Name: ____________________

Instructor: ANSWERS

MATH 10250, Practice Exam 2

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 20 minutes.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 14 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(•)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(•)
3.	(a)	(b)	(•)	(d)	(e)
4.	(•)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(•)	(e)

Please do NOT write in this box.

Multiple Choice	______
6.	______
7.	______
8.	______
9.	______
10.	______
11.	______
12.	______
13.	______
14.	______
15.	______
Total	______

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Multiple Choice

1.(4 pts) Compute the **elasticity of demand** when $p = \frac{1}{2}$ given the demand equation:

$$x = \underbrace{4 - 2p}_{f(p)} \quad \Rightarrow f(p) = 4 - 2p \Rightarrow f'(p) = -2$$

(a) $E\left(\frac{1}{2}\right) = \frac{1}{7}$ ~~(b)~~ $E\left(\frac{1}{2}\right) = \frac{1}{3}$ (c) $E\left(\frac{1}{2}\right) = -\frac{1}{3}$

(d) $E\left(\frac{1}{2}\right) = 2$ (e) $E\left(\frac{1}{2}\right) = 1$

$$E(p) = \frac{-p f'(p)}{f(p)} = \frac{-p(-2)}{4-2p} = \frac{2p}{4-2p}$$

$$E\left(\tfrac{1}{2}\right) = \frac{2\left(\frac{1}{2}\right)}{4-2\left(\frac{1}{2}\right)} = \frac{1}{4-1} = \boxed{\frac{1}{3}}$$

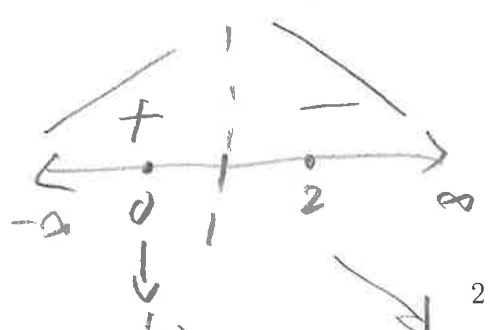
2.(4 pts) Find the interval of **increasing** of $f(x) = 4x - 2x^2$

(a) $(-1, \infty)$ (b) $(-\infty, -1)$ (c) $(0, 1)$

(d) $(1, \infty)$ ~~(e)~~ $(-\infty, 1)$

$f'(x) = 4 - 4x$

$4 - 4x = 0 \Rightarrow 4 = 4x \Rightarrow x = 1$



$\rightarrow$ $\boxed{f \text{ is increasing on } (-\infty, 1)}$

$f'(0) = 4 > 0$ $f'(2) = 4 - 8 < 0$

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3.(4 pts) If we know that the demand when $p = 40$ is **inelastic**, then which **ONE** of the following statement is **TRUE**?

↳ price and revenue go the same direction

(a) Increasing the price will not affect the revenue.

(b) Decreasing the price will cause an increase of revenue.

~~(c)~~ Decreasing the price will cause a decrease of revenue. ←

(d) Increasing the price will cause a decrease of revenue.

(e) Not enough information to tell.

4.(4 pts) Compute the **marginal revenue** when $x = 10$ given

$$R(x) = x^3 - 20x$$

~~(a)~~ 280 (b) 90 (c) 800 (d) 100 (e) 8000

$R'(x) = 3x^2 - 20$

$R'(10) = 3 \cdot 10^2 - 20 = 300 - 20 = \boxed{280}$

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5.(4 pts) Find the **vertical asymptote** of $\frac{x}{x^2+4x+4}$. $= \frac{x}{(x+2)(x+2)}$ $\Rightarrow$ VA is $x = -2$

(a) $x = 2$ and $x = -2$ (b) $x = 2$ (c) $x = 0$

(d) $x = -2$ (e) $x = 0$ and $x = -2$

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Partial Credit

You must show your work on the partial credit problems to receive credit!

6.(x pts.) (a) Compute $\frac{dy}{dx}$ given

$$x^3y - x^2 = y^3 + 1$$

$$\frac{d}{dx}(x^3y - x^2) = \frac{d}{dx}(y^3+1)$$

$$\Rightarrow 3x^2y + x^3\frac{dy}{dx} - 2x = 3y^2\frac{dy}{dx}$$

$$\Rightarrow x^3\frac{dy}{dx} - 3y^2\frac{dy}{dx} = 2x - 3x^2y$$

$$\Rightarrow \left(\frac{dy}{dx}\right)(x^3 - 3y^2) = 2x - 3x^2y$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{2x - 3x^2y}{x^3 - 3y^2}}$$

(b) Find the **slope** of the tangent line of the curve above at $x = 0$ and $y = -1$.

↳ compute $\frac{dy}{dx}$ when $x = 0, y = -1$

$$\frac{dy}{dx} = \frac{2\cdot 0 - 3\cdot 0^2\cdot(-1)}{0^3 - 3(-1)^2} = \frac{0}{-3} = \boxed{0}$$

Slope of the tangent line of the curve above at $x=0$ and $y=-1$

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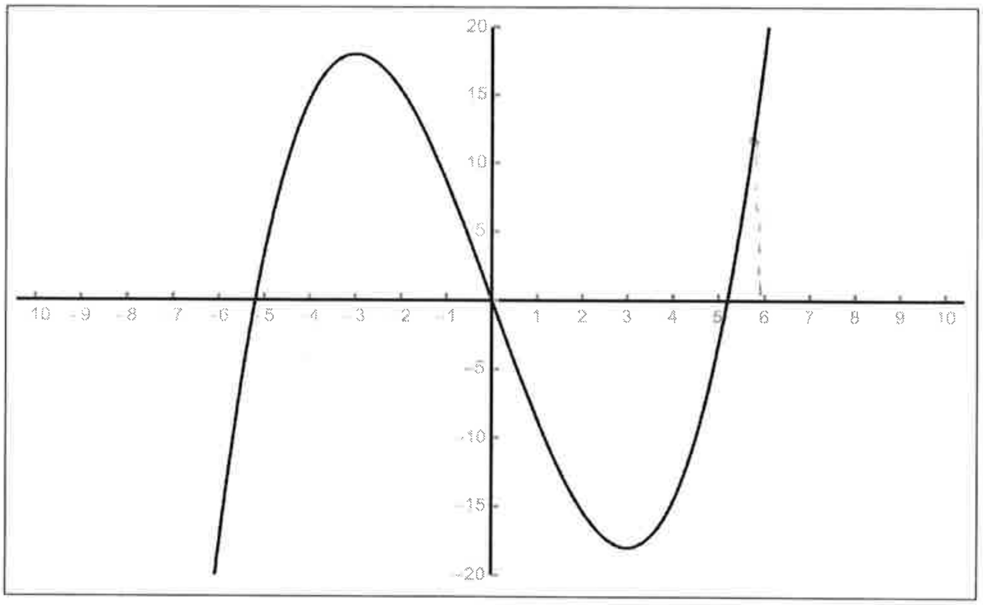
7.(x pts.) Given the graph of f



Fill in the blanks:

- The interval(s) on which f is **decreasing** is: $(-3, 3)$.
- The interval(s) on which f is **increasing** is: $(-\infty, -3) \cup (3, \infty)$
- All the critical numbers of f are $x =$ $-3, 3$.
- $f'(x)$ $>$ 0 on $(-\infty, -3)$.
- f is concave upward on $(0, 6)$.
- f has an inflection point at $x =$ 0.
- $f''(x)$ $<$ 0 on $(-\infty, 0)$ because f is concave downward on $(-\infty, 0)$.
- $f'(3)$ $=$ 0. (since 3 is a critical number of f)
- f has a relative maximum at $x = -3$.
- $f(0) =$ 0.

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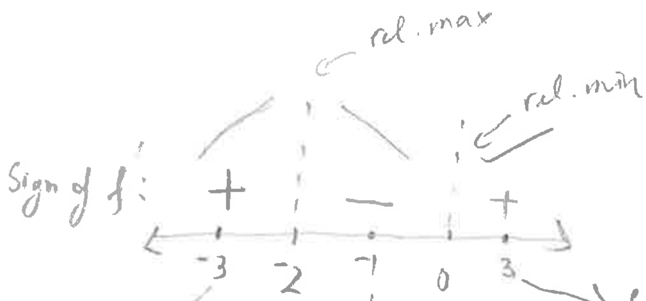
8.(x pts.) Given $f(x) = x^3 + 3x^2 + 1$

(a) Find the interval of **increasing** and **decreasing** of f.

$f'(x) = 3x^2 + 6x$

$3x^2 + 6x = 0 \Rightarrow 3x(x+2) = 0 \Rightarrow x = 0$ or $x = -2$



$f'(3) = 3 \cdot 9 + 18 > 0$

$f'(-3) = 3(9) - 18 > 0$ $\quad f'(-1) = 3 - 6 < 0$

$\boxed{\begin{array}{l} f \text{ is increasing on } (-\infty, -2) \cup (0, \infty) \\ f \text{ is decreasing on } (-2, 0) \end{array}}$

(b) Fill in the blanks:

- f has a relative **maximum** at $x =$ <u>-2</u>. And the value of the relative maximum is $f(-2) = (-2)^3 + 3(-2)^2 + 1 = -8 + 12 + 1 = \boxed{5}$
- f has a relative **minimum** at $x =$ <u>0</u>. And the value of the relative minimum is $f(0) = \boxed{1}$.

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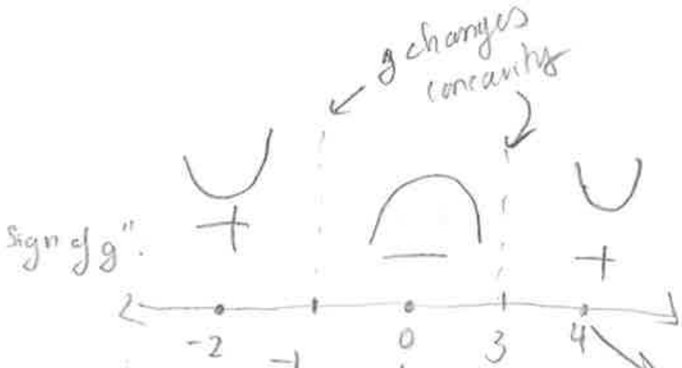
9.(x pts.) Find all the **inflection points** of $g(x) = \frac{1}{4}x^4 - x^3 - \frac{9}{2}x^2 + 3$.

$g'(x) = x^3 - 3x^2 - 9x$

$g''(x) = 3x^2 - 6x - 9$

$3x^2 - 6x - 9 = 0 \Rightarrow 3(x^2 - 2x - 3) = 0 \Rightarrow 3(x-3)(x+1) = 0$

$\Rightarrow x = 3$ or $x = -1$



$g''(4) = 3(4-3)(4+1) > 0$

$g''(0) = -9 < 0$

$g''(-2) = 3\underbrace{(-2-3)}_{-5}\underbrace{(-2+1)}_{-1} > 0$

So, g has inflection points at $x = -1$ and $x = 3$

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10.(x pts.) The Custom Office makes a line of executive desks. It estimated that the total cost for making x units of the Junior Executive model is

$$C(x) = 80x + 1500$$

dollars/year. Also, it revenue (in dollars) per year is given by

$$R(x) = x^3 - 20x.$$

(a) Find the **marginal profit** function.

$$P(x) = R(x) - C(x) = x^3 - 20x - (80x + 1500)$$

$$\Rightarrow P(x) = x^3 - 20x - 80x - 1500$$

$$\Rightarrow P(x) = x^3 - 100x - 1500$$

The marginal profit is : $\boxed{P'(x) = 3x^2 - 100}$

(b) Compute the marginal profit when $x = 20$.

$\hookrightarrow P'(20)$

$$P'(20) = 3(20^2) - 100 = 3(400) - 100 = 1200 - 100$$

$$= \boxed{1100}$$

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11.(x pts.) Given the function $f(x) = x + \frac{1}{x}$. $= x + x^{-1}$

(a) Find all the **critical numbers** ~~of~~ of f.

$f'(x) = 1 - x^{-2} = 1 - \frac{1}{x^2}$

$1 - \frac{1}{x^2} = 0 \Rightarrow x^2\left(1 - \frac{1}{x^2}\right) = x^2 \cdot 0 \Rightarrow x^2 - 1 = 0$

$\Rightarrow (x-1)(x+1) = 0$

$\Rightarrow x = 1,\ x = -1$

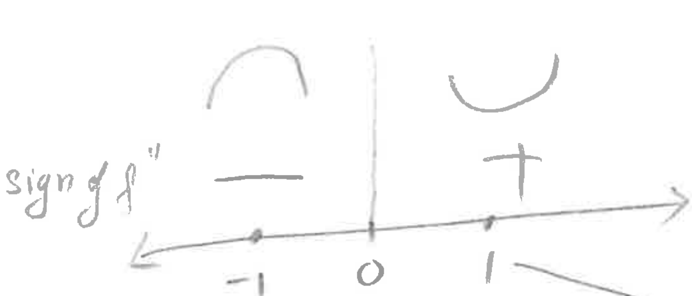
So, all the critical numbers of f are : $\boxed{x = 1,\ x = -1}$

(b) Find the interval of **concavity** of f.

$f''(x) = +2x^{-3} = \frac{2}{x^3}$

$\frac{2}{x^3} = 0 \Leftrightarrow 2 = 0 \rightarrow$ can't be . So the eq $\frac{2}{x^3} = 0$ has NO solution.

But, $f(x) = x + \frac{1}{x}$ is undefined at $x = 0$. So, we get



So, f concave downward on $(-\infty, 0)$

f concave upward on $(0, \infty)$

$f''(1) = \frac{2}{1^3} = \frac{2}{1} > 0$

$f''(-1) = \frac{2}{(-1)^3} = \frac{2}{-1} < 0$

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12.(x pts.) Find the **absolute maximum value** and the **absolute minimum value** of the function

$$g(x) = x^2 - 2x - 3 \qquad \text{on } [-2, 3].$$

$g'(x) = 2x - 2$

$2x - 2 = 0 \Rightarrow x = 1 \longrightarrow$ inside $[-2, 3] \rightarrow$ take.

x	$y = x^2 - 2x - 3$
-2	$(-2)^2 - 2(-2) - 3 = 4 + 4 - 3 = 5 \leftarrow$ abs max
1	$1^2 - 2(1) - 3 = 1 - 2 - 3 = -4 \leftarrow$ abs min
3	$3^2 - 2(3) - 3 = 9 - 6 - 3 = 0$

The **absolute maximum value** of g is: $\boxed{5}$

The **absolute minimum value** of g is: $\boxed{-4}$

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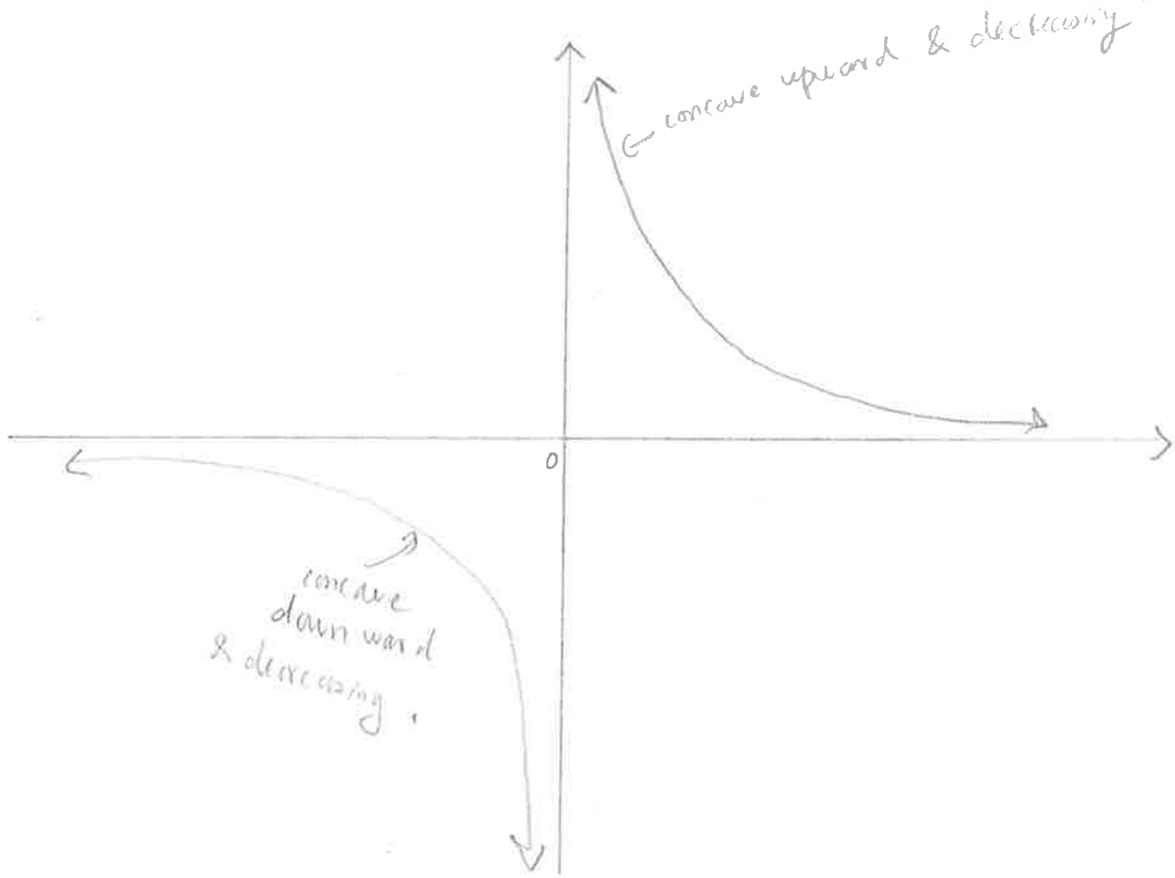
13.(x pts.) Sketch the graph of a function with the following properties

- It has a **vertical asymptote** at $x = 0$.
- It has a **horizontal asymptote** at $y = 0$.
- It is **decreasing** on $(-\infty, 0)$ and $(0, \infty)$.
- It is **concave upward** on $(0, \infty)$.
- It is **concave downward** on $(-\infty, 0)$.



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14.(x pts.) The quantity demanded each month of the Walter Serkin recording of Beethoven's Moonlight Sonata, manufactured by Phonola Record Industries, is related to the price per compact disc. The demand equation is given by

$$\text{price} \rightarrow p = -x + 6 \qquad x = \text{number of disc}$$

where p denotes the unit price in dollars and x is the number of disc demanded.

(a) Phonola wants to maximize its revenue in selling this recording. How many discs should the company produce to maximize its revenue?

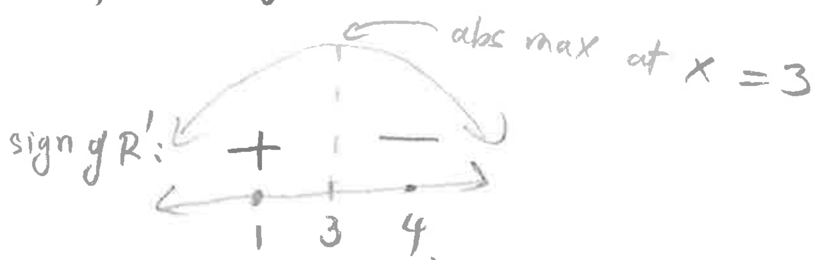
Q: $x = ?$ so that revenue is maximize

$R(x) = xp \Rightarrow R(x) = x(-x+6) \Rightarrow R(x) = -x^2 + 6x$

$R'(x) = -2x + 6$

$-2x + 6 = 0 \Rightarrow x = 3$

Now, see if $x = 3$ gives maximum value for revenue



$R'(1) = 4 > 0 \qquad R'(4) = -8 + 6 < 0$

So, the company should produce $\boxed{\text{3 discs}}$ per month to maximize revenue

(b) What is the maximum revenue realizable?

$\rightarrow R(3)$

$R(3) = -3^2 + 6(3) = -9 + 18 = \boxed{9}$

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15.(x pts.) Suppose the whole sale price of a certain brand of eggs, p (in dollars per carton), is related to the weekly supply, x , by the equation

$$10p^2 - x^2 = 65$$

If 5 cartons of eggs, $x = 5$, are available at the beginning of a certain week and the price is decreasing at the rate of \$2 per carton per week, at what rate is the weekly supply changing?

x = quantity supplied

p = unit price.

Summary : $x = 5$, $\frac{dp}{dt} = -2$, then $\frac{dx}{dt} = ?$

$10p^2 - x^2 = 65$

$\frac{d}{dt}\left[10p^2 - x^2\right] = \frac{d}{dt}[65]$

$20p\left(\frac{dp}{dt}\right) - 2x\left(\frac{dx}{dt}\right) = 0$ ← now, plug in $x = 5$, $\frac{dp}{dt} = -2$

$\Rightarrow 20p(-2) + 2(5)\left(\frac{dx}{dt}\right) = 0$

$\Rightarrow -40p + 10\left(\frac{dx}{dt}\right) = 0$ ← need p in order to find $\frac{dx}{dt}$ → use $10p^2 - x^2 = 65$ when $x = 5$

$\Rightarrow 10p^2 - 25 = 65$

$\Rightarrow 10p^2 = 90$

$\Rightarrow p^2 = 9$

$\Rightarrow p = 3$ or $p = -3$ ignore

$\Rightarrow -40(3) + 10\left(\frac{dx}{dt}\right) = 0$

$\Rightarrow 10\left(\frac{dx}{dt}\right) = 120$

$\boxed{\frac{dx}{dt} = 12}$