Theorem - An equation of the circle w/ center $C(a, b)$ and radius r is given by

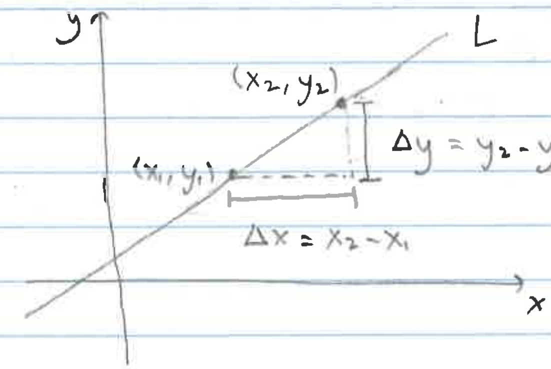
$$(x-a)^2 + (y-b)^2 = r^2$$

Ex: Find equation of circle w/ radius 6 and center $(-3, 8)$

$$(x+3)^2 + (y-8)^2 = 36$$

1.4 Straight lines

Let L be the unique straight line passing through (distinct) points (x_1, y_1), (x_2, y_2)



Def. If $x_1 \neq x_2$ the slope m of L is given by

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

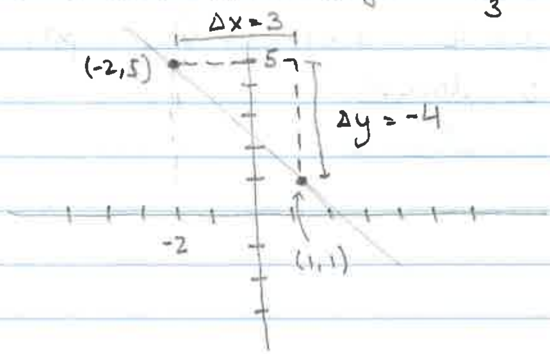
delta = "change in"

- If $x_1 = x_2$, then L is a vertical line and its slope is undefined

Note - Slope of a straight line remains the same no matter what 2 points you choose

$\Rightarrow$ slope m of straight line L is a measure of the rate of change of y with respect to x.

Example - Sketch straight line passing through point $(-2,5)$ with slope $-\frac{4}{3}$



Def. Two distinct lines are parallel if their slopes are equal or both undefined. } later

Equations of Lines

Point-slope form: an equation of the line with slope m and passing through (x_1, y_1) is given by

$$y - y_1 = m(x - x_1)$$

Proof/ For any point (x,y) on the line

$$m = \frac{y-y_1}{x-x_1}.$$

Multiplying both sides of this equation by $(x-x_1)$ yields

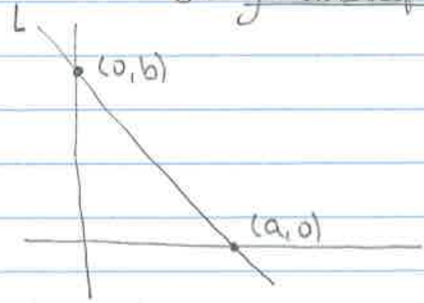
$$y - y_1 = m(x - x_1).$$

Ex: Equation of line passing through $(-3, 4)$ with slope -1 is

$$y - 4 = -1(x+3)$$

Def. { Straight line L that is neither horizontal nor vertical cuts x-axis and y-axis at some points $(a,0)$, $(0,b)$, resp. }

$a =$ x-intercept (of L)
$b =$ y-intercept



Ex: Let L be a line w/ slope m and y-intercept b (so L goes through $(0,b)$).
Equation of L is

$$y - b = m(x-0)$$
$$\Rightarrow \quad y = mx + b$$

Slope-intercept form: An equation of a line w/ slope m and y-intercept b is given by
$y = mx + b$

Ex: Determine slope and y-intercept of the line whose equation is

$$3x - 4y = 8$$

- Write in slope-intercept form:

$$3x - 4y = 8 \Rightarrow -4y = 8 - 3x$$
$$\Rightarrow \quad y = \frac{8 - 3x}{-4} = \frac{8}{-4} + \frac{-3x}{-4}$$
$$\Rightarrow \quad y = \underbrace{\frac{3}{4}}_{\text{slope}}x \underbrace{- 2}_{\text{y-intercept}}$$

General form of linear eq: $ax + by + c = 0$, a, b not both zero due Friday

HW: • Sketch the straight line represented by the equation

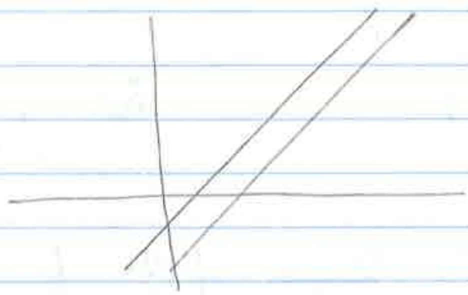
$$-8x + 4y + 16 = 0$$

• Find equation of circle w/ center at ~~origin~~ (0,1) and passing through (2,3)

Parallel Lines

Def. Two distinct lines are parallel if their slopes are ① equal
② both undefined.

Ex:



Def. If L_1, L_2 are distinct nonvertical lines w/ slopes m_1, m_2 resp., then L_1 is perpendicular to L_2 ($L_1 \perp L_2$) if

$$m_1 = \frac{-1}{m_2}$$

Ex: Find equation of line through $(3,1)$ and $\perp$ to $2x - y + 1 = 0$.

$$2x - y - 1 = 0 \Rightarrow y = \underset{\uparrow \text{ slope } = 2}{2}x - 1$$

Need: equation of line through $(3,1)$ w/ slope $\frac{-1}{2}$:

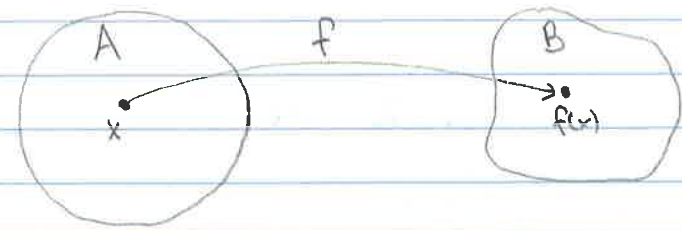
$$y - 1 = \frac{-1}{2}(x-3)$$

General form? $2y + 2 = x - 3$

$$\Rightarrow 2y - x + 5 = 0$$

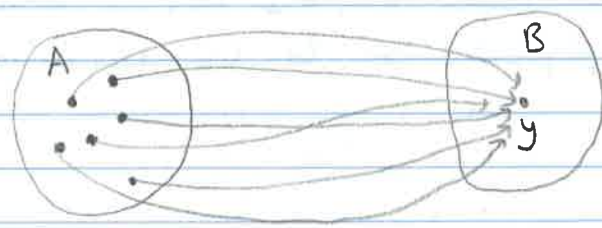
2.1 Functions and Their Graphs

Def. A function is a rule that assigns to each element in a set A one and only one element in a set B.



Set A is the domain of the function.
Set of all f(x) for all x in A is the range of the function.

Ex:



Is this a function? Yes!
What is its domain? Range?

Ex: A rule associating football team w/ # of touchdowns for the season (?)
Function? Yes!
Domain? All football teams.
Can two teams have the same # of touchdowns? Yes!
Can the same team have two different #s? No!

Ex: Consider function $f(x) = x^2 + x + 4$.
What is $f(2)$?
$f(2) = 2^2 + 2 + 4 = 4 + 2 + 4 = 10$

Determining Domain of a Function

Def. Consider function $y = f(x)$.
$x :=$ independent variable
$y :=$ dependent variable

{To find domain, we must determine restriction placed on x by function $f(x)$.}

- domain is all values of x for which $f(x)$ is a real #.

Note: 1) We can't divide by 0
2) The even root of a negative # is not real

Ex: Find domain

1) $f(x) = \sqrt{x-1}$
$\Rightarrow$ domain: $[1, \infty)$

2) $f(x) = \sqrt[3]{x}$
$\Rightarrow$ domain: $(-\infty, +\infty)$

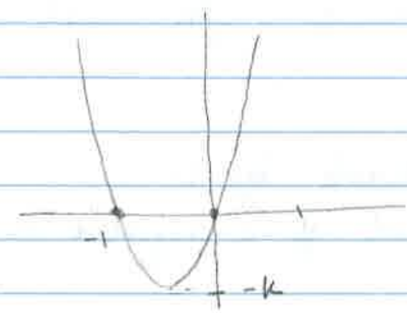
3) $f(x) = \dfrac{1}{x^2 - 9}$
$\Rightarrow$ domain: all $x \neq 3$

4) $f(x) = x^2$
$\Rightarrow$ domain: all real #'s

Graphs of Functions

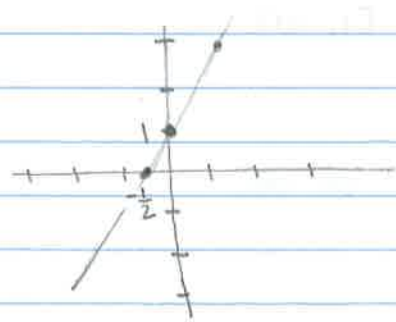
Def. The graph of function f is the set of points (x, y) in the xy-plane s.t.
- x is in the domain of f
- $y = f(x)$

Ex: 1) Graph $f(x) = x^2 + x$
$= x(x+1)$ → zeros at $x = 0, -1$



domain : all real #s
range : $[-k, \infty)$

2) $f(x) = 2x + 1$



x-intercept:
$0 = 2x + 1$
$x = -1/2$

$m = \frac{\text{rise}}{\text{run}} = \frac{2}{1}$

domain, range } = all real #s