You are encouraged to work with other students on this assignment but you are expected to write and work on your own answers. You don't need to provide the name of students you worked with.

You are expected to submit

(a) a zipped file containing all your work online on Sakai so the grader can reproduce your results using only your online submission. Due on Sakai, Wednesday, Oct. 2.

(b) a print-out copy of your obtained results for each problem. Due in class, Wednesday, Oct. 2.

Problem 1 : the bisection method

Use the bisection method to find a root of the function $f(x) = \dfrac{\sin x}{x} - 0.33$ in $(0, \pi]$ with accuracy tolerance of 10^{-6}.
For this problem, please report the initial interval, the approximate root p, the value $f(p)$ and the number of iterations needed in your print-out submission.

Problem 2 : fixed point iterations

The value of $\sqrt[3]{21}$ can be approximated by fixed-point iterations using the following functions:
(i) $g_1(x) = \frac{1}{21}\left(20x + \frac{21}{x^2}\right)$

(ii) $g_2(x) = x - \frac{x^3-21}{3x^2}$

(iii) $g_3(x) = \sqrt{\frac{21}{x}}$

Write a program to compute the sequence $\{x_n\}$ generated by each of these fixed point iteration algorithm, starting with $x_1 = 1$. For the stopping criterion, use

$$|x_n - x_{n-1}| \leqslant 10^{-7}.$$

For this problem, please report the approximate root p, the error $|\sqrt[3]{21} - p|$ and the number of iterations needed for each case in your print-out submission.

Problem 3 : Newton basin in one dimension

Newton's method solves a nonlinear equation starting from an initial approximation. The method converges to a solution that is (surprisingly) dependent of that initial approximation. Two different starting points might produce sequences of approximations that converge to two different roots of the equation at hand. The set of points that will make the method converge to a given root is called the *basin of attraction* of that root. For this problem, you will classify a group of possible starting points for the algorithm by color, according to the basin they belong to.

Use Newton's method to solve the equation

$$x^3 - 6x^2 + 11x - 6 = 0$$

with tolerance 10^{-12}, 10 maximum number of iterations and starting points from 0 to 4 with step size 0.1, i.e., $0, 0.1, 0.2, ..., 3.9, 4$. The roots of the given equation are three:

$$x_1 = 1,\ x_2 = 2,\ x_3 = 3$$

For each starting point x0, plot a dot at $(x0, 0)$ in red, green or blue color, if the method converged to x_1, x_2 or x_3 respectively. If your method fails or does not converge, plot a black diamond. Plot the function on the

same axis in a solid black line. The resulting figure will look similar to Figure 1.
For this problem, please include your obtained figure in your print-out submission.

Note: the example in the figure is for a different equation. The one you will generate will look similar.

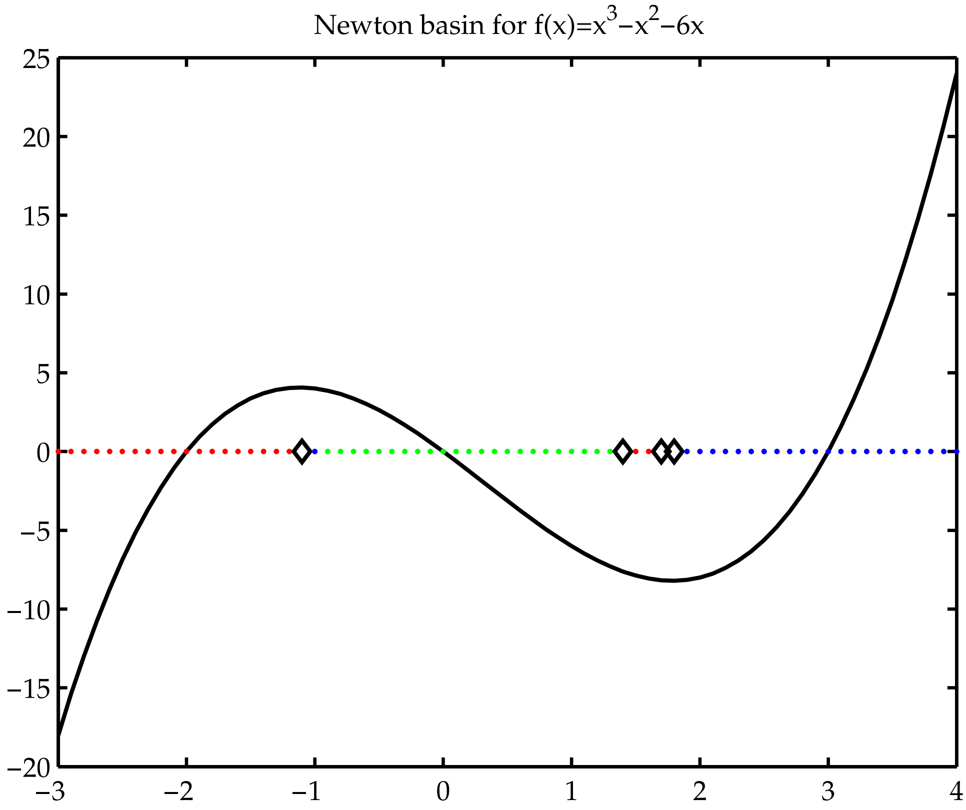

Figure 1: Example of a Newton basin for the equation $x^3 - x^2 - 6x = 0$ for initial approximations in $[-3, 4]$ with step 0.1, tolerance 10^{-12} and 10 maximum number of iterations. The roots of this equation are $-2, 0$ and 3. If the initial approximation of Newton's method is one of the red dots, the method will converge to -2; one of the green dots, will converge to 0; and one of the blue dots, will converge to 3. On the black diamonds, the method either did not converge in the maximum iterations given or it failed.