Section 4.4 Composite Numerical Integration

Motivation:

1) on large interval, use low order Newton-Cotes formulas are not accurate.
2) on large interval, interpolation using high degree polynomial is unsuitable because of oscillatory nature of high degree polynomials.

Main idea: divide integration interval $[a, b]$ into subintervals and use simple integration rule for each subinterval.

Example 1.

a) Use Simpson's rule to approximate $\int_0^4 e^x dx$. The exact value is 53.59819.

b) Divide $[0,4]$ into $[0,2]$, $[2,4]$. Use Simpson's rule to approximate $\int_0^2 e^x dx$, $\int_2^4 e^x dx$. Then approximate $\int_0^4 e^x dx$ by adding the approximations.

Solution:

a) $\int_0^4 e^x dx \approx \frac{2}{3}\left(e^0 + 4e^2 + e^4\right) = 56.76958.$

Error= $|53.59819$ $-56.76958|$ = 3.17143

b) $\int_0^4 e^x dx = \int_0^2 e^x dx + \int_2^4 e^x dx \approx \frac{1}{3}\left(e^0 + 4e^1 + e^2\right) + \frac{1}{3}(e^2 + 4e^3 + e^4) = 53.86385$

Error=$|53.59819$ $-53.61622|$ = 0.265

- **b)** is much more accurate than **a)**.

Composite Trapezoidal rule (Thm 4.5)

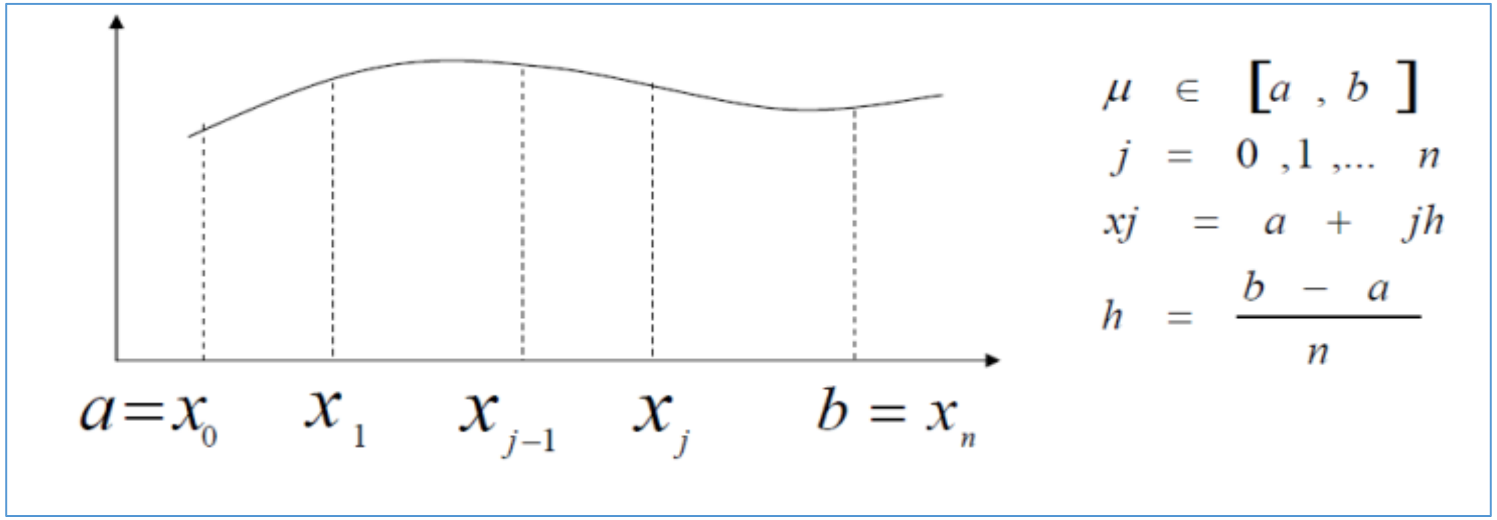

Figure 1 Composite Trapezoidal Rule

$$\int_a^b f(x)dx = \frac{h}{2}\left[f(x_0) + 2\sum_{j=1}^{n-1} f(x_j) + f(x_n)\right] - \frac{b-a}{12}h^2 f''(\mu)$$

- $h = \frac{b-a}{n}$, $x_j = a + j\,h, j = 0,1,\ldots,n,\ \mu \in (a,b)$

Composite Simpson’s rule (Thm 4.4)

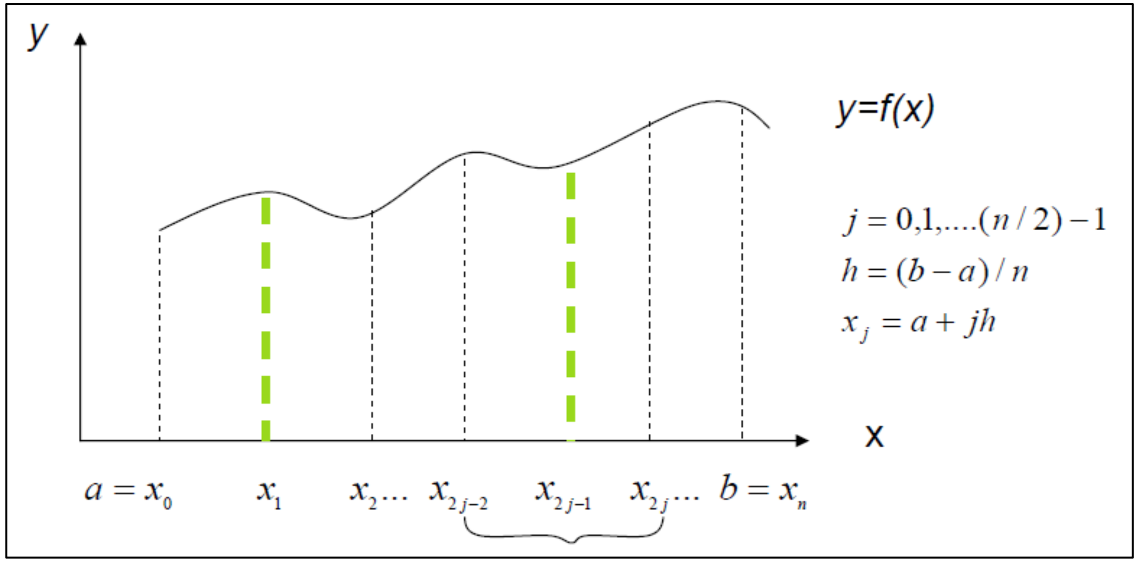

Figure 2 Composite Simpson's rule

$$\int_a^b f(x)dx = \frac{h}{3}\left(f(x_0) + 2\sum_{j=1}^{\left(\frac{n}{2}\right)-1} f(x_{2j}) + 4\sum_{j=1}^{\left(\frac{n}{2}\right)} f(x_{2j-1}) + f(x_n) \right) - \frac{b-a}{180} h^4 f^{(4)}(\mu)$$

- $h = \frac{b-a}{n}$, $\quad x_j = a + j\,h, j = 0,1,\ldots,n,\ \mu \in (a,b)$
- n is even

Composite Midpoint rule (Thm 4.6)

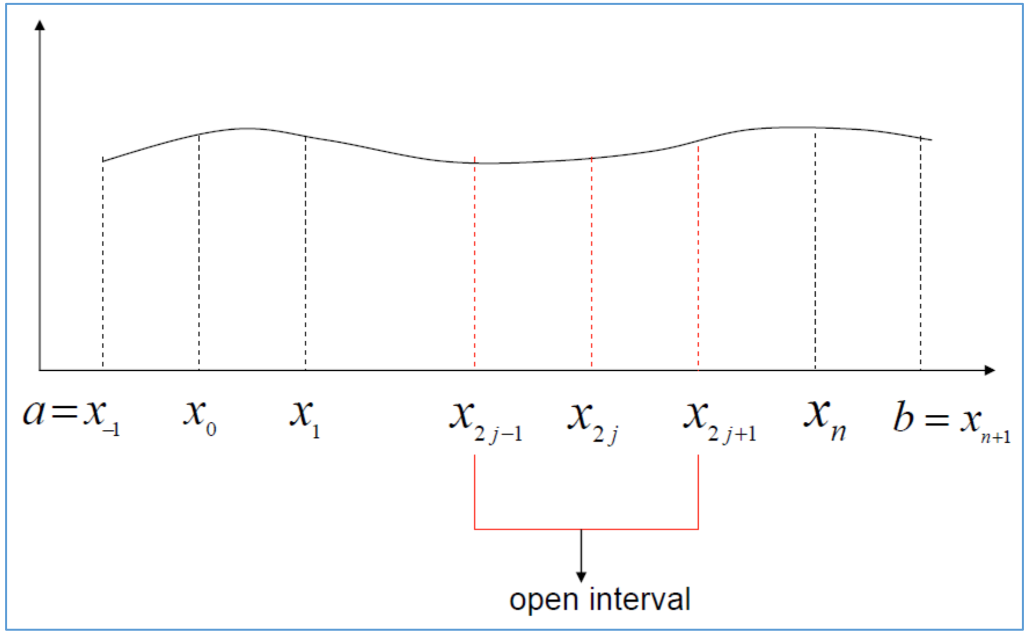

Figure 3 Composite Midpoint rule

$$\int_a^b f(x)dx = 2h\sum_{j=0}^{\frac{n}{2}} f(x_{2j}) + \frac{b-a}{6}h^2 f''(\mu)$$

- $h = \frac{b-a}{n+2}$, $\quad x_j = a + (j+1)h, j = -1,0,\dots,n+1,\ \mu \in (a,b)$

Example 2. Approximate $\int_0^4 x^3 dx$ using

a) Composite Trapezoidal rule with $n = 4$.

b) Composite Simpson's rule with $n = 4$.

c) Composite Midpoint rule with $n = 6$.

Example 3. Determine the values of n required to approximate $\int_0^2 \frac{1}{x+4}\,dx$ to within 10^{-5}. Use

a) Composite Trapezoidal rule.

b) Composite Simpson's rule.

c) Composite Midpoint rule.

Soln:

$a)\;\; n \geq 46$

$b)\;\; n \geq 6$

$c)\;\; n \geq 64$