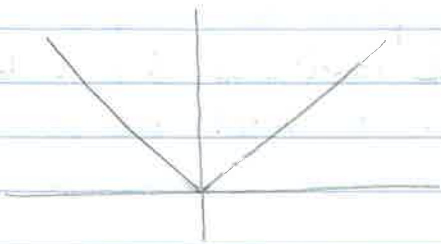
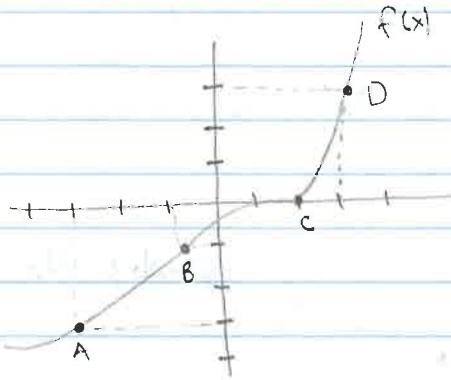
3) Graph: $f(x) = |x|$ $\Longrightarrow$ $f(x) = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$



domain = all real #s

range = all nonneg. #s

4)



A = (-3, -3)

B = (-1, -1)

C = (2, 0)

D = (3, 3)

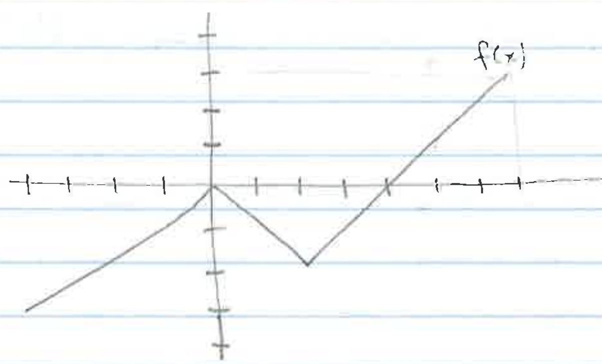
$f(-3) = -3$

$f(2) = 0$

$f(-1) = -1$

$f(3) = 3$

5)



$f(x) = 0$ when $x = \cdots 0$

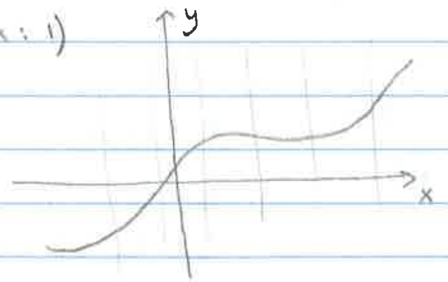
$f(x) = 3$ when $x = \cdots 7$

$f(x) = -1$ when $x = \cdots -1$

Vertical Line Test

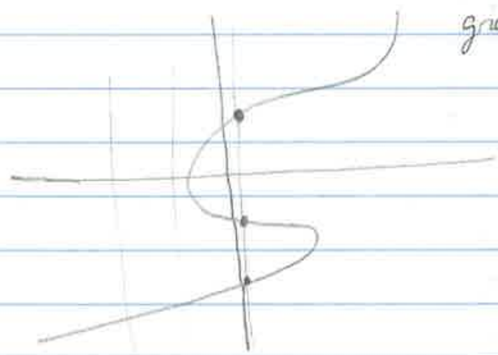
A curve in the xy-plane is the graph of a function $y = f(x)$ if (and only if) each vertical line intersects it in at most one point.

Ex: 1)



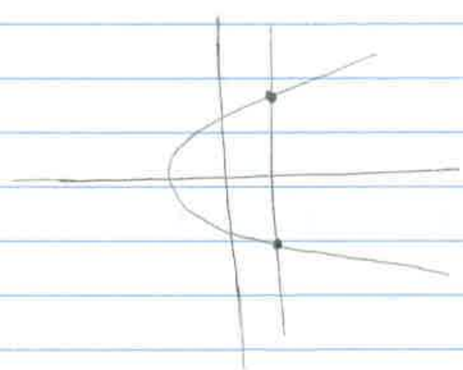
graph? Yes!

2)



graph? No!

3)



graph? No!

Problem - During a sale at (insert store), every item is 30% off.

$C(x) :=$ cost of item whose original price is x.

Express $C(x)$ as a function.

$$C(x) = \frac{7}{10} x$$

What is the cost of an item w/ original price \$30?

$$C(30) = \frac{7}{10} \cdot 30 = 21$$

2.2 The Algebra of Functions

Sum, Difference, Product, and Quotient of Functions

Let f and g be functions w/ domains A, B resp. Then the sum f+g, difference f-g and product fg of f and g are functions w/ domain $A \cap B$ (A intersected w/ B: all points common to both A and B)

and rule

1) $(f+g)(x) = f(x) + g(x)$
2) $(f-g)(x) = f(x) - g(x)$
3) $(fg)(x) = f(x)g(x)$

The quotient f/g has domain $A \cap B$ excluding all numbers x s.t. $g(x) = 0$, and rule

4) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

Ex: $f(x) = \sqrt{x}$, $g(x) = x^2 + 2$

1) $(f+g)(x) = f(x) + g(x) = \sqrt{x} + (x^2 + 2)$
2) $(f-g)(x) = f(x) - g(x) = \sqrt{x} - (x^2 + 2)$
3) $(fg)(x) = f(x)g(x) = \sqrt{x}(x^2 + 2)$

* domain of f(x) ? all nonneg #, $[0, \infty)$
g(x) ? all real numbers $(-\infty, \infty)$
$(f \pm g)(x)$, $(fg)(x)$? $[0, \infty)$

4) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x}}{x^2+2}$

* domain of $\left(\frac{f}{g}\right)(x)$? $[0, \infty)$

5) Suppose $g(x) = x^2 - 2$ instead.

* domain of g(x) ? $(-\infty, \infty)$
$\left(\frac{f}{g}\right)(x)$? $[0, \infty) \cap (-\infty, \infty) - \{\sqrt{2}\}$
$= [0, \sqrt{2}) \cup (\sqrt{2}, \infty)$

Def. $A \cup B$:= all points in either A or B

or := just A, just B, or both A and B.

Ex: $A = \{1, 2, 4, 7.8\}$

$B = \{0, 1\}$

$A \cup B = \{0, 1, 2, 4, 7.8\}$

Composition of Functions

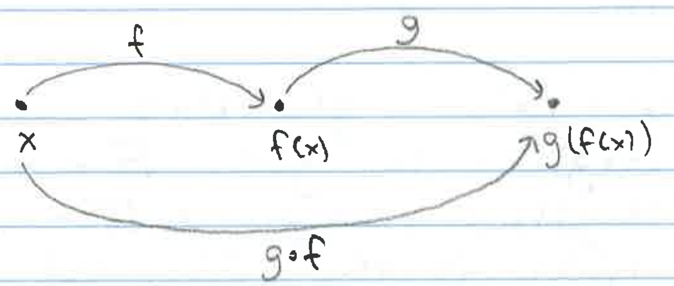
Let f and g be functions. Then the composition of g and f is

$(g \circ f)(x) = g(f(x))$

"g of f".

The domain of $g \circ f$ is the set of all x in the domain of f s.t. f(x) is in the domain of g.

Def. $g \circ f$ is called a composite function.



Ex: $f(x) = x^2 - 1$, $g(x) = \sqrt{x} + 1$

1) $(g \circ f)(x) = g(f(x)) = \sqrt{x^2 - 1} + 1$

• domain? all x s.t. $x^2 - 1$ is in domain of $g(x) \Rightarrow (-\infty, -1] \cup [1, \infty)$

* draw # line to check

2) $(f \circ g)(x) = f(g(x)) = (\sqrt{x} + 1)^2 - 1$

$= x + 2\sqrt{x}$

• domain? $[0, \infty)$

Ex: Find functions f, g s.t. $h = f \circ g$, where

$$h(x) = \frac{1}{x^2+2}$$

$$f = \frac{1}{x}, \quad g = x^2+2$$

2.3 Functions and Mathematical Models

Def. A polynomial function of degree 1 is called a linear function:

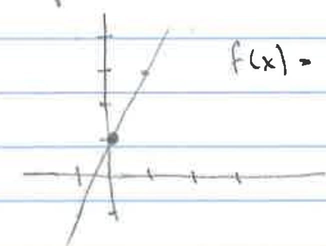
$$y = f(x) = ax + b$$

A polynomial function of degree 2 is called a quadratic function:

$$y = f(x) = ax^2 + bx + c$$

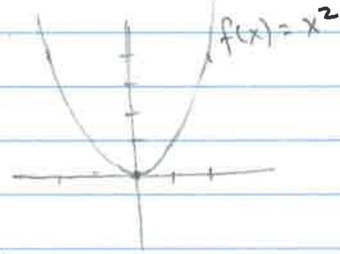
Note: 1) Graph of a linear function is a line.

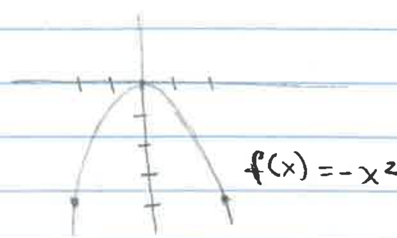
Ex:



2) Graph of a quadratic function is a parabola.

Ex





* ∪ if coeff of x^2 is +

∩ " " " " " −