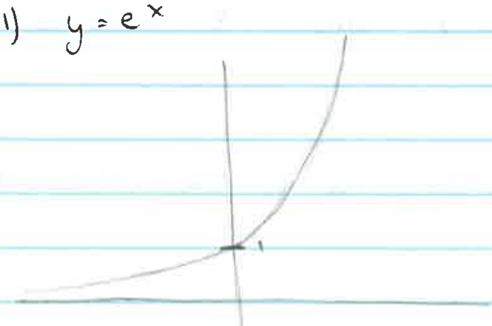
The Base e

Def. $e = \lim_{x \to \infty} \left(1 + \frac{1}{m}\right)^m$

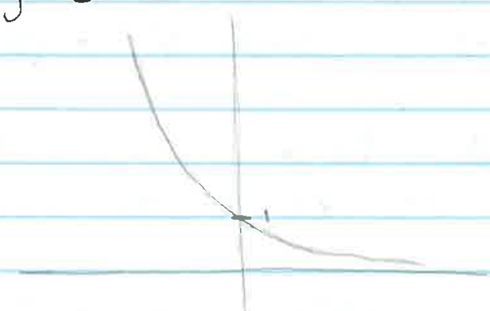
≈ 2.7182818

Ex: Sketch a graph of the function

1) $y = e^x$



2) $y = e^{-x}$



5.2 Logarithmic Functions

Given equation $3^x = 81$, how do you solve for x?

Def. Logarithm of x to base b

$$y = \log_b x \quad \text{iff} \quad x = b^y \qquad (b>0,\ b\neq 1,\ x>0)$$

- $\log_b x$ is the power to which b must be raised to obtain x.

⚠ $\log_b x$ is defined only for positive values of x. (Why?)

Ex: 1) $\log_{10} 100 = 2$, since $10^2 = 100$
2) $\log_5 125 = 3$, since $5^3 = 125$
c) $\log_3 \frac{1}{27} = -3$, since $3^{-3} = \frac{1}{27}$

Ex: Solve the equation for x.

1) $\log_3 x = 4 \longrightarrow x = 3^4 = 81$

2) $\log_{16} 4 = x \longrightarrow 16^x = 4 \Rightarrow x = \frac{1}{2}$

3) $\log_x 8 = 3 \longrightarrow x^3 = 8 \Rightarrow x = 2$

Logarithmic Notation

$\log x = \log_{10} x$ common logarithm

$\ln x = \log_e x$ natural logarithm

Laws of Logarithms

If $m, n > 0$ and $b > 0$, $b \neq 1$, then

1) $\log_b mn = \log_b m + \log_b n$
2) $\log_b \frac{m}{n} = \log_b m - \log_b n$

3) $\log_b m^n = n \log_b m$
4) $\log_b 1 = 0$
5) $\log_b b = 1$

Ex: 1) $\log(2 \cdot 3) = \log 2 + \log 3$

2) $\ln \frac{5}{3} = \ln 5 - \ln 3$

3) $\log \sqrt{7} = \log 7^{1/2} = \frac{1}{2} \log 7$

4) $\log_5 1 = 0$

5) $\log_{45} 45 = 1$

Ex: Given that $\log 2 \approx 0.3$, $\log 3 \approx 0.5$, $\log 5 \approx 0.7$, use the laws of Logarithms to evaluate the following

1) $\log 15 = \log 5 \cdot 3$

$= \log 5 + \log 3$

$\approx 0.7 + 0.5$

$= 1.2$

2) $\log 7.5 = \log \frac{15}{2} = \log 15 - \log 2$

$\approx 1.2 - 0.3$

$= 0.9$

3) $\log 81 = \log 3^4 = 4 \log 3 \approx 4(0.5) = 2$

Ex: Expand and simplify the following expressions

1) $\log_3 x^2y^3 = \log_3 x^2 + \log_3 y^3$

$= 2\log_3 x + 3\log_3 y$

2) $\log_2 \frac{x^2+1}{2^x} = \log_2 (x^2+1) - \underbrace{\log_2 2^x}_{= x\log_2 2 = x(1) = x}$

$= \log_2 (x^2+1) - x$

3) $\ln \frac{x^2\sqrt{x^2-1}}{e^x} = \ln x^2\sqrt{x^2-1} - \ln e^x$

$= \ln x^2 + \ln (x^2-1)^{1/2} - \overbrace{\ln e^x}^{= x\ln e = x}$

$= 2\ln x + \frac{1}{2}\ln (\underbrace{x^2-1}_{(x+1)(x-1)}) - x$

$= 2\ln x + \frac{1}{2}\left(\ln(x+1) + \ln(x-1)\right) - x$