Name: _____

Instructor: _____

MATH 10250, Practice Exam 1
June 29, 2018

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- No calculators.
- The exam lasts for 1 hour and 20 minutes.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 13 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!				
1. (a)	(b)	(c)	(d)	(e)
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Please do NOT write in this box.	
Multiple Choice	_____
6.	_____
7.	_____
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12.	_____
13.	_____
14.	_____
Total	_____

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Multiple Choice

1.(4 pts) Which formula below is the equation of a circle of radius $\frac{1}{2}$ and center at $(-1, 0)$?

(a) $(x+1)^2 + y^2 = \frac{1}{4}$ (b) $x^2 + (y+1)^2 = \frac{1}{4}$

(c) $(x-1)^2 + y^2 = \frac{1}{4}$ (d) $(x+1)^2 + (y+1)^2 = \frac{1}{2}$

(e) $(x+1)^2 + y^2 = \frac{1}{2}$

equation of a circle: $r^2 = (x-a)^2 + (y-b)^2$, r = radius, (a,b) = center

$\Rightarrow \left(\frac{1}{2}\right)^2 = (x+1)^2 + (y-0)^2$

$\Rightarrow \frac{1}{4} = (x+1)^2 + y^2$

2.(4 pts) Write the slope intercept form of the line through $(1, 2)$ and $(-3, 4)$.

(a) $y = 2x + 10$ (b) $y = x + 1$ (c) $y = \frac{1}{2}x$

(d) $y = \frac{1}{2}x + \frac{3}{2}$ (e) $y = -\frac{1}{2}x + \frac{5}{2}$

$m = \frac{4-2}{-3-1} = \frac{2}{-4} = -\frac{1}{2}$

$\Rightarrow y - 2 = -\frac{1}{2}(x-1)$

$\Rightarrow y = -\frac{1}{2}x + \frac{1}{2} + 2$

$= -\frac{1}{2}x + \frac{5}{2}$

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3.(4 pts) Which of the following limits do not exist?

(a) $\lim_{x\to 0} \frac{x^2+2x}{x}$ **(b)** $\lim_{x\to 3} \frac{x-3}{x^2+2}$ **(c)** $\lim_{x\to 1} \frac{3}{x-1}$

(d) $\lim_{x\to 0^+} \sqrt{x}$ **(e)** $\lim_{x\to 1} |x-2|$

(a) $\lim_{x\to 0} \frac{x^2+2x}{x} = \lim_{x\to 0} x+2 = 2$

(b) $\lim_{x\to 3} \frac{x-3}{x^2+2} = \frac{3-3}{3^2+2} = 0$

★ (c) $\lim_{x\to 1} \frac{3}{x-1}$ DNE

(d) $\lim_{x\to 0^+} \sqrt{x} = 0$

(e) $\lim_{x\to 1} |x-2| = |1-1| = 1$

4.(4 pts) The population of a certain bacteria culture is modeled by the function

$$f(t) = t^3 + 3t^2 + 2.$$

Which of the following is the average growth rate of the bacteria between $t = 1$ and $t = 2$?

(a) 16 **(b)** 15 **(c)** 24 **(d)** 9 **(e)** 18

→ slope of the line through points $(1, f(1)), (2, f(2))$

$f(1) = 1+3+2 = 6$

$f(2) = 8+12+2 = 22$

$m = \frac{f(2)-f(1)}{2-1} = \frac{22-6}{1} = 16$

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5.(4 pts) If

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	0	2	6	6
3	4	2	-2	2

which of the following is $(f \circ g)'(3)$?

(a) 24 **(b)** 8 **(c)** 4 (d) 12 **(e)** -4

$$\frac{d}{dx}\left[f(g(x))\right]\Big|_{x=3} = f'(g(x)) \cdot g'(x)\Big|_{x=3}$$

$$= f'(g(3)) \cdot g'(3)$$

$$= f'(2) \cdot 2$$

$$= 6 \cdot 2 = 12$$

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Partial Credit

You must show your work on the partial credit problems to receive credit!

6.(x pts.)The equation for the line L is given by

$$4x + 2y = 4$$

$\Rightarrow \quad 2y = 4 - 4x$

$y = 2 - 2x$

$= -2x + 2$

(a) What is the slope and y-intercept of line L?

$m = -2$

$y\text{-int} = 2$

(b) Which of the following line below is parallel to line L? Explain why.

$-y + 1 = -2x$ $\qquad$ $\frac{1}{2}y = x$ $\qquad$ $y - 1 = \frac{1}{2}x$ $\qquad$ $y = -\frac{1}{2}x$ $\qquad$ none

$y = 2x + 1$ $\qquad$ $y = 2x$ $\qquad$ $y = \frac{1}{2}x + 1$

none of these lines have slope -2

(c) Does the point $(0, 1)$ lie on line L?

$4(0) + 2(1) = 2 \neq 4$ $\qquad$ No, because the equation of the line is not satisfied for $(0,1)$

(d) Suppose line L' is perpendicular to line L, what is the slope of line L'?

$m' = \frac{-1}{m} = \frac{-1}{-2} = \frac{1}{2}$

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7.(x pts.) **Evaluate**

$$\lim_{x \to 4} \frac{x^2 + x - 20}{3x - 12}$$

$$= \lim_{x \to 4} \frac{(x+5)(x-4)}{3(x-4)}$$

$$= \lim_{x \to 4} \frac{x+5}{3}$$

$$= \frac{9}{3} = \boxed{3}$$

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8.(x pts.) **Compute the derivative of f from its limit definition.**

$$f(x) = x^2 - 1$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{[(x+h)^2 - 1] - [x^2 - 1]}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{h(2x + h)}{h}$$

$$= \lim_{h \to 0} 2x + h$$

$$= 2x$$

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9.(x pts.)Given $f(x) = \begin{cases} \dfrac{x}{x+x^2} & \text{if } x < 0 \\ x^2 - 1 & \text{if } x \geq 0 \end{cases}$

Find

(a) $\lim_{x \to 0^-} f(x) =$ $\lim_{x \to 0^-} \dfrac{x}{x+x^2} = \lim_{x \to 0^-} \dfrac{1}{1+x} = 1$

(b) $\lim_{x \to 0^+} f(x) =$ $\lim_{x \to 0^+} x^2 - 1 = -1$

(c) $\lim_{x \to 0} f(x) =$ DNE because $\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$

(d) $f(0) =$ $0^2 - 1 = -1$

(e) Is f continuous at $x = 0$? Justify your answer.

No, because $\lim_{x \to 0} f(x) \neq f(0)$, since $\lim_{x \to 0} f(x)$ DNE.

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10.(x pts.) **The cost in dollars incurred by a record company in pressing x CDs is given by**

$$C(x) = 1.8x + 2300$$

(a) What **are the fixed costs** of production?

(b) **Find a formula for the average cost per disk, $\bar{C}(x)$, in pressing** x CDs.

(c) Evaluate $\lim_{x \to \infty} \bar{C}(x)$

You don't need to know this... yet...

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11.(x pts.)Let $f(x) = (2x-1)^3$.

(a) **Find the slope of the tangent line to the graph of f at $x = 1$.**

$f'(x) = 3(2x-1)^2 \cdot 2 = 6(2x-1)^2$ ← $= f'(1)$

$f'(1) = 6(2(1)-1)^2 = \boxed{6}$

(b) Write the **equation** of the tangent line to the graph of f at $x = 1$.

$m = 6$

$(x_0, y_0) = (1, f(1))$, $f(1) = (2(1)-1)^3 = 1$

$= (1, 1)$

$y - y_0 = m(x - x_0)$

$\Rightarrow y - 1 = 6(x-1)$

$\Rightarrow y = 6x - 5$

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12.(x pts.)

(a) Given $f(x) = (x^2+1)^3(2x^3-x)$, compute $f'(x)$

$p(x)$ $q(x)$

$p'(x) = 3(x^2+1)^2 \cdot 2x$

$= 6x(x^2+1)^2$

$q'(x) = 6x^2 - 1$

$f'(x) = p'(x)\,q(x) + p(x)\,q'(x)$

$= 6x(x^2+1)^2(2x^3-x) + (x^2+1)^3(6x^2-1)$

(b) Given $g(x) = \dfrac{2x-1}{x^4+1}$, compute $g'(x)$ $\quad \frac{p(x)}{q(x)}$

$p'(x) = 2$

$q'(x) = 4x^3$

$g'(x) = \dfrac{p'(x)\,q(x) - p(x)\,q'(x)}{[q(x)]^2}$

$= \dfrac{2(x^4+1) - (2x-1)(4x^3)}{(x^4+1)^2}$

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13.(x pts.) Let $f(x) = \sqrt{1-x}$, find $f''(0)$

$$f'(x) = \frac{1}{2}(1-x)^{-1/2} \cdot (-1)$$

$$= -\frac{1}{2}(1-x)^{-1/2}$$

$$f''(x) = \frac{1}{4}(1-x)^{-3/2} \cdot (-1)$$

$$= -\frac{1}{4}(1-x)^{-3/2}$$

$$f''(0) = -\frac{1}{4}(1-0)^{-3/2}$$

$$= -\frac{1}{4}$$

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14.(x pts.) A coffee company will make 6400 bags of coffee when the price per bag is \$10, and 10000 bags of coffee when the price per bag is \$14. The supply function is known to have the form

$$p(x) = a\sqrt{x} + b$$

where x is the number of bags made, p is the price per bag, and a, b are real numbers.

(a) Determine the supply function.

You don't need to know this.

(b) What unit price will induce the company to make 4900 bags of coffee?

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