MATH 10250 Extra Credit Problems

1. Find the **slope** of the tangent line of $g(x) = (2-x)^3(e^5 + 5\ln(2x+3))$ at the point $x = 0$.

2. Which integral do you obtain if you preform the u substitution $u = \sqrt{2x} + 1$ to the integral $\int \frac{1}{\sqrt{2x}+1} dx$

3. Compute
$$\lim_{x \to 3} \frac{x-3}{x^2 - x - 6}$$

4. Compute $\frac{dy}{dx}$ given
$$e^{x^3} y - \ln(x)x^2 = y + 1$$

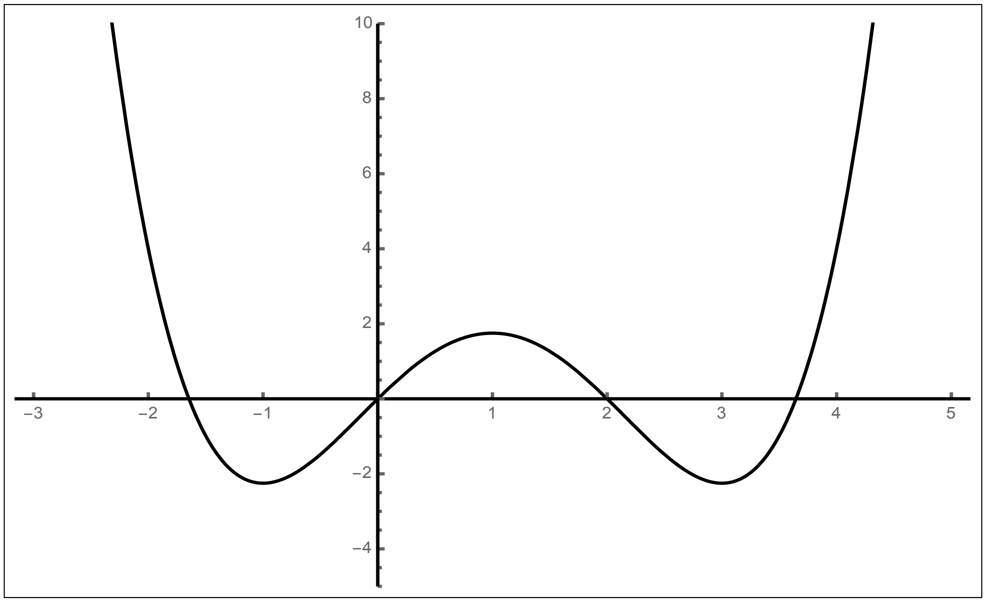
5. The weekly demand for the LectroCopy photocopying machine is given by the demand equation
$$p(x) = 20 - 4x$$
where p denotes the wholesale unit price in dollars and x denotes the quantity demanded. The weekly total cost function for manufacturing these copiers is given by
$$C(x) = 2x^3 - x^2 + 10x + 120$$
where $C(x)$ denotes the total cost incurred in producing x units.

   Find the revenue function, the profit function, the average cost function, and the marginal profit function.

6. The demand equation for a certain product is
$$p(x) = 10 - 2x$$
Compute the elasticity of demand when $p = 15$. Is the demand elastic, unitary, or inelastic at this price?

7. Suppose the whole sale price of a certain brand of eggs, p (in dollars per carton), is related to the weekly supply, x (in thousands of carton), by the equation
$$10p^2 - x^2 = 186$$
If 8,000 ($x = 8$) carton of eggs are available at the beginning of a certain week and the price is failing at the rate of \$0.02 per carton per week, at what rate is the weekly supply changing?

8. Given the graph of f



- The intervals on which f is decreasing are:
- The intervals on which f is increasing are:
- The intervals on which f is concave upward are:
- The interval on which f is concave downward is:
- The critical numbers of f are: $x =$
- The inflection points of f are: $x =$
  Fill in the blanks:
- $f'(x)$ ___ 0 for x inside $(-1, 1)$
- $f'(x)$ ___ 0 for x inside $(-\infty, -1)$
- $f''(x)$ ___ 0 for x inside $(0, 2)$
- $f''(x)$ ___ 0 for x inside $(2, \infty)$

9. Let $g(x) = \dfrac{x}{x-2}$

   (a) **Domain** of g is:

   (b) The y**-intercept** of g is: (0, ___)

   (c) The x**-intercept** of g is: (___, 0)

   (d) Find the **vertical asymptote** of g:

   (e) Find the **horizontal asymptote** of g by computing:

   $$\lim_{x \to -\infty} \frac{x}{x-2} =$$

   $$\lim_{x \to \infty} \frac{x}{x-2} =$$

(f) Find the interval of **increasing** and **decreasing** of g:
g is increasing on:
g is decreasing on:

(g) Find the interval of concavity of g:
g is concave downward on:
g is concave upward on:

10. Odyssey Travel Agency's monthly profit, P (**in thousands of dollars**), depends on the amount of money spent on advertising each month, denote x (**in thousands of dollars**). The relationship between P and x is given by:

$$P(x) = -x^2 + 8x + 20$$

(a) To maximize its monthly profits, what should be Odyssey's monthly advertising budget? (**note**: your answer should be more than a thousand of dollars).
(b) What is the maximum monthly profit realizable?

11. The temperature of a cup of coffee t minutes after it is poured is given by

$$T = 70 + 100e^{-.05t}$$

a) What was the temperature of the coffee when it was poured?

b) When will the coffee be cool enough to drink, that is, when will it reach 120 degrees?

12. A culture of bacteria that initially contained 3000 bacteria has a count of 18000 bacteria after 2 hours.

(a) Determine the function $Q(t) = Q_0 e^{kt}$, where $Q(t)$ is the population of the culture of bacteria after t hours.
(b) Find the number of bacteria present after 4 hours

13. Given $f'(x) = e^{-6x} + 1$, find $f(x)$ given that $f(0) = \dfrac{5}{6}$.

14. Compute the following integrals:

(a) $\displaystyle\int 2x^3 + \frac{1}{x^2} - x + 3\sqrt{x} - \frac{1}{x} - 4\,dx$

(b) $\displaystyle\int 3t^2 \left(t^3 + 3\right)^{14}\,dt$

15. Given that

$$\int_2^3 f(x)\,dx = 6 \quad \text{and} \quad \int_2^3 g(x)\,dx = -2.$$

Compute $\displaystyle\int_2^3 f(x) - 4g(x)\,dx$.

16. Compute the following definite integrals:

(a) $\int_1^4 \frac{1}{\sqrt{x}} - 3\,dx$

(b) $\int_0^1 xe^{x^2}\,dx$

17. Find the AREA underneath the curve $h(x) = \frac{2}{x^2}$ on the interval $[1, 2]$.

18. Given $f(x) = x^2 + 3x$, compute

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

(For this one I want to see the whole computation.)

19. Approximate the area under the curve $f(x) = e^x + 3$ between $x = 0$ and $x = 6$ using 3 rectangles and midpoints (the height of each rectangle should be computed using the midpoint of its interval base).

20. What is the area between $f(x) = 2x + 3$ and $g(x) = x$ on the interval $[0, 1]$.
Hint: Draw a picture.

21. Each integral on the left is equal to exactly one integral on the right. Match them.

(1) $\int_a^a e^x dx$	$\int_{-2}^2 xdx + \int_{-2}^2 4dx$ (A)
(2) $\int_1^5 e^x dx$	$\int_0^1 u^5 du$ (B)
(3) $\int_1^e \frac{(\ln x)^5}{x} dx$	$\int_1^4 \frac{2}{u^2} du$ (C)
(4) $\int_{-1}^1 \sqrt{x+1} dx$	$-\int_5^1 e^x dx$ (D)
(5) $\int_0^1 e^x dx + \int_1^2 e^x dx$	$\int_0^2 \sqrt{u} du$ (E)
(6) $\int_{-2}^2 x + 4dx$	$\int_0^2 e^x dx$ (F)
(7) $2\int_0^1 \frac{3x^2+2x+1}{(x^3+x^2+x+1)^2} dx$	0 (G)

22. Find the average value of

(a) e^x on [0,10]

(b) $\frac{1}{x-1}$ on [2,10]

23. Given that:

A) The area under $f(x)$ on [0,1] is 2.
B) The area under $f(x)$ on [0,5] is 7.
C) The area under $g(x)$ on [0,1] is 1.
D) The area under $g(x)$ on [1,5] is 2.

Either find the following or explain why there is not enough information.

a) $\int_0^5 f(x) - g(x)dx$
b) $\int_0^1 f(x) - g(x)dx$
c) $\int_1^5 f(x) + 4g(x)dx$
d) $\int_2^5 6f(x)dx$
e) $\int_2^5 6dx$
f) $\int_1^5 2f(x) - 3g(x)dx$

24. Use the laws of logs to expand and simplify.

a) $\log(x(x+1)^4)$
b) $\ln \dfrac{e^x}{1+e^x}$
c) $\ln(xe^{-x^2})$