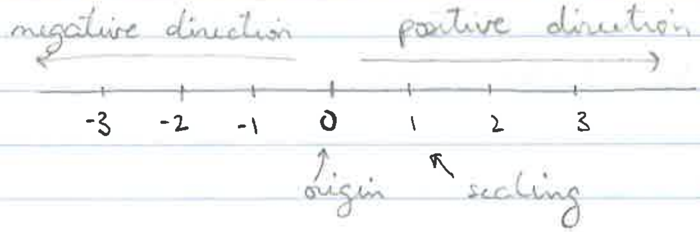
Precalc Review

Real number line:



Intervals

Finite Intervals

Finite Intervals	Graph	Example
Open: (a,b)		(-1, 2)
Closed: [a,b]		[0, 1]
Half-open: (a,b]		(-3, 0]
[a,b)		[8, 10)

Infinite Intervals

(a, ∞)

[a, ∞)

(-∞, a)

(-∞, a]

Exponents and Radicals

Def. b = real number

n = positive integer

"b to the power of n" means

$$b^n = \underbrace{b \cdot b \cdot \dots \cdot b}_{n \text{ times}}$$

base

power

* If $b \neq 0$, we define

$b^0 = 1$

Ex: $\left(\frac{3}{5}\right)^3 = \left(\frac{3}{5}\right)\left(\frac{3}{5}\right)\left(\frac{3}{5}\right) = \frac{3^3}{5^3} = \frac{27}{125}$

$\left(\frac{3}{5}\right)^0 = 1$

Def. n = positive integer

$b^{1/n}$:= number that, when raised to the n^{th} power, is equal to b

Called the <u>n^{th} root of b</u>, written $\sqrt[n]{b}$

Ex: $27^{1/3} = \sqrt[3]{27} = 3$

⚠ n^{th} root of a negative number is not always defined in the real line

Note - More than one number might satisfy the definition of n^{th} root

Ex: $(-4)^2 = (4)^2 = 16$

⇝ We define $b^{1/n}$ to be the <u>positive</u> n^{th} root of b.

Rules for Defining b^n (for $b>0$)

Rule	Example
• If n is a positive integer, $b^n = \underbrace{b \cdot b \cdot \cdots \cdot b}_{n \text{ times}}$	$3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$
• If $n=0$, then $b^0 = 1$ (0^0 not defined)	$\left(\frac{7}{13}\right)^0 = 1$
Negative exponent: • If n is a positive integer, $b^{-n} = \frac{1}{b^n}$	$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$
• Fractional exponent: If n is a p.i., $b^{1/n} = \sqrt[n]{b}$ denotes n^{th} root of b	$16^{\frac{1}{4}} = \sqrt[4]{16} = 2$ (since $2 \cdot 2 \cdot 2 \cdot 2 = 16$)
• If m, n are p.i. $b^{\frac{m}{n}} = \sqrt[n]{b^m} = (\sqrt[n]{b})^m$	$8^{2/3} = (\sqrt[3]{8})^2 = 2^2 = 4$
• If $m, n > 0$ $b^{-\frac{m}{n}} = \frac{1}{b^{\frac{m}{n}}}$	$9^{-3/2} = \frac{1}{9^{3/2}} = \frac{1}{(\sqrt[2]{9})^3} = \frac{1}{27}$

Laws of Exponents

Law	Example
$b^m \cdot b^n = b^{m+n}$	$x^5 \cdot x^3 = x^{5+3} = x^8$
$\frac{b^m}{b^n} = b^{m-n} \quad (b \neq 0)$	$\frac{y^8}{y^2} = y^{8-2} = y^6$
$(b^m)^n = b^{m \cdot n}$	$(z^3)^4 = z^{3 \cdot 4} = z^{12}$
$(ab)^n = a^n b^n$	$(3y)^2 = 3^2 y^2 = 9y^2$
$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad (b \neq 0)$	$\left(\frac{x}{10}\right)^3 = \frac{x^3}{10^3} = \frac{x}{1000}$

Ex: Simplify the expressions:

1) $\frac{2^{\frac{1}{3}}}{2^{4/3}} = 2^{1/3 - 4/3} = 2^{-3/3} = 2^{-1} = \frac{1}{2}$

2) $(18^{4/3})^{1/4} = 18^{4/3 \cdot 1/4} = 18^{1/3}$

3) $(z^{-3} y^8)^{-1/2} = (z^{-3})^{-1/2} \cdot (y^8)^{-1/2}$

$= z^{3/2} \cdot y^{-4} = \frac{z^{3/2}}{y^4}$

4) $\sqrt[3]{16zy^2} \cdot \sqrt[3]{8y} = (16zy^2)^{1/3} \cdot (8y)^{1/3}$

$= (16zy^2 \cdot 8y)^{1/3}$

$= (128zy^3)^{1/3}$

$= 128^{1/3} z^{1/3} y$

$= \sqrt[3]{128z}\, y$

Def. If a radical appears in the numerator or denominator of an algebraic expression, we can simplify the expression by eliminating the radical from the numerator or denominator.

This process is called rationalization

Ex: Rationalize the denominator of the expressions:

1) $\frac{9x}{2\sqrt{x}} = \frac{9x}{2\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \frac{9x\sqrt{x}}{2x} = \frac{9}{2}\sqrt{x}$

2) $\frac{3}{8}x^{-1/3} = \frac{3}{8} \cdot \frac{1}{x^{1/3}} = \frac{3}{8} \cdot \frac{1}{\sqrt[3]{x}}$

$= \frac{3}{8} \cdot \frac{1}{x^{1/3}} \cdot \frac{x^{2/3}}{x^{2/3}}$

$= \frac{3x^{2/3}}{8x} = \frac{3\sqrt[3]{x^2}}{8x}$

Operations w/ Algebraic Expressions

Def. A monomial is an algebraic expression of the form $ax^m y^n$ (a real, m, n nonnegative integers)

Ex: $3xy$

A polynomial is a monomial or the sum of 2 or more monomials

Ex: $8x^2y + 4y + 3x^2$

The degree of a polynomial is the highest power of the variables appearing in the polynomial

Ex: $8x^2y + 4y + 3x^2$ has degree 3.

Def. Constant terms and terms containing same variable factor are called like terms

We can combine like terms by adding coefficients:

Ex: 1) $4x + 2x = 6x$

2) $(3y^3 + 6y^2 + 8) + (7y^3 + y + 4)$
$= 10y^3 + 6y^2 + y + 12$

Multiplying algebraic expressions:

Ex: $(3x + 4)(x^2 + 2x + 1)$
$= 3x(x^2 + 2x + 1) + 4(x^2 + 2x + 1)$
$= (3x^3 + 6x^2 + 3x) + (4x^2 + 8x + 4)$
$= 3x^3 + 10x^2 + 11x + 4$

Useful Product Formulas

Formula	Example
$(a+b)^2 = a^2 + 2ab + b^2$	$(x+3y)^2 = x^2 + 6xy + 9y^2$
$(a-b)^2 = a^2 - 2ab + b^2$	$(2x-y)^2 = 4x^2 - 4xy + y^2$
$(a+b)(a-b) = a^2 - b^2$	$(x+2)(x-2) = x^2 - 4$

Factoring

Def. Factoring is the process of expressing an algebraic expression as the product of other algebraic expressions.

Ex: 1) $2x^{3/2} - 3x^{1/2} = x^{1/2}(2x-3)$

2) $4x(x+1)^{1/2} - x^2(x+1)^{-1/2}$
$= x(x+1)^{-1/2}(4(x+1) - x)$
$= x(x+1)^{-1/2}(3x+4)$

3) $2ax + 2ay + bx + by$
$= 2a(x+y) + b(x+y)$
$= (2a+b)(x+y)$

Product Formulas Used in Factoring

Formula	Example
$x^2-y^2=(x+y)(x-y)$	$x^2-81=(x+9)(x-9)$
$x^2+2xy+y^2=(x+y)^2$	$x^2+8x+16=(x+4)^2$
$x^2-2xy+y^2=(x-y)^2$	$4x^2-4xy+y^2=(2x-y)^2$
$x^3+y^3=(x+y)(x^2-xy+y^2)$	$z^3+27=z^3+3^3$ $=(z+3)(z^2-3z+9)$
$x^3-y^3=(x-y)(x^2+xy+y^2)$	$8x^3-y^6=(2x)^3-(y^2)^3$ $=(2x-y^2)(4x^2+2xy^2+y^4)$

The factors of 2nd degree polynomial w/ integer coefficient

$$px^2+qx+r$$

are

$$(ax+b)(cx+d)$$

such that

$$ac=p, \quad ad+bc=q, \quad bd=r$$

Ex: 1) x^2-2x-3

$=(x\quad)(x\quad)$ product $=-3$, sum $=-2$ $\Rightarrow -3, 1$

$=(x-3)(x+1)$

2) $-3t^2+192t+195$

$=-3(t^2-64t-65)$

$=-3(t\quad)(t\quad)$ $P=-65$, $S=-64$ $\Rightarrow -65, 1$

$=-3(t+1)(t-65)$