MATH 10250 Practice Exam 1 - Extras

1. Factor: (a) $x^2 + x - 2$ (b) $x^2 - 9 = (x+3)(x-3)$

$= (x+2)(x-1)$

2. Write the **slope intercept form** of the equation of a line that passes through $(-1, 4)$ and $(2, 5)$

$m = \frac{5-4}{2-(-1)} = \frac{1}{3}$

$(x_0, y_0) = (-1, 4)$ (or $(2,5)$, either works)

$y - y_0 = m(x - x_0)$

$\Rightarrow y - 4 = \frac{1}{3}(x - (-1))$

$\Rightarrow y = \frac{1}{3}x + \frac{1}{3} + 4 \Rightarrow \boxed{y = \frac{1}{3}x + \frac{13}{3}}$

3. Given $f(x) = \begin{cases} \dfrac{x^2 - 2x + 1}{x - 1} & \text{if } x < 1 \\ 0 & \text{if } x \geq 1 \end{cases}$

Find

(a) $\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^-} \dfrac{x^2 - 2x + 1}{x - 1} = \lim\limits_{x \to 1^-} \dfrac{(x-1)^{\cancel{2}}}{\cancel{x-1}}$

$= \lim\limits_{x \to 1^-} x - 1 = 1 - 1 = 0$

(b) $\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1^+} 0 = 0$

(c) $\lim_{x\to 1} f(x) = 0$

(d) $f(1) = 0$

Is f continuous at $x = 1$? Justify your answer.

Yes, because $\lim_{x\to 1} f(x) = f(1)$

4. Let $f(x) = x^2 - 4\sqrt{x}$. Find the (instantaneous) rate of change of f at $x = 4$.

$f'(4)$

$f'(x) = 2x - 4\left(\frac{1}{2}x^{-1/2}\right)$

$= 2x - 2x^{-1/2}$

$f'(4) = 2(4) - 2(4)^{-1/2} = 8 - \frac{2}{\sqrt{4}} = 8 - 1 = 7$

5. Find derivative of

(a) $f(x) = \frac{1}{2}\sqrt{x} + x^2 - \sqrt{2}$

$f'(x) = \frac{1}{4}x^{-1/2} + 2x$

(b) $g(x) = \frac{2}{x^2} - x^{-1} + \sqrt[5]{x}$ $\rightarrow$ $g(x) = 2x^{-2} - x^{-1} + x^{1/5}$

$g'(x) = -4x^{-3} + x^{-2} + \frac{1}{5}x^{-4/5}$

(c) $h(x) = 2\sqrt{100 - x^2}$

$h'(x) = 2 \cdot \frac{1}{2}(100 - x^2)^{-1/2} \cdot (-2x)$

$= -2x(100 - x^2)^{-1/2}$

6. **Find the second derivative of**

$$f(x) = (x^3 - 3)^{-1}$$

$$f'(x) = -(x^3-3)^{-2} \cdot 3x^2$$

$$= -3x^2(x^3-3)^{-2}$$

$$f''(x) = (-6x)(x^3-3)^{-2} + (-3x^2) \cdot -2(x^3-3)^{-3} \cdot 3x^2$$

$$= -6x(x^3-3)^{-2} + 18x^4(x^3-3)^{-3}$$

7. Given

$$f(x) = \frac{\sqrt{2x}}{x^2-1}$$

What is $f'(2)$?

$$f'(x) = \frac{\frac{1}{2}(2x)^{-1/2} \cdot 2\,(x^2-1) - \sqrt{2x} \cdot 2x}{(x^2-1)^2}$$

$$= \frac{(2x)^{-1/2}(x^2-1) - (2x)^{3/2}}{(x^2-1)^2}$$

$$f'(2) = \frac{\left(\frac{1}{\sqrt{2(2)}}\right)((2)^2-1) - \left(\sqrt{2(2)}\right)^3}{((2)^2-1)^2}$$

$$= \frac{\left(\frac{1}{4}\right)(3) - 8}{9}$$

$$= \frac{\frac{3}{4} - \frac{32}{4}}{9} = \frac{-29}{36}$$

8. The demand for VR-glasses is modeled by the function

$$p(x) = -x^2 - 4x + 64$$

where p is measured in dollars and x is measured in thousands of glasses.

1. Find the average rate of change in the unit price of VR-glasses if the quantity demanded is between 0 and 5 thousand.
2. What is the instantaneous change in the unit price of VR-glasses at a demand of 3 thousand units ($x = 3$)?

1) $p(0) = -(0)^2 - 4(0) + 64 = 64$

$p(5) = -(5)^2 - 4(5) + 64$

$= -25 - 20 + 64$

$= 19$

average rate of change = slope of line through $(0,64)$, $(5,19)$

$= \frac{64-19}{0-5} = \frac{45}{-5} = -9$

2) $p'(x) = -2x - 4$

instantaneous rate of change

$p'(3) = -2(3) - 4 = -6 - 4 = -10$

9. Given $f(x) = \sqrt{2x}$ and $g(x) = x^2 - x$. Compute the following (Simplify your answers if possible; but, don't combine anything in part (a)):

(a) $(f+g)(x) =$ $\sqrt{2x} + (x^2 - x)$

(a) $(f-g)(x) =$ $\sqrt{2x} - (x^2 - x) = \sqrt{2x} - x^2 + x$

(b) $(fg)(x) =$ $\sqrt{2x}\,(x^2 - x)$

(c) $\left(\frac{f}{g}\right)(x) =$ $\dfrac{\sqrt{2x}}{x^2 - x}$

(e) $(f \circ g)(x) =$ $f(g(x)) = f(x^2 - x) = \sqrt{2(x^2 - x)}$

(f) $(g \circ f)(x) =$ $g(f(x)) = g(\sqrt{2x}) = (\sqrt{2x})^2 - \sqrt{2x}$

$= 2x - \sqrt{2x}$

(g) $(g \circ f)(2) =$ $2(2) - \sqrt{2(2)}$

$= 4 - 2$

$= 2$