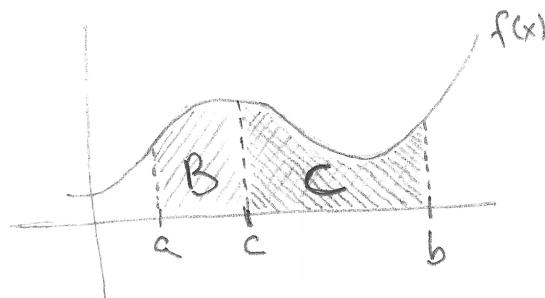
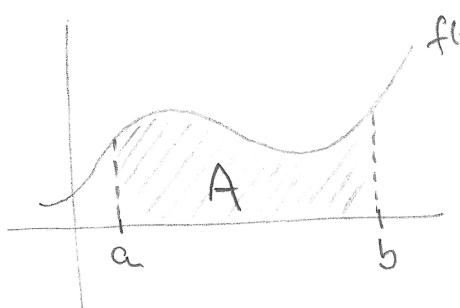


Written HW 5

1)



Suppose we have some continuous function $f(x)$ defined on $[a, b]$.

Then, $\int_a^b f(x) dx = A$, $\int_a^c f(x) dx = B$, and $\int_c^b f(x) dx = C$.

Note that $A = B + C$, so we conclude

$$\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx.$$

2) Suppose $f(x)$ is negative and integrable on $[a, b]$. Then

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \Delta x (f(x_1) + f(x_2) + \dots + f(x_n)),$$

where $\Delta x = \frac{b-a}{n}$, and x_i is inside the i^{th} subinterval.
by limit definition
of an integral

Since $f(x)$ is negative on $[a, b]$, then $f(x_1) < 0, f(x_2) < 0, \dots, f(x_n) < 0$, which implies that

$$\lim_{n \rightarrow \infty} \Delta x (f(x_1) + f(x_2) + \dots + f(x_n)) < 0$$

↑ positive ↑ negative

$$\Rightarrow \int_a^b f(x) dx < 0.$$