

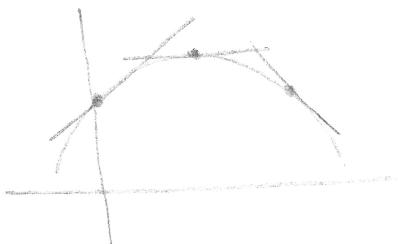
### Written HW 3

$$1) \lim_{h \rightarrow 0^+} \left. \frac{|x+h| - |x|}{h} \right|_{x=0} = \lim_{h \rightarrow 0^+} \frac{|h|}{h} = \lim_{h \rightarrow 0^+} \frac{h}{h} = 1$$

$$\lim_{h \rightarrow 0^-} \left. \frac{|x+h| - |x|}{h} \right|_{x=0} = \lim_{h \rightarrow 0^-} \frac{|h|}{h} = \lim_{h \rightarrow 0^-} \frac{-h}{h} = -1$$

Since  $\lim_{h \rightarrow 0^+} \left. \frac{|x+h| - |x|}{h} \right|_{x=0} \neq \lim_{h \rightarrow 0^-} \left. \frac{|x+h| - |x|}{h} \right|_{x=0}$ , we conclude that  $\lim_{h \rightarrow 0} \left. \frac{|x+h| - |x|}{h} \right|_{x=0}$  does not exist.

2) If  $f''(x) < 0$  on  $(a, b)$ , then  $f'(x)$  must be decreasing on  $(a, b)$ . We observe that a function  $f$  is concave down if and only if  $f'$  is decreasing:

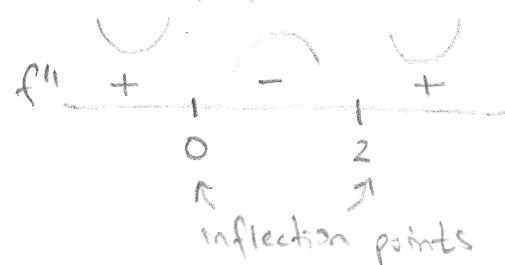
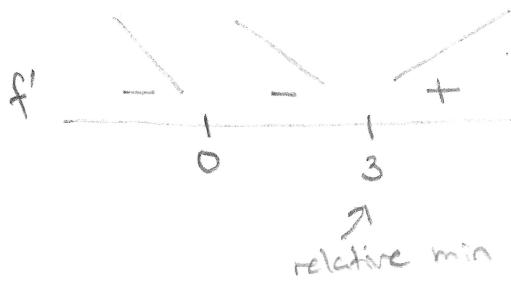


Thus,  $f(x)$  is concave down on  $(a, b)$ .

$$3) f(x) = x^4 - 4x^3 \Rightarrow y\text{-int: } (0, 0), \text{ x-int: } (0, 0), (4, 0)$$

$$f'(x) = 4x^3 - 12x^2 = 4x^2(x-3) \Rightarrow \text{critical points @ } x=0, x=3$$

$$f''(x) = 12x^2 - 24x = 12x(x-2) \Rightarrow \begin{matrix} \text{critical points @ } x=0, x=2 \\ \text{of } f \\ \text{of } f' \end{matrix}$$



$$f(2) = (2)^4 - 4(2)^3 = 16 - 32 = -16$$

$$f(3) = 3^4 - 4(3)^3 = 3^3(3-4) = -27$$

