Name: ______________________

Instructor: ______________________

MATH 10250, Exam 1
June 29, 2018

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 20 minutes.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 16 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice	
6.	
7.	
8.	
9.	
10.	
11.	
12.	
13.	
14.	
15.	
Total	

Name: ____________________

Instructor: ____________________

Multiple Choice

Choose **one** answer for each problem.

1.(4 pts) Which formula below is the equation of a circle of radius 3 with center $(-2, 1)$?

(a) $(x+2)^2 + (y-1)^2 = 3$

(b) $(x-2)^2 + (y-1)^2 = 9$

(c) $(x-1)^2 + (y+2)^2 = 9$

(d) $(x-1)^2 + (y+2)^2 = 3$

(e) $(x+2)^2 + (y-1)^2 = 9$

$9 = (x+2)^2 + (y-1)^2$

2.(4 pts) The equation of line L is given by:

$$y = -\frac{1}{3}x + 1$$

Which of the following statements is **true** about the line L?

~~(a)~~ The y-intercept of line L is 0

(b) Line L is parallel to the line $3y + x = 0$ $y = -\frac{1}{3}x$

~~(c)~~ The slope of line L is 1

(d) The point $(0, 0)$ is on line L

(e) None of the above

Name: ____________________

Instructor: ____________________

3.(4 pts) Which of the following limits **does not** exist?

(a) $\lim\limits_{x\to 3} \dfrac{3}{x-3}$ (b) $\lim\limits_{x\to -1} \dfrac{x+1}{x-1}$ (c) $\lim\limits_{x\to 2} \dfrac{x^2-4x+4}{x-2}$

(d) $\lim\limits_{x\to -1} \sqrt{(x+1)^2}$ (e) $\lim\limits_{x\to 0^-} |x|$

a) $\lim\limits_{x\to 3} \dfrac{3}{x-3}$ DNE

d) $\lim\limits_{x\to -1} \sqrt{(x+1)^2} = \sqrt{(-1+1)^2} = 0$

b) $\lim\limits_{x\to -1} \dfrac{x+1}{x-1} = \dfrac{-1+1}{-1-1} = 0$

e) $\lim\limits_{x\to 0^-} |x| = |0| = 0$

c) $\lim\limits_{x\to 2} \dfrac{x^2-4x+4}{x-2} = \lim\limits_{x\to 2} \dfrac{(x-2)^{\cancel{2}}}{\cancel{x-2}} = \lim\limits_{x\to 2} x-2 = 0$

4.(4 pts) Which of the following is the derivative of $f(x) = \frac{1}{4}x^2 + \sqrt[3]{x} + \frac{1}{x^4} + 2\sqrt{3}$

$= \frac{1}{4}x^2 + x^{1/3} + x^{-4} + 2\sqrt{3}$

(a) $\frac{1}{2}x + \frac{1}{3}x^{-2/3} - 4x^{-5} + 2\sqrt{3}$ (b) $\frac{1}{4}x + \frac{1}{2}x^{-1/2} - 4x^{-5}$

(c) $\frac{1}{2}x + \frac{1}{3}x^{-2/3} - 4x^{-5}$ (d) $\frac{1}{8}x + \frac{1}{3}x^{-2/3} - 4x^{-5}$

(e) $\frac{1}{4}x + \frac{1}{3}x^{-2/3} - 4x^{-3}$

$f'(x) = \frac{1}{2}x + \frac{1}{3}x^{-2/3} - 4x^{-5}$

Name: ______________________

Instructor: ______________________

5.(4 pts) Suppose

$$f(x) = x^2 + 3 \quad \text{and} \quad g(x) = \frac{1}{\sqrt{x}}$$

Which of the following is $(f \circ g)(2)$?

(a) $\frac{7\sqrt{2}}{2}$ (b) $\frac{1}{\sqrt{7}}$ (c) 5 (d) $2\sqrt{2} + 3$ (e) $\frac{7}{2}$

$f(g(x)) = f\left(\frac{1}{\sqrt{x}}\right) = \left(\frac{1}{\sqrt{x}}\right)^2 + 3 = \frac{1}{x} + 3$

$(f \circ g)(2) = \frac{1}{2} + 3 = \frac{7}{2}$

Name: ______________________

Instructor: ____________________

Partial Credit

You must show your work on the partial credit problems to receive credit!

6. (6 pts.) Write the **slope intercept form** of the line through the points $(4, 2)$ and $(11, 9)$.

slope of the line: $\frac{9-2}{11-4} = \frac{7}{7} = 1$

point slope form:

$y - y_0 = m(x - x_0)$

$\Rightarrow y - 2 = 1(x - 4)$

$\Rightarrow \boxed{y = x - 2}$ $\leftarrow$ slope intercept form

Name: ____________________

Instructor: ____________________

7. (6 pts.) Given

$$f(x) = x^3 \quad \text{and} \quad g(x) = \sqrt[3]{x+1}$$

compute the following:

(a) $(f+g)(x) =$ $x^3 + \sqrt[3]{x+1}$

(b) $(fg)(x) =$ $x^3 \sqrt[3]{x+1}$

(c) $(f \circ g)(x) =$ $f(\sqrt[3]{x+1}) = (\sqrt[3]{x+1})^3 = x+1$

Name: ______________________

Instructor: ______________________

8. (6 pts.) Use the limit definition of a derivative to compute $f'(x)$ for

$$f(x) = 2x^2 + 3$$

You must compute the limit, you are not allowed to use rules of differentiation.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{[2(x+h)^2 \cancel{+3}] - [2x^2 \cancel{+3}]}{h}$$

$$= \lim_{h \to 0} \frac{2(\cancel{x^2} + 2xh + h^2) - \cancel{2x^2}}{h}$$

$$= \lim_{h \to 0} \frac{4x\cancel{h} + 2h^{\cancel{2}}}{\cancel{h}}$$

$$= \lim_{h \to 0} 4x + 2h$$

$$= 4x$$

Name: ______________________

Instructor: ______________________

9. (8 pts.) Find all values of x that satisfy the inequality

$$\frac{|x^2-4|}{x-2} \geq 0$$

Need: ① $\dfrac{|x^2-4|}{x-2} = 0$

or

② $\dfrac{|x^2-4|}{x-2} > 0$

① $\dfrac{|x^2-4|}{x-2} = 0 \iff |x^2-4| = 0$ and $\dfrac{|x^2-4|}{x-2}$ is defined

$\iff \boxed{x=-2}$

② $\dfrac{|x^2-4|}{x-2} > 0 \iff x-2>0$

$\iff \boxed{x>2}$

The inequality is satisfied when $x=-2$ or $x>2$.

In interval notation: $\{-2\} \cup (2, \infty)$

Name: ____________________

Instructor: ____________________

10. (12 pts.) Given $f(x) = \begin{cases} \dfrac{x^2 - x - 2}{x - 2} & \text{if } x < 2 \\ x + 1 & \text{if } x \geq 2 \end{cases}$

Compute the following:

(a) $\lim_{x \to 2^-} f(x) =$ $\lim_{x \to 2^-} \dfrac{x^2 - x - 2}{x - 2} = \lim_{x \to 2^-} \dfrac{\cancel{(x-2)}(x+1)}{\cancel{x-2}}$

$= \lim_{x \to 2^-} x + 1 = \boxed{3}$

(b) $\lim_{x \to 2^+} f(x) =$ $\lim_{x \to 2^+} x + 1 = \boxed{3}$

(c) $\lim_{x \to 2} f(x) =$ $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = \boxed{3}$

(d) $f(2) =$ $2 + 1 = \boxed{3}$

(e) Is f continuous at $x = 2$? Justify your answer.

Yes, because $\lim_{x \to 2} f(x) = f(2)$

Name: ______________________

Instructor: ______________________

11. (8 pts.) Find all the roots of the polynomial

$$p(x) = x^3 + x^2 - x$$

Need to find values of x satisfying

$$0 = x^3 + x^2 - x.$$

We have

$$x^3 + x^2 - x = x(x^2 + x - 1),$$

so $\boxed{x=0}$ is a root.

Now, we use the quadratic equation to find the roots of $x^2 + x - 1$:

$$\text{roots} = \frac{-1 \pm \sqrt{1 - 4(1)(-1)}}{2(1)}$$

$$= \boxed{\frac{-1 \pm \sqrt{5}}{2}}$$

So, the roots of $p(x)$ are: $0, \frac{-1+\sqrt{5}}{2}, \frac{-1-\sqrt{5}}{2}$.

Name: ________________________

Instructor: ________________________

12. (8 pts.) Simplify

$$\frac{\left(\frac{2x^2+3}{x^3}\right)+\left(\frac{1}{x}\right)}{\left(\frac{x^4+3}{5x^2}\right)(x^3)}$$

into a single rational expression (that is, the result should be the quotient of two polynomials).

$$\frac{\left(\frac{2x^2+3}{x^3}\right)+\left(\frac{1}{x}\right)}{\left(\frac{x^4+3}{5x^2}\right)(x^3)} = \frac{\frac{2x^2+3}{x^3}+\frac{x^2}{x^3}}{\frac{x^4+3}{5\cancel{x^2}}\cdot\frac{x^{\cancel{3}}}{1}}$$

$$= \frac{\frac{2x^2+x^2+3}{x^3}}{\frac{x^5+3x}{5}}$$

$$= \frac{3x^2+3}{x^3} \div \frac{x^5+3x}{5}$$

$$= \frac{3x^2+3}{x^3} \cdot \frac{5}{x^5+3x}$$

$$= \boxed{\frac{15x^2+15}{x^8+3x^4}}$$

Name: ______________________________

Instructor: ___________________________

13. (8 pts.) Under a set of controlled laboratory conditions, the size of the population of a certain bacteria culture at time t (in minutes) is described by the function

$$f(t) = 3t^2 + 2t + 1$$

(a) Find the **average** growth rate of the bacteria between $t = 0$ to $t = 1$.

$f(0) = 1$

$f(1) = 6$

average growth rate between $t=0$ and $t=1$ = slope of line through $(0,1)$ and $(1,6)$ = $\frac{6-1}{1-0}$ = $\boxed{5}$

(b) Find the rate of growth (this is **instantaneous** growth rate) at $t = 10$.

$f'(t) = 6t + 2$

$f'(10) = 6(10) + 2 = \boxed{62}$

Name: ______________________

Instructor: ______________________

14. (10 pts.) Let $f(x) = 4x^4 + 2x + 3$

(a) Give the **slope** of the tangent line of f at $x = 1$.

$f'(x) = 16x^3 + 2$

$f'(1) = 16 + 2 = \boxed{18}$

(b) Give the **equation** of the tangent line to f at $x = 1$. Give the equation in **slope-intercept** form.

point on the line: $(1, f(1))$, $f(1) = 4 + 2 + 3 = 9$

$= (1, 9)$

point slope form:

$y - y_0 = m(x - x_0)$

$y - 9 = 18(x - 1) = 18x - 18$

slope intercept form:

$\boxed{y = 18x - 9}$

Name: ______________________

Instructor: ____________________

15. (10 pts.) Determine the domain of the following functions:

(a) $f(x) = \sqrt{x^2 - 1}$

domain = all x such that $x^2 - 1 \geq 0$

$= (-\infty, -1] \cup [1, \infty)$

(b) $g(x) = 7x^5 + 13x^3 + x$

domain = $(-\infty, \infty)$

(c) $h(x) = \dfrac{15x^2 + x}{x^3 - 4x}$

domain = all x such that $\underbrace{x^3 - 4x}_{x(x^2-4) = x(x+2)(x-2)} \neq 0$

$= \boxed{\text{all } x \text{ except } 0, 2, -2.}$

(d) $p(x) = \sqrt[3]{2x + 3}$

domain = $(-\infty, \infty)$

(e) $q(x) = 6$

domain = $(-\infty, \infty)$

Name: ____________________

Instructor: ____________________

Extra Credit

1. (5 pts.) Prove that if a function $f(x)$ is differentiable at x_0, then it must be continous at x_0.

If f is differentiable at x_0, then the limit

$$\lim_{h\to 0} \frac{f(x_0+h)-f(x_0)}{h}$$

exists. Therefore, the product of limits

$$\left(\lim_{h\to 0} \frac{f(x_0+h)-f(x_0)}{h}\right)\left(\lim_{h\to 0} h\right)$$

also exists and equals zero, since $\lim_{h\to 0} h = 0$.

Thus, we have

$$0 = \left(\lim_{h\to 0} \frac{f(x_0+h)-f(x_0)}{h}\right)\left(\lim_{h\to 0} h\right)$$

$$= \lim_{h\to 0}\left[\left(\frac{f(x_0+h)-f(x_0)}{\cancel{h}}\right)(\cancel{h})\right]$$

$$= \lim_{h\to 0} f(x_0+h)-f(x_0)$$

$$\Rightarrow \lim_{h\to 0} f(x_0+h) = f(x_0)$$

We do a change of variables by letting $h = x - x_0$. Then

$$\lim_{(x-x_0)\to 0} f(x_0+(x-x_0)) = f(x_0)$$

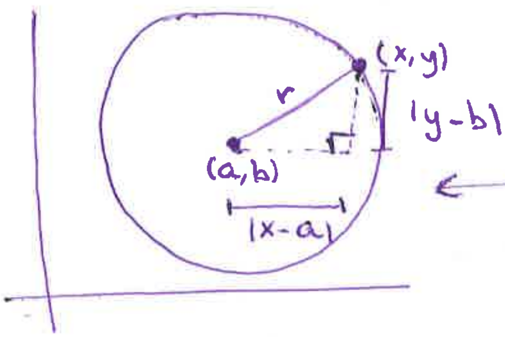
$\Rightarrow \boxed{\lim_{x\to x_0} f(x) = f(x_0)}$, which is exactly the definition of continuity of f at x_0. Thus, f is continuous at x_0.

Name: ______________________

Instructor: ______________________

2. (5 pts.) Prove that any point (x, y) on a circle with center (a, b) and radius r must satisfy the equation

$$r^2 = (x-a)^2 + (y-b)^2$$



Consider a circle with center (a,b) and a point (x,y) on the circle. We can form a right triangle as shown, with sides $|x-a|$ and $|y-b|$ and hypotenuse r.

By the Pythagorean Theorem, we get

$$r^2 = (|x-a|)^2 + (|y-b|)^2$$

$$\Rightarrow r^2 = (x-a)^2 + (y-b)^2.$$

Alternatively, we could just apply the distance formula:

distance between (a,b) and (x,y) $= r = \sqrt{(x-a)^2 + (y-b)^2}$

$$\Rightarrow r^2 = (x-a)^2 + (y-b)^2$$