Math 20580 Name:________________________

Midterm 1 Instructor:________________________

September 19, 2017 Section:________________________

Calculators are NOT allowed. Do not remove this answer page – you will return the whole exam. You will be allowed 75 minutes to do the test. You may leave earlier if you are finished.

There are 8 multiple choice questions worth 7 points each and 4 partial credit questions each worth 11 points. Record your answers by placing an $\times$ through one letter for each problem on this answer sheet.

Sign the pledge. "On my honor, I have neither given nor received unauthorized aid on this Exam":

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1. a b c d e
2. a b c d e
3. a b c d e
4. a b c d e
5. a b c d e
6. a b c d e
7. a b c d e
8. a b c d e

Multiple Choice.

9.

10.

11.

12.

Total.

Part I: Multiple choice questions (7 points each)

1. Consider the linear system

$$\begin{cases} x_1 - 3x_2 = 5 \\ x_2 + x_3 = 0 \end{cases}$$

Which of the following (x_1, x_2, x_3) is a solution of the system?

(a) $(-3, -1, 1)$ and $(2, -1, 1)$ (b) $(-1, -2, 2)$ and $(-3, -1, 1)$

(c) $(2, -1, 1)$ and $(-1, -2, 2)$ (d) $(-5, 0, 0)$ and $(3, 1, -1)$

(e) none of the above

2. For which values of h and k is the matrix below in reduced echelon form?

$$A = \begin{bmatrix} 1 & 2 & h & 1 \\ 0 & 0 & k & -2 \end{bmatrix}$$

(a) $h = 1$ and $k = 0$ (b) $h = 1$ and any k

(c) $k = 1$ and any h (d) $h = 0$ and $k = 1$

(e) none of the above

3. Consider the vectors

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad \vec{v}_2 = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} \qquad \vec{v}_3 = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} \qquad \vec{v}_4 = \begin{bmatrix} 3 \\ 4 \\ 7 \end{bmatrix}$$

Which of the following statements are true?

A. $\{\vec{v}_1, \vec{v}_2\}$ are linearly dependent.

B. $\{\vec{v}_2, \vec{v}_3, \vec{v}_4\}$ are linearly independent.

C. $\vec{v}_4$ is in $\text{Span}\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$.

(a) A,B only (b) A,C only (c) B,C only (d) A,B,C (e) A only

4. Find the product AB where

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 2 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}$$

(a) $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 3 \\ 1 & -1 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & -1 \\ 4 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2 \\ -1 & 1 \\ 2 & 3 \end{bmatrix}$ (e) $\begin{bmatrix} 2 & 4 \\ -1 & 2 \end{bmatrix}$

5. Which of the following matrices is invertible?

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & -1 \\ 2 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 2 \\ 0 & 3 \end{bmatrix}$$

(a) A only (b) A,B,C only (c) A,B only (d) D only (e) B, C only

6. Consider the vectors $\vec{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$, and a linear transformation $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ with the property that

$$T(\vec{u}) = \begin{bmatrix} 4 \\ 2 \end{bmatrix} \qquad \text{and} \qquad T(\vec{v}) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

The image of the vector $\vec{u} + 2\vec{v}$ under the transformation T is

(a) $\begin{bmatrix} 6 \\ 6 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ (c) $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$ (d) $\begin{bmatrix} 9 \\ 6 \end{bmatrix}$ (e) $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$

7. Let $\mathcal{B} = \{\vec{b}_1, \vec{b}_2\}$ be a basis for a subspace of H in $\mathbb{R}^4$ where

$$\vec{b}_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 3 \end{bmatrix} \quad \text{and} \quad \vec{b}_2 = \begin{bmatrix} -2 \\ 2 \\ 3 \\ -1 \end{bmatrix}.$$

If $[\vec{x}]_{\mathcal{B}} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ is the coordinate vector (relative to $\mathcal{B}$) of some element $\vec{x}$ in H then

(a) $\vec{x} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 5 \end{bmatrix}$ (b) $\vec{x}$ is in $\mathbb{R}^2$ (c) $\vec{x} + \vec{b}_2 = 2\vec{b}_1$

(d) $\vec{x}, \vec{b}_1, \vec{b}_2$ are linearly independent (e) none of the above.

8. The ranks of the matrices

$$A = \begin{bmatrix} 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 3 \\ 1 & -1 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 2 & 5 & -3 & -4 & 8 \\ 0 & -3 & 2 & 5 & 7 \\ 0 & 0 & 0 & 4 & -6 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

are given by

(a) $\text{rank}(A) = 2$, $\text{rank}(B) = 3$, $\text{rank}(C) = 3$

(b) $\text{rank}(A) = 1$, $\text{rank}(B) = 2$, $\text{rank}(C) = 3$.

(c) $\text{rank}(A) = 2$, $\text{rank}(B) = 3$, $\text{rank}(C) = 4$.

(d) $\text{rank}(A) = 1$, $\text{rank}(B) = 3$, $\text{rank}(C) = 3$.

(e) $\text{rank}(A) = 2$, $\text{rank}(B) = 2$, $\text{rank}(C) = 3$.

Part II: Partial credit questions (11 points each). Show your work.

9. Consider the vectors

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 2 \\ 1 \\ t \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} 0 \\ 1 \\ -t \end{bmatrix}.$$

Find the values of t for which $\vec{v}_1$ is contained in $\text{Span}\{\vec{v}_2, \vec{v}_3\}$.

10. Consider the linear transformation $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^4$ given by

$$T(x_1, x_2) = \begin{bmatrix} x_1 + x_2 \\ -x_1 \\ 2x_1 + 3x_2 \\ x_1 + 2x_2 \end{bmatrix}.$$

(a) Find the standard matrix of T.

(b) Write down four distinct vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4$ that are in the range of T.

(c) Is the vector $\vec{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ in the range of T?

11. Find the inverse of the matrix

$$A = \begin{bmatrix} 2 & 0 & 5 \\ 0 & 1 & 0 \\ 3 & 0 & 7 \end{bmatrix}$$

12. Find a basis for Col(A) and a basis for Nul(A) where

$$A = \begin{bmatrix} 1 & -2 & 0 & 1 & 1 \\ 2 & -4 & 1 & 4 & 1 \\ -1 & 2 & -1 & -3 & 0 \end{bmatrix}$$