Extra Credit Problems SOLUTIONS

1) $g'(x) = 3(2-x)^2(-1)\left(e^5 + 5\ln(2x+3)\right) + (2-x)^3\left(\frac{5}{2x+3}(2)\right)$

$= -3(2-x)^2\left(e^5 + 5\ln(2x+3)\right) + (2-x)^3\left(\frac{10}{2x+3}\right)$

$\boxed{g'(0) = -3(2)^2\left(e^5 + 5\ln(3)\right) + (2)^3\left(\frac{10}{3}\right)}$

slope of the tangent line of $g(x)$ at $x=0$.

2) $u = \sqrt{2x} + 1$

$du = \frac{1}{2}(2x)^{-1/2} \cdot 2\,dx = \frac{1}{\sqrt{2x}}\,dx$

$\Rightarrow \sqrt{2x}\,du = dx$, where $u = \sqrt{2x} + 1$

$\Rightarrow \sqrt{2x} = u - 1$

$\Rightarrow (u-1)\,du = dx$

$\Rightarrow \int \frac{1}{\sqrt{2x}+1}\,dx = \int \frac{1}{u}\cdot(u-1)\,du = \boxed{\int \frac{u-1}{u}\,du}$

3) $\lim_{x\to3} \frac{x-3}{x^2-x-6} = \lim_{x\to3} \frac{\cancel{x-3}}{\cancel{(x-3)}(x+2)} = \lim_{x\to3} \frac{1}{x+2} = \frac{1}{3+2} = \boxed{\frac{1}{5}}$

4) $\frac{d}{dx}\left(e^{x^3}y - \ln(x)\,x^2 = y+1\right)$

$\Rightarrow e^{x^3}\cdot 3x^2\cdot y + e^{x^3}\frac{dy}{dx} - \left(\frac{1}{x}\cdot x^2 + \ln(x)\cdot 2x\right) = \frac{dy}{dx}$

$\Rightarrow 3x^2e^{x^3}y + e^{x^3}\frac{dy}{dx} - x - 2x\ln(x) = \frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx}\left(e^{x^3}-1\right) = x + 2x\ln(x) - 3x^2e^{x^3}y$

$\Rightarrow \boxed{\frac{dy}{dx} = \frac{x + 2x\ln(x) - 3x^2e^{x^3}y}{e^{x^3}-1}}$

5) revenue: $R(x) = x \cdot p(x) = x(20-4x) = 20x - 4x^2$

Profit: $P(x) = R(x) - C(x)$

$$= (20x - 4x^2) - (2x^3 - x^2 + 10x + 120)$$

$$= -2x^3 - 3x^2 + 10x - 120$$

average cost: $\bar{C}(x) = \dfrac{C(x)}{x} = \dfrac{2x^3 - x^2 + 10x + 120}{x}$

$$= 2x^2 - x + 10 + \frac{120}{x}$$

marginal profit: $P'(x) = -6x^2 - 6x + 10$

---

6) $p = 10 - 2x \Rightarrow x = \dfrac{p-10}{-2} = -\frac{1}{2}p + 5$

$\Rightarrow f(p) = -\frac{1}{2}p + 5 \rightsquigarrow f'(p) = -\frac{1}{2}$

$$E(p) = \frac{-p\,f'(p)}{f(p)} = \frac{-p\left(-\frac{1}{2}\right)}{-\frac{1}{2}p+5} = \frac{\frac{1}{2}p}{5-\frac{1}{2}p}$$

$$E(15) = \frac{\frac{1}{2}(15)}{5-\frac{1}{2}(15)} = \frac{15/2}{10/2 - 15/2} = \frac{15/2}{-5/2} = \frac{15}{-5} = \boxed{-3}$$

The demand is elastic, since $|-3| > 1$.

---

7) $\dfrac{d}{dt}(10p^2 - x^2 = 186)$

$$\begin{array}{r} 186 \\ 64 \\ \hline 250 \end{array}$$

$\Rightarrow 10 \cdot 2p \dfrac{dp}{dt} - 2x \dfrac{dx}{dt} = 0$, $x = 8$

$p = \sqrt{\dfrac{186 + x^2}{10}} = \sqrt{\dfrac{186+64}{10}} = \sqrt{\dfrac{250}{10}}$
$= 5$

$\Rightarrow \dfrac{dx}{dt} = \dfrac{20p\dfrac{dp}{dt}}{2x}$

weekly supply change

$\dfrac{dp}{dt} = -0.02$

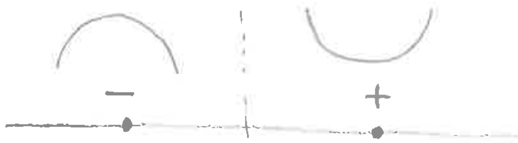
$$= \frac{20(5)(-0.02)}{2(8)} = \frac{-2}{2 \cdot 8} = \boxed{-\frac{1}{8}}$$

9) $g'(x) = \dfrac{(1)(x-2) - (x)(1)}{(x-2)^2} = \dfrac{-2}{(x-2)^2}$ crit pt. @ $x = 2$

$g'(1) = \dfrac{-2}{(1-2)^2} < 0$ 2 $g'(3) = \dfrac{-2}{(3-2)^2} < 0$

$g'(x) = -2(x-2)^{-2}$

$g''(x) = (-2)(-2)(x-2)^{-3} = \dfrac{4}{(x-2)^3}$ crit pt @ $x=2$



$g''(1) = \dfrac{4}{(1-2)^3} < 0$ 2 $g''(3) = \dfrac{4}{(3-2)^3} > 0$

---

10) $P'(x) = -2x + 8 = -2(x-4)$ crit. pt. @ $x=4$

$P''(x) = -2$, so $P(x)$ is always concave down and thus has an absolute max at $x=4$

(a) $\Rightarrow$ max profit when advertising budget is $\boxed{\$\,4{,}000}$

(b) $P(4) = -(4)^2 + 8(4) + 20 = -16 + 32 + 20 = 36$

$\Rightarrow$ max profit realizable is $\boxed{\$\,36{,}000}$

---

11) a) $T(0) = 70 + 100e^{-0.05(0)} = 170$

b) $120 = 70 + 100e^{-0.05t}$

$\Rightarrow e^{-0.05t} = \dfrac{120-70}{100} = \dfrac{50}{100} = \dfrac{1}{2}$

$\Rightarrow \ln(e^{-0.05t}) = \ln\left(\frac{1}{2}\right)$

$\Rightarrow -0.05t = \ln\left(\frac{1}{2}\right)$ $\Rightarrow t = \dfrac{\ln\left(\frac{1}{2}\right)}{-0.05} = \dfrac{-\ln(2)}{-0.05} = \boxed{\dfrac{\ln(2)}{0.05}}$

2^{-1}

12) $Q_0 = 3000$, $Q(2) = 18000$

$18000 = 3000 e^{k(2)}$

$\Rightarrow \quad 6 = e^{2k} \quad \Rightarrow \quad \ln(6) = 2k \quad \Rightarrow \quad k = \frac{\ln(6)}{2}$

a) $Q(t) = 3000 e^{\ln(6)/2 \, t}$

b) $Q(4) = 3000 e^{\ln(6)/2 \cdot (4)} = 3000 e^{2\ln(6)}$

---

13) $\int e^{-6x} + 1 \, dx = -\frac{1}{6} e^{-6x} + x + C$

$\Rightarrow f(x) = -\frac{1}{6} e^{-6x} + x + C$, where $f(0) = \frac{5}{6}$

$\Rightarrow \boxed{f(x) = -\frac{1}{6} e^{-6x} + x + 1}$

$\Rightarrow f(0) = -\frac{1}{6} e^{-6(0)} + 0 + C$

$= -\frac{1}{6} + C$

$\Rightarrow C = 1$

---

14) a) $\int 2x^3 + x^{-2} - x + 3x^{1/2} - \frac{1}{x} - 4 \, dx$

$= \frac{2}{4} x^4 + \frac{x^{-1}}{-1} - \frac{x^2}{2} + \frac{3x^{3/2}}{3/2} - \ln|x| - 4x + C$

$= \boxed{\frac{1}{2} x^4 - \frac{1}{x} - \frac{1}{2} x^2 + 2x^{3/2} - \ln|x| - 4x + C}$

b) $\int 3t^2 \underbrace{(t^3+3)}_{u}{}^{14} \, dt$ $\qquad u = t^3 + 3$, $du = 3t^2 \, dt$

$= \int u^{14} \, du = \frac{u^{15}}{15} + C = \boxed{\frac{1}{15} (t^3+3)^{15} + C}$

---

15) $\int_2^3 f(x) - 4g(x) \, dx = \underbrace{\int_2^3 f(x) \, dx}_{6} - 4 \underbrace{\int_2^3 g(x) \, dx}_{-2}$

$= 6 - 4(-2)$

$= \boxed{14}$

16) a) $\int_1^4 x^{-1/2} - 3\,dx = \dfrac{x^{1/2}}{1/2} - 3x \Big|_1^4$

$= 2\sqrt{x} - 3x \Big|_1^4 = \left(2\sqrt{4} - 3(4)\right) - \left(2\sqrt{1} - 3(1)\right)$

$= (4 - 12) - (2 - 3)$

$= \boxed{-7}$

b) $\int_0^1 x e^{x^2} dx$ $\qquad u = x^2$

$du = 2x\,dx \implies \frac{1}{2} du = x\,dx$

$= \frac{1}{2}\int_{u(0)}^{u(1)} e^u\,du = \frac{1}{2}\int_0^1 e^u\,du = \frac{1}{2} e^u \Big|_0^1 = \frac{1}{2}(e^1 - e^0)$

$= \boxed{\frac{1}{2}(e - 1)}$

---

17) Area $= \int_1^2 h(x)\,dx = \int_1^2 \frac{2}{x^2}\,dx = \int_1^2 2x^{-2}\,dx = \dfrac{2x^{-1}}{-1}\Big|_1^2$

$= 2\left(\frac{-1}{2} - (-1)\right) = \boxed{1}$

---

18) $\lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h\to 0} \dfrac{[(x+h)^2 + 3(x+h)] - [x^2 + 3x]}{h}$

$= \lim\limits_{h\to 0} \dfrac{(\cancel{x^2} + 2xh + \cancel{3x} + 3h) - (\cancel{x^2} + \cancel{3x})}{h}$

$= \lim\limits_{h\to 0} \dfrac{2xh + 3h}{h}$

$= \lim\limits_{h\to 0} \dfrac{\cancel{h}(2x+3)}{\cancel{h}}$

$= \boxed{2x + 3}$

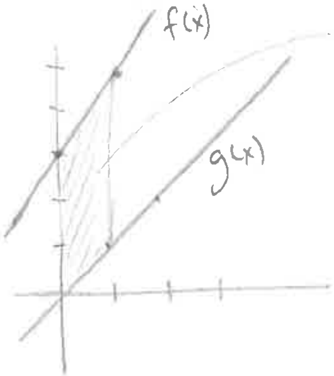
19) $\text{Area} \approx 2(f(1) + f(3) + f(5))$

$= 2((e^1+3) + (e^3+3) + (e^5+3))$

$= 2(e + e^3 + e^5 + 9)$

---

20)



$\text{Area} = (\text{area under } f(x)) - (\text{area under } g(x))$

$= \int_0^1 f(x)\,dx - \int_0^1 g(x)\,dx$

$= \int_0^1 2x+3\,dx - \int_0^1 x\,dx$

$= \int_0^1 (2x+3) - x\,dx$

$= \int_0^1 x+3\,dx = \frac{x^2}{2} + 3x \Big|_0^1 = \frac{1}{2} + 3 = \boxed{\frac{7}{2}}$

---

21) (1) $\int_a^a e^x\,dx = 0 \rightsquigarrow \boxed{G}$

(2) $\int_1^5 e^x\,dx = -\int_5^1 e^x\,dx \rightsquigarrow \boxed{D}$

(3) $\int_1^e \frac{[\ln(x)]^5}{x}\,dx$ $\qquad u = \ln(x)\,dx$, $\quad du = \frac{1}{x}\,dx$

$= \int_{u(1)}^{u(e)} u^5\,du = \int_0^1 u^5\,du \rightsquigarrow \boxed{B}$

(4) $\int_{-1}^1 \sqrt{x+1}\,dx$ $\qquad u = x+1$, $\quad du = dx$

$= \int_{u(-1)}^{u(1)} \sqrt{u}\,du = \int_0^2 \sqrt{u}\,du \rightsquigarrow \boxed{E}$

(5) $\int_0^1 e^x\,dx + \int_1^2 e^x\,dx = \int_0^2 e^x\,dx \rightsquigarrow \boxed{F}$

(6) $\int_{-2}^{2} x + 4\, dx = \int_{-2}^{2} x\, dx + \int_{-2}^{2} 4\, dx \rightsquigarrow \boxed{A}$

(7) $2 \int \frac{3x^2 + 2x + 1}{x^3 + x^2 + x + 1}\, dx$ $\quad u = x^3 + x^2 + x + 1$

$du = 3x^2 + 2x + 1\, dx$

$= 2 \int \frac{1}{u^2}\, du \rightsquigarrow \boxed{C}$

---

22) a) average value of e^x on $[0,10]$ $= \frac{1}{10-0} \int_0^{10} e^x\, dx$

$= \frac{1}{10} e^x \Big|_0^{10} = \frac{1}{10}(e^{10} - e^0) = \boxed{\frac{1}{10}(e^{10} - 1)}$

b) average value of $\frac{1}{x-1}$ on $[2,10]$ $= \frac{1}{10-2} \int_2^{10} \frac{1}{x-1}\, dx$ $\quad u = x-1$

$du = dx$

$= \frac{1}{8} \int_{u(2)}^{u(10)} \frac{1}{u}\, du = \frac{1}{8} \int_1^9 \frac{1}{u}\, du$

$= \frac{1}{8} \ln|u| \Big|_1^9 = \frac{1}{8}(\ln(9) - \cancel{\ln(1)})$

$= \boxed{\frac{1}{8} \ln(9)}$

---

23) (a) $\int_0^5 f(x) - g(x)\, dx = \int_0^5 f(x)\, dx - \left[\int_0^1 g(x)\, dx + \int_1^5 g(x)\, dx \right]$

$= 7 - [1 + 2]$

$= \boxed{4}$

(b) $\int_0^1 f(x) - g(x)\, dx = \int_0^1 f(x)\, dx - \int_0^1 g(x)\, dx = 2 - 1 = \boxed{1}$

(c) $\int_1^5 f(x) + 4g(x)\, dx = \int_1^5 f(x)\, dx + 4\int_1^5 g(x)\, dx$

$= \left[\int_0^5 f(x)\, dx - \int_0^1 f(x)\, dx \right] + 4 \int_1^5 g(x)\, dx$

$= [7 - 2] + 4(2) = \boxed{13}$

(d) $\displaystyle\int_2^5 6\,f(x)\,dx = 6\underbrace{\int_2^5 f(x)\,dx}$

we don't have enough information to determine this integral

(e) $\displaystyle\int_2^5 6\,dx = 6x\Big|_2^5 = 6(5-2) = 6(3) = \boxed{18}$

(f) $\displaystyle\int_1^5 2f(x) - 3g(x)\,dx = 2\int_1^5 f(x)\,dx - 3\int_1^5 g(x)\,dx$

$\displaystyle = 2\left[\int_0^5 f(x)\,dx - \int_0^1 f(x)\,dx\right] - 3\int_1^5 g(x)\,dx$

$= 2(7-2) - 3(2) = 10 - 6 = \boxed{4}$

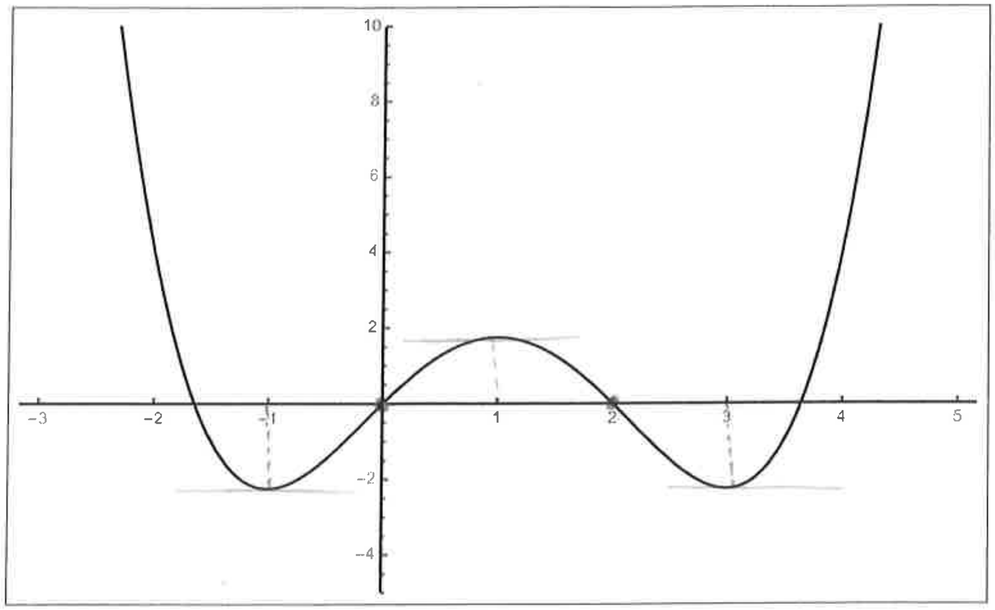
---

24) (a) $\log(x(x+1)^4) = \log(x) + \log((x+1)^4)$

$= \boxed{\log(x) + 4\log(x+1)}$

(b) $\ln\left(\dfrac{e^x}{1+e^x}\right) = \ln(e^x) - \ln(1+e^x) = \boxed{x - \ln(1+e^x)}$

(c) $\ln(xe^{-x^2}) = \ln(x) + \ln(e^{-x^2}) = \boxed{\ln(x) - x^2}$



- The intervals on which f is decreasing are: $(-\infty,-1) \cup (1,3)$
- The intervals on which f is increasing are: $(-1,1) \cup (3,\infty)$
- The intervals on which f is concave upward are: $(-\infty,0) \cup (2,\infty)$
- The interval on which f is concave downward is: $(0,2)$
- The critical numbers of f are: $x = -1, 1, 3$
- The inflection points of f are: $x = 0, 2$

Fill in the blanks:

- $f'(x) \geq 0$ for x inside $(-1, 1)$
- $f'(x) \leq 0$ for x inside $(-\infty, -1)$
- $f''(x) \leq 0$ for x inside $(0, 2)$
- $f''(x) \geq 0$ for x inside $(2, \infty)$

9. Let $g(x) = \dfrac{x}{x-2}$

   (a) **Domain** of g is: $(-\infty, 2) \cup (2, \infty)$, since $g(x)$ is undefined at $x=2$

   (b) The **y-intercept** of g is: $(0, \underline{0})$, since $g(0) = 0$

   (c) The **x-intercept** of g is: $(\underline{0}, 0)$, since $g(x) = 0$ iff $x = 0$

   (d) Find the **vertical asymptote** of g: $x=2$

   (e) Find the **horizontal asymptote** of g by computing:

$$\lim_{x\to-\infty} \frac{x}{x-2} = \lim_{x\to-\infty} \frac{x}{x-2} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to-\infty} \frac{1}{1-\frac{2}{x}} = \boxed{1}$$

$$\lim_{x\to\infty} \frac{x}{x-2} = \boxed{1}$$ by the same process as above

(f) Find the interval of **increasing** and **decreasing** of g:
g is increasing on: none
g is decreasing on: $(-\infty, 2) \cup (2, \infty)$

(g) Find the interval of concavity of g:
g is concave downward on: $(-\infty, 2)$
g is concave upward on: $(2, \infty)$

10. Odyssey Travel Agency's monthly profit, P (**in thousands of dollars**), depends on the amount of money spent on advertising each month, denote x (**in thousands of dollars**). The relationship between P and x is given by:

$$P(x) = -x^2 + 8x + 20$$

(a) To maximize its monthly profits, what should be Odyssey's monthly advertising budget? (**note**: your answer should be more than a thousand of dollars).
(b) What is the maximum monthly profit realizable?

11. The temperature of a cup of coffee t minutes after it is poured is given by

$$T = 70 + 100e^{-.05t}$$

a) What was the temperature of the coffee when it was poured?

b) When will the coffee be cool enough to drink, that is, when will it reach 120 degrees?

12. A culture of bacteria that initially contained 3000 bacteria has a count of 18000 bacteria after 2 hours.

(a) Determine the function $Q(t) = Q_0e^{kt}$, where $Q(t)$ is the population of the culture of bacteria after t hours.
(b) Find the number of bacteria present after 4 hours

13. Given $f'(x) = e^{-6x} + 1$, find $f(x)$ given that $f(0) = \frac{5}{6}$.

14. Compute the following integrals:

(a) $\displaystyle\int 2x^3 + \frac{1}{x^2} - x + 3\sqrt{x} - \frac{1}{x} - 4\,dx$

(b) $\displaystyle\int 3t^2\left(t^3+3\right)^{14}\,dt$

15. Given that

$$\int_2^3 f(x)\,dx = 6 \quad \text{and} \quad \int_2^3 g(x)\,dx = -2.$$

Compute $\displaystyle\int_2^3 f(x) - 4g(x)\,dx$.

16. Compute the following definite integrals: