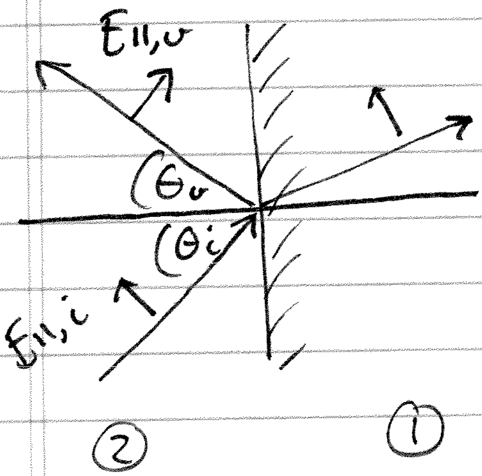
Optical Spectroscopy

"Oblique" reflection

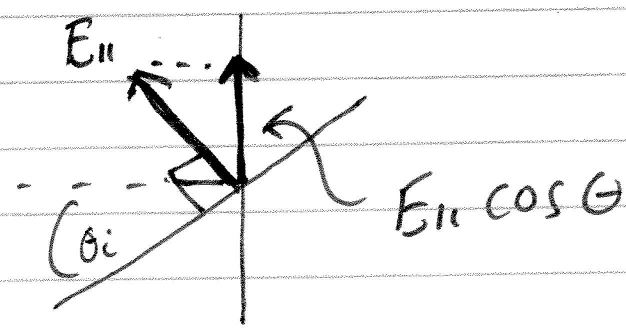
(1) $\vec{E}$ $\parallel$ to the plane of incidence



"p-polarization"

~ tangential components of $\vec{E}$ + $\vec{H}$ must be continuous at the interface

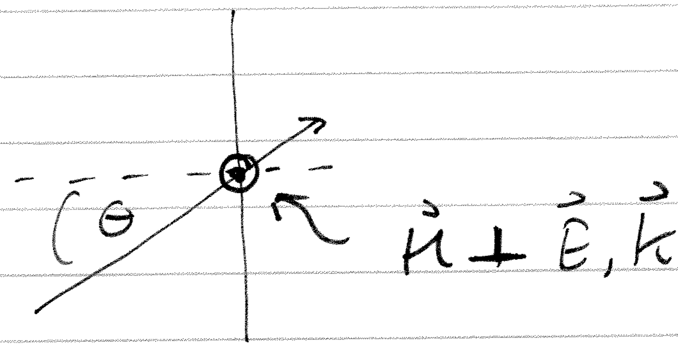
Look at $\vec{E}$:



$$\Rightarrow \boxed{E_{\parallel,i} \cos\theta_i + E_{\parallel,r} \cos\theta_r = E_{\parallel,t} \cos\theta_t}$$

1

Look at $\vec{H}$:



$\Rightarrow H_{\parallel,i} + H_{\parallel,r} = H_{\perp,t}$

$\Rightarrow k_i E_{\parallel,i} - k_i E_{\parallel,r} = k_t E_{\parallel,t}$

$$\Rightarrow \boxed{E_{\parallel,t} - E_{\parallel,r} = \frac{n_1}{n_2} E_{\parallel,t} = m E_{\parallel,t}} \quad (2)$$

$m = \frac{n_1}{n_2}$ ← lets not worry about absorption for now

We also have: (i) $\theta_i = \theta_r$

(ii) $\sin\theta_t = \frac{\sin\theta_i}{m}$

Snell's Law

$(n_1 \sin\theta_1 = n_2 \sin\theta_2)$

1 becomes $(E_{\parallel,i} + E_{\parallel,r})\cos\theta_i = E_{\parallel,t}\cos\theta_t$

$$\Rightarrow (1+r)\cos\theta_i = t\cos\theta_t$$

$$\Rightarrow (1+r) = t\,\frac{\cos\theta_t}{\cos\theta_i}$$

2 becomes $(1-r) = mt$

add $$2 = t\left(m + \frac{\cos\theta_t}{\cos\theta_i}\right)$$

$$\Rightarrow t = \frac{2\cos\theta_i}{m\cos\theta_i + \cos\theta_t}$$

$$\Rightarrow (1+r) = t\,\frac{\cos\theta_t}{\cos\theta_i}$$

becomes $$(1+r) = \frac{2\cos\theta_t}{m\cos\theta_i + \cos\theta_t}$$

$$\Rightarrow r = \frac{2\cos\theta_t}{m\cos\theta_i + \cos\theta_t} - 1$$

$$\boxed{r = \frac{\cos\theta_t - m\cos\theta_i}{\cos\theta_t + m\cos\theta_i}}$$

in this equation we have to calculate θ_t from θ_i using Snell's Law

$$\sin\theta_t = \frac{\sin\theta_i}{n}$$

$$\Rightarrow \quad \theta_t = \arcsin\left(\frac{\sin\theta_i}{n}\right)$$

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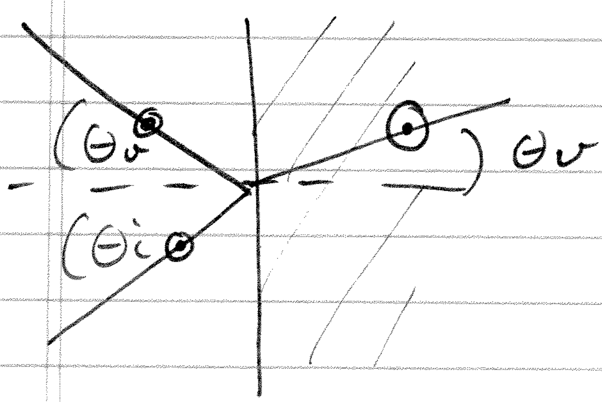
(2) $\vec{E} \perp$ to plane of incidence



"s-polarization"

now $\vec{E}_{\perp,i} + \vec{E}_{\perp,r} = \vec{E}_{\perp,t}$

& $H_{\perp,i}\cos\theta_i + H_{\perp,r}\cos\theta_r = H_{\perp,t}\cos\theta_t$

$$\Rightarrow \quad 1 + r = t \qquad \text{from } \vec{E}_\perp$$

$$n_2 (1 - r) \cos\theta_i = n_1 t \cos\theta_t \qquad \text{from } \vec{H}_\perp$$

$$\Rightarrow (1 - r) = m t \frac{\cos\theta_t}{\cos\theta_i}$$

add $$2 = t\left(1 + \frac{m\cos\theta_t}{\cos\theta_i}\right)$$

$$\Rightarrow t = \frac{2\cos\theta_i}{\cos\theta_i + m\cos\theta_t}$$

$$r = t - 1 = \frac{2\cos\theta_i}{\cos\theta_i + m\cos\theta_t} - 1$$

$$\boxed{r = \frac{\cos\theta_i - m\cos\theta_t}{\cos\theta_i + m\cos\theta_t}}$$

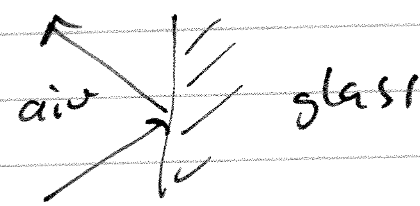
Compare $\parallel$ ("p") and $\perp$ ("s") pol^n

$$r_\parallel = r_p = \frac{\cos\theta_t - m\cos\theta_i}{\cos\theta_t + m\cos\theta_i}$$

$$r_\perp = r_s = \frac{\cos\theta_i - m\cos\theta_t}{\cos\theta_i + m\cos\theta_t}$$

θ_i + θ_t are interchanged

~ consider $n_1 > n_2$



$$\Rightarrow m = \frac{n_1}{n_2} > 1$$

r_s/r_p can be +ve or -ve, depending on the angles

t_s/t_p ~ always positive

~ a negative reflection coefficient means that there is a change in phase

~ at normal incidence $r_p = r_s = \frac{1-m}{1+m}$

< 0

~ reflected wave is always out-of-phase w/ the incident wave

2 special angles:

(i) Brewster's angle (for $n_1 > n_2$)

(ii) Total Internal Reflection (TIR)

~ for $n_2 > n_1$ so $m < 1$

(i) Brewster's angle

$n > 1$

Snell's Law: $\theta_t < \theta_i \Rightarrow \cos\theta_t > \cos\theta_i$

$$r_p = \frac{\cos\theta_t - n\cos\theta_i}{\cos\theta_t + n\cos\theta_i}$$ ← can be +ve or −ve

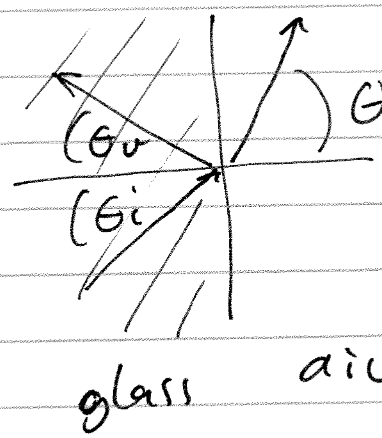
angle where $r_p = 0$ is called "Brewster's angle"

corresponds to $\cos\theta_t = n\cos\theta_i$

$$r_s = \frac{\cos\theta_i - n\cos\theta_t}{\cos\theta_i + n\cos\theta_i}$$ ~ always −ve (no minimum)

~ Brewster's angle is used in laser cavities to reduce losses due to ~~[illegible]~~ reflections c(ii) Total Internal Reflection

consider $n_2 > n_1$, $m < 1$



Snell's Law

$\theta_t > \theta_i$

$$\sin\theta_t = \frac{\sin\theta_i}{m}$$

$$\Rightarrow \sin\theta_t > \sin\theta_i$$

~ as we increase θ_i we can get to a critical angle where $\theta_t = 90°$

$\Rightarrow \sin\theta_t = 1$

critical angle (θ_c) is given by

$$\sin\theta_c = m$$

or $\theta_c = \arcsin m$

$\theta_i \geq \theta_c$ ~ no transmitted wave