2) $\dfrac{12x^2}{\sqrt{2x^2+3}} + \dfrac{6\sqrt{2x^2+3}}{1}$

$= \dfrac{12x^2}{\sqrt{2x^2+3}} + \dfrac{6(2x^2+3)}{\sqrt{2x^2+3}}$

$= \dfrac{12x^2+6(2x^2+3)}{\sqrt{2x^2+3}}$ } $\rightarrow$ $12x^2+12x^2+18$

$= \dfrac{24x^2+18}{\sqrt{2x^2+3}}$ } $\rightarrow$ $6(4x^2+3)$

=

Rationalizing Algebraic Fractions

Trick : $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b}) = (\sqrt{a})^2 - (\sqrt{b})^2$

$= a-b$

Ex: Rationalize denominator : $\dfrac{1}{1+\sqrt{x^2+2}}$

$\dfrac{1}{1+\sqrt{x^2+2}} \cdot \dfrac{(1-\sqrt{x^2+2})}{(1-\sqrt{x^2+2})}$

$= \dfrac{1-\sqrt{x^2+2}}{1-(x^2+2)}$

$= \dfrac{1-\sqrt{x^2+2}}{-x^2-1}$

Inequalities

Properties: If A, B, C are real numbers or polynomials with real coeff$^{(*)}$, then

1) $A \underline{<} B$ and $B \underline{<} C \Rightarrow A \underline{<} C$
2) $A \underline{<} B \Rightarrow A + C \underline{<} B + C$
3) $A \underline{<} B$ and C real number $> 0 \Rightarrow AC \underline{<} BC$
4) $A \underline{<} B$ and C " " $< 0 \Rightarrow AC \underline{>} BC$

Ex: 1) Find all values of x that satisfy

$-1 \leq 2x - 5 < 7$

$\Rightarrow -1 + 5 \leq 2x - 5 + 5 < 7 + 5$

$\Rightarrow 4 \leq 2x < 12$

$\Rightarrow 4/2 \leq 2x/2 < 12/2$

$\Rightarrow 2 \leq x < 6$

solution: $[2, 6)$

2) Solve inequality $\frac{x+1}{x-1} \geq 0$

$\frac{x+1}{x-1} > 0$ if $x+1$ and $x-1$ have same sign

$(x+1)$	$-$	$-$	0	$+$	$+$	$+$	...			
$(x-1)$	$-$	$-$	$-$	$-$	0	$+$	$+$	$+$	...	
	-4	-3	-2	-1	0	1	2	3	4	

$\Rightarrow$ same sign when x is in $(-\infty, -1)$ and $(1, \infty)$

$\frac{x+1}{x-1} = 0$ if $x + 1 = 0 \Rightarrow x = -1$

$\Rightarrow \frac{x+1}{x-1} \geq 0$ when x is in $(-\infty, -1]$ or $(1, \infty)$

Absolute Value

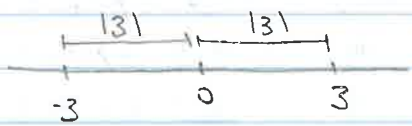
Def. The absolute value of A is

$$|A| = \begin{cases} A & \text{if } A \geq 0 \\ -A & \text{if } A < 0 \end{cases}$$

} A can be polynomial

{ Note: $|A|$ is the distance between A and the origin, so it is always nonnegative }

Ex:



Properties

1) $|-A| = |A|$
2) $|AB| = |A||B|$
3) $\left|\frac{A}{B}\right| = \frac{|A|}{|B|} \quad (B \neq 0)$
4) $|A+B| \leq |A| + |B|$ (triangle identity)

Ex: $|3 + (-4)| = |-1| = 1$

$\leq |3| + |-4| = 7$

Ex: Evaluate $|\sqrt{3} - 2| + |2 - \sqrt{3}|$.

$\sqrt{3} < 2 \Rightarrow \sqrt{3} - 2 < 0$

$\Rightarrow |\sqrt{3} - 2| = -(\sqrt{3} - 2) = 2 - \sqrt{3}$

$= |2 - \sqrt{3}|$

$\Rightarrow |\sqrt{3} - 2| + |2 - \sqrt{3}| = (2 - \sqrt{3}) + (2 - \sqrt{3})$

$= 4 - 2\sqrt{3}$

Ex: 1) Solve inequalities $|x| \le 5$ and $|x| \ge 5$

$|x| \le 5$

$\rightarrow \; -5 \le x \le 5$

$|x| \ge 5$

$\rightarrow \; x \ge 5$ or $x \le -5$

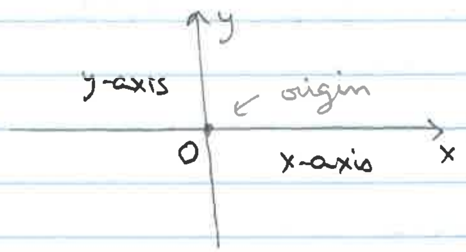
2) $|2x - 3| \le 1 \overset{\text{by (1)}}{\Rightarrow} -1 \le 2x - 3 \le 1$

$\Rightarrow -1 + 3 \le 2x - 3 + 3 \le 1 + 3$

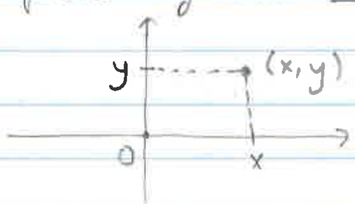
$\Rightarrow \dfrac{2}{2} \le \dfrac{2x}{2} \le \dfrac{4}{2}$

$\Rightarrow 1 \le x \le 2$

1.3 Cartesian Coordinate System

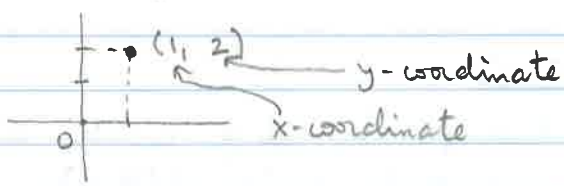


Def. Can uniquely represent a point in the plane by an ordered pair of numbers (x, y)

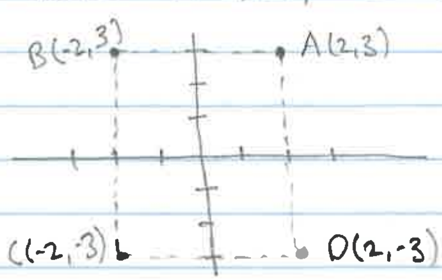


- Can also denote point (x, y) by $P(x, y)$.

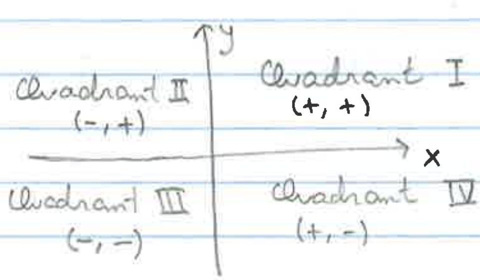
Ex: 1) Plot point $(1, 2)$



2) Plot $A(2,3)$, $B(-2,3)$, $C(-2,-3)$, $D(2,-3)$



Def.

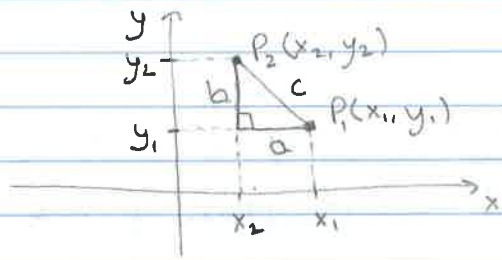


Distance Formula

The distance d between two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Proof /



Pythagorean Theorem: $a^2 + b^2 = c^2$

$$a = |x_2 - x_1|$$

$$b = |y_2 - y_1|$$

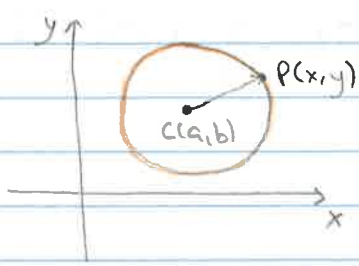
$$\Rightarrow c = \sqrt{a^2 + b^2}$$

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Equation of a Circle

Ex: Let $P(x,y)$ be a point lying on the circle with radius r and center $C(a,b)$.

What is the relationship between x and y?



$$r = \sqrt{(x-a)^2 + (y-b)^2}$$

$$\Rightarrow r^2 = (x-a)^2 + (y-b)^2$$

{* every point on the circle must satisfy this equation.}

Theorem - An equation of the circle w/ center $C(a, b)$ and radius r is given by

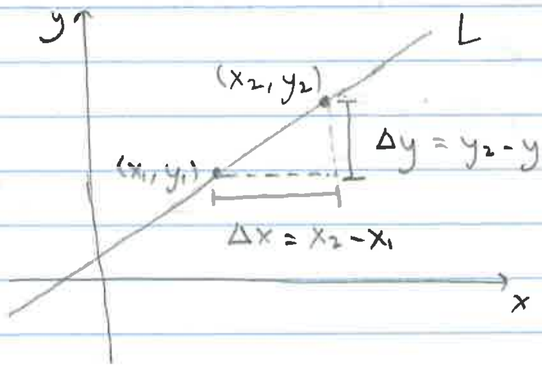
$$(x-a)^2 + (y-b)^2 = r^2$$

Ex: Find equation of circle w/ radius 6 and center $(-3, 8)$

$$(x+3)^2 + (y-8)^2 = 36$$

1.4 Straight lines

Let L be the unique straight line passing through (distinct) points (x_1, y_1), (x_2, y_2)



Def. If $x_1 \neq x_2$ the slope m of L is given by

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

delta = "change in"

- If $x_1 = x_2$, then L is a vertical line and its slope is undefined

Note - Slope of a straight line remains the same no matter what 2 points you choose

$\Rightarrow$ slope m of straight line L is a measure of the rate of change of y with respect to x.