powers are similar (see errors). d. Spectral reconstruction of a broadband signal by ptychoscopy. The multi-color curve is the reconstruction where each color corresponds to a different comb line. The dashed line represents the ground truth, as measured on an RF spectrum analyzer (averaged). e. Comparison of the distribution of residuals between the ptychoscopic measurement (over the valid range) and the spectrum analyzer measurement when the measurements are taken with the same RBW and measurement time. f. Comparison of the two signal-to-noise ratios (SNRs). The ptychoscopic measurement is better on account of the non-ergodicity of the incoherent waveform. Modified from Ref. 1.

we utilize combs with repetition rates of 25 MHz and 24.95 MHz, respectively. We chose a frequency-modulated (FM) voltage controlled oscillator (VCO) as the input signal. In order to generate an interesting incoherent spectrum, we FM modulate the VCO using a random noise voltage with a desired distribution. This random modulation is chosen to generate an incoherent spectrum that spans multiple comb lines, approximately eight repetition rates wide, and consists of both broad and narrow features (Figure 3d). First, the spectrum is measured with a spectrum analyzer (100 averages, 10 ms acquisition time, 1 MHz resolution bandwidth, resulting in 25 points per repetition rate). Ptychoscopy is performed on this signal, but the raw spectra are essentially flat and do not resemble the true spectrum in any way; they also have poor signal-to-noise ratio (under 5 dB). This is due to the fact that the underlying signals are much broader than the comb repetition rate. Nevertheless, using this data and following the reconstruction procedure yields a spectrum that agrees well with the ground truth (Fig. 3d). The spectrum has also been accurately reconstructed over eight repetition rates, despite the fact that we are only using information up to half the combs' repetition rates (12.5 MHz).

One can compare the signal-to-noise ratio (SNR) of ptychoscopy with that of a conventional RF spectrum analyzer (Figure 3e and 3e). One may expect that because the spectrum analyzer utilizes a tunable LO, its SNR would be superior. However, one finds that for the same resolution bandwidth (1 MHz) and measurement time (10 ms), the ptychoscopic reconstruction is actually superior—having less root-mean-square (RMS) error, thinner tails, and much higher SNR. This is ultimately because the VCO is not a true stationary incoherent source. It switches frequency randomly every 4 μs, and the result of this is that single sweeps of the spectrum analyzer produce highly variable results. Ptychoscopy, by contrast, lacks this problem entirely. It is capable of accurately resolving spectra even in single-shot mode, as all of the signal is always recorded.

I will discuss this result and discuss the prospects for passive high-speed spectroscopy of essentially any dynamically-varying electromagnetic source. One can imagine applications in reaction kinetics,[35] in biology,[36,37] pharmaceuticals,[34] or millimeter-wave systems.[38]

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