Optics Course — Lecture 1

Ray or Geometrical Optics

~ light travels in straight lines ~ Rays

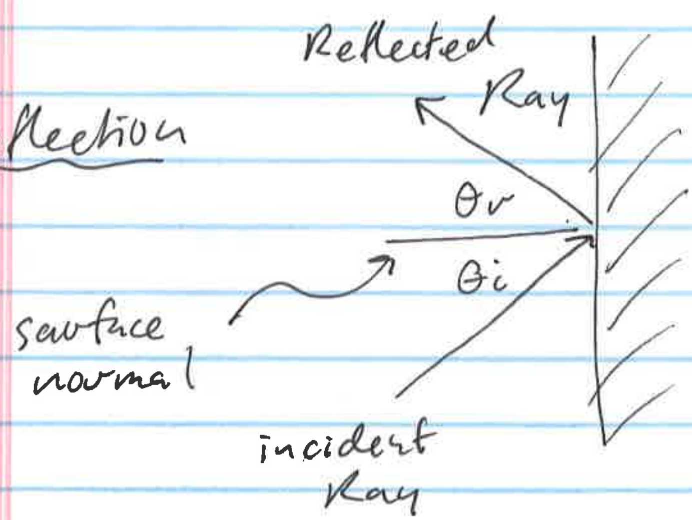
~ describe the medium by the **refractive index** $n \geq 1$

$$n = \frac{\text{speed of light in free space}}{\text{speed of light in the medium}} = \frac{c_0}{c}$$

i.e $c = c_0/n$

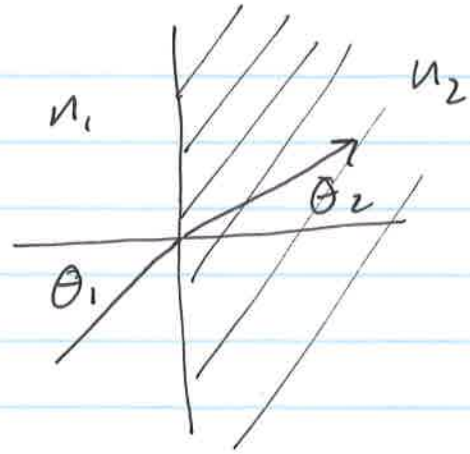
~ a familiar application of Ray Optics is **reflection** and **refraction**

Reflection



$$\boxed{\theta_i = \theta_r}$$

Refraction:



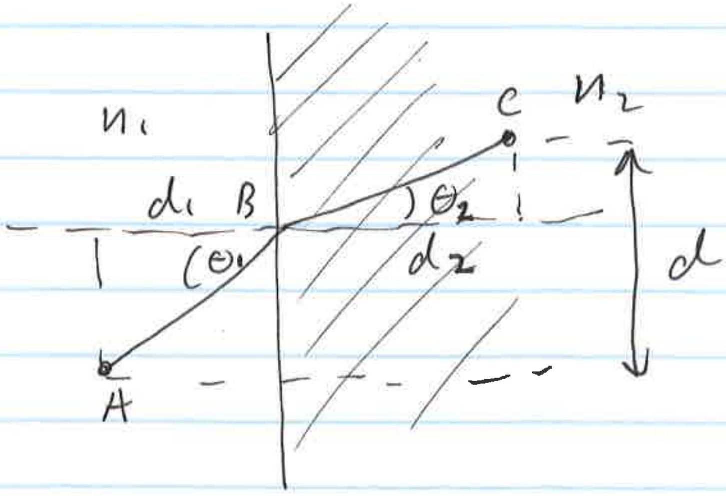
Snell's Law

$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

usually presented as an empirical observation, can be derived from "Fermat's Principle"

light travels along the path of least time

Consider the following construct:



~ time taken to get from $A \to C$

$$t = \frac{\overline{AB}}{c_1} + \frac{\overline{BC}}{c_2} = \frac{n_1 \overline{AB}}{c_0} + \frac{n_2 \overline{BC}}{c_0}$$

$\Rightarrow$ need to minimize $n_1 \overline{AB} + n_2 \overline{BC}$

subject to the constraint $d_1 \tan\theta_1 + d_2 \tan\theta_2 = d$

Note: $n_1 \overline{AB} + n_2 \overline{BC} = \dfrac{n_1 d_1}{\cos\theta_1} + \dfrac{n_2 d_2}{\cos\theta_2}$

~ this is a problem in Lagrange multipliers

Function to be minimize: $f(\theta_1, \theta_2) = \dfrac{n_1 d_1}{\cos\theta_1} + \dfrac{n_2 d_2}{\cos\theta_2}$

Constraint: $g(\theta_1, \theta_2) = d_1 \tan\theta_1 + d_2 \tan\theta_2 - d$

Lagranian: $F(\theta_1, \theta_2) = f(\theta_1, \theta_2) - \lambda g(\theta_1, \theta_2)$

~ determine λ & $\therefore$ correct minimum of f by $\dfrac{\partial F}{\partial \theta_1} = 0 = \dfrac{\partial F}{\partial \theta_2}$

$$\frac{\partial F}{\partial \theta_1} = \frac{n_1 d_1 \sin\theta_1}{\cos^2\theta_1} - \frac{\lambda d_1}{\cos^2\theta_1} = 0$$

$$\Rightarrow n_1 d_1 \sin\theta_1 = \lambda d_1 \Rightarrow \lambda = n_1 \sin\theta_1$$

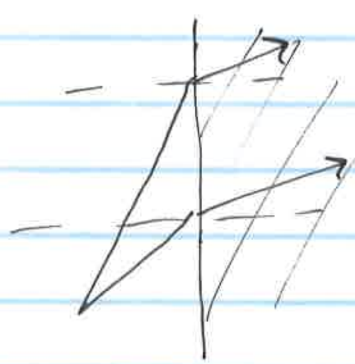
$$\frac{\partial F}{\partial \theta_2} = \frac{n_2 d_2 \sin\theta_2}{\cos^2\theta_2} - \frac{\lambda d_2}{\cos^2\theta_2} = 0$$

$$\Rightarrow n_2 d_2 \sin\theta = \lambda d_2$$

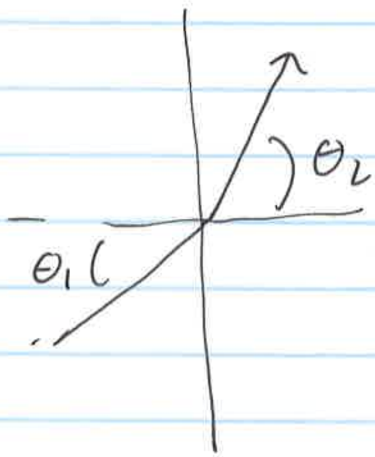
$$\Rightarrow \boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

when $n_2 > n_1 \Rightarrow \theta_2 < \theta_1$

~ normal case



when $n_2 < n_1$ then $\theta_2 > \theta_1$



~ as we ↑ θ_1 we can get to a critical angle where $\theta_2 = 90°$

~ after this there is no more refraction, light is totally reflected
"Total Internal Reflection"

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$\theta_2 = 90°$ when $\theta_1 = \theta_c$ ~ "critical angle"

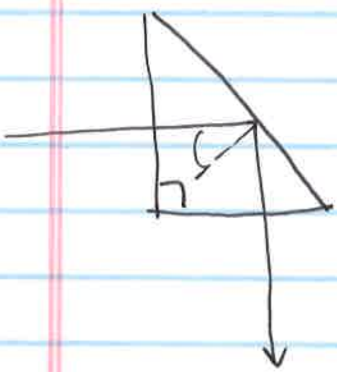
$$n_1 \sin\theta_c = n_2 \quad \Rightarrow \quad \theta_c = \sin^{-1}\frac{n_2}{n_1}$$

eg air - glass $\quad \theta_c = \sin^{-1}\frac{1}{1.55} = 40.2°$

air - water $\quad \theta_c = \sin^{-1} 1/1.33 = 48.7°$

~ for glass $\theta_c < 45°$

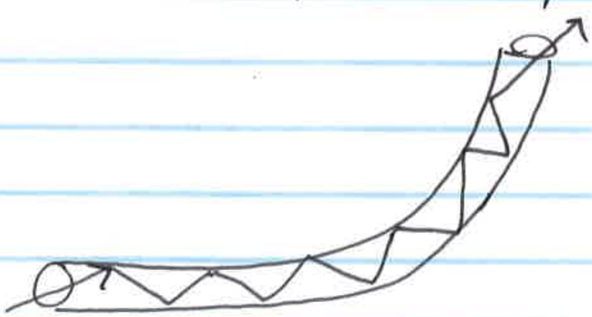
⇒ an application for this is to use a prism to steer laser beams



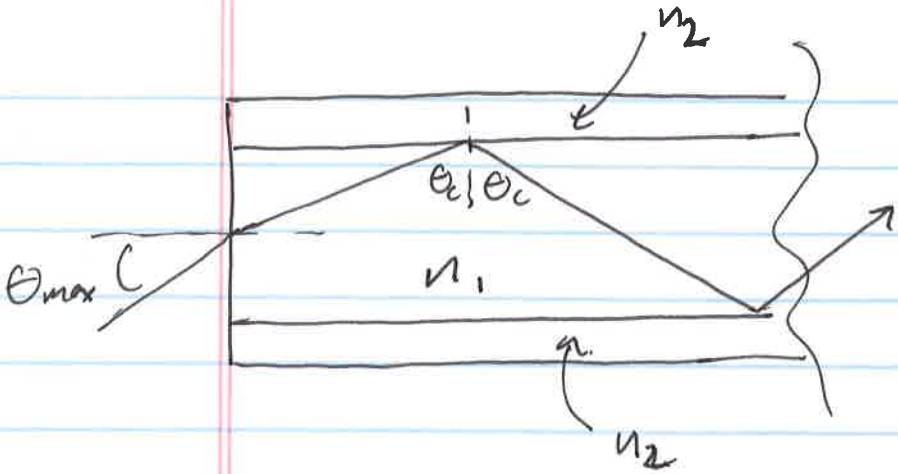
$\theta_i = 45° > \theta_c$

⇒ beam is totally internally reflected

~ another example is optical fibers.



~ lets calculate the "acceptance angle" of a fiber



~ for angles $> \theta_{max}$ we have $\theta < \theta_c$ $\Rightarrow$ leakage

core $= n_1$
cladding $= n_2$
outside $= n$

at the entrance

$$n \sin\theta_{max} = n_1 \sin(90° - \theta_c)$$

$$\Rightarrow n \sin\theta_{max} = n_1 \cos\theta_c$$

$$\Rightarrow \frac{n}{n_1} \sin\theta_{max} = \cos\theta_c$$

$$\Rightarrow \frac{n^2}{n_1^2} \sin^2\theta_{max} = \cos^2\theta_c = 1 - \sin^2\theta_c$$

$$\Rightarrow \frac{n^2}{n_1^2} \sin^2\theta_{max} = 1 - \frac{n_2^2}{n_1^2}$$

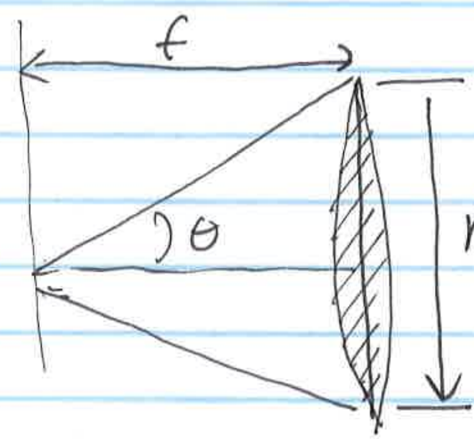
$$\Rightarrow \boxed{n \sin\theta_{max} = \sqrt{n_1^2 - n_2^2}}$$

~ the quantity $n \sin\theta$ is the "numerical aperture"

i.e. $NA = n \sin\theta_{max} = \sqrt{n_1^2 - n_2^2}$

This quantity is very important in microscopy because it gives the resolving power of an objective (lens)

eg thin lens



$f\# = f/D$

"tighter focus" for smaller $f\#$

$\sin\theta = D/2f \Rightarrow f\# = 1/2NA$