

Reflection at Interfaces

Lecture ⑤

Boundary Conditions

~ tangential components of $\vec{E}$ & $\vec{H}$
must be continuous across the
~~open~~ interface

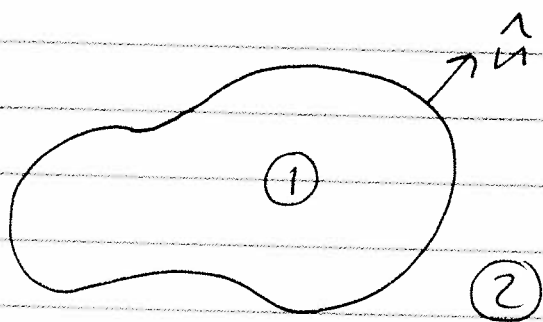
Mathematically:

$$(\vec{E}_2 - \vec{E}_1) \times \hat{n} = 0$$

$$(\vec{H}_2 - \vec{H}_1) \times \hat{n} = 0$$

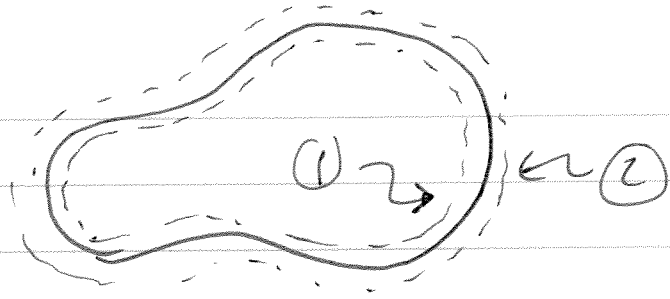
$\hat{n}$ = outward normal vector

Imagine a surface:



~ now imagine 2 surfaces just inside & just outside this surface

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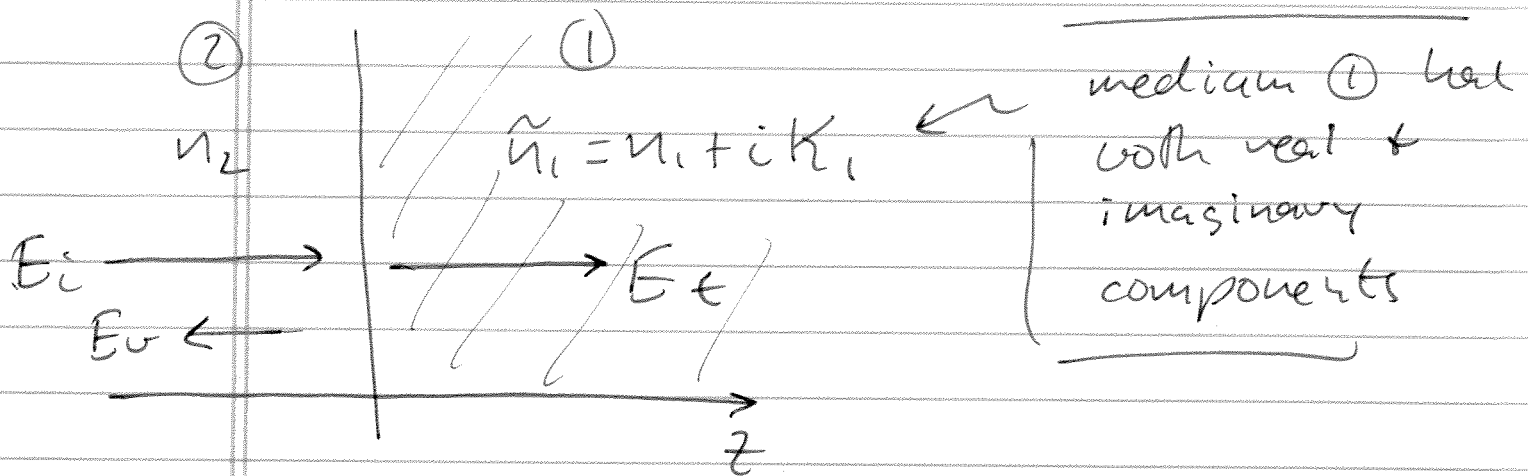
Energy flow across ①: $\int_{A_1} \vec{S}_1 \cdot \hat{n} dA = \int_{A_1} \hat{n} \cdot (\vec{E}_1 \times \vec{H}_1) dA$

Energy flow across ②: $\int_{A_2} \vec{S}_2 \cdot \hat{n} dA = \int_{A_2} \hat{n} \cdot (\vec{E}_2 \times \vec{H}_2) dA$

$$\begin{aligned}
 \int \hat{n}_1 \cdot (\vec{E}_1 \times \vec{H}_1) dA &= \int \vec{H}_1 \cdot (\vec{n} \times \vec{E}_1) dA \quad \leftarrow \overline{A \cdot (B \times C)} \\
 &= \int \vec{H}_1 \cdot (\vec{n} \times \vec{E}_2) dA \quad \left| \begin{array}{l} = B \cdot (C \times A) \\ = C \cdot (A \times B) \end{array} \right. \\
 &= \int \vec{E}_2 \cdot (\vec{H}_1 \times \vec{n}) dA \quad \leftarrow \overline{B \cdot C} \quad \left| \begin{array}{l} \vec{E}_1 \times \vec{n} = \vec{E}_2 \times \vec{n} \\ \vec{H}_1 \times \vec{n} = \vec{H}_2 \times \vec{n} \end{array} \right. \\
 &= \int \vec{E}_2 \cdot (\vec{H}_2 \times \vec{n}) dA \quad \leftarrow \overline{\vec{H}_1 \times \vec{n} = \vec{H}_2 \times \vec{n}} \\
 &= \int \hat{n} \cdot (\vec{E}_2 \times \vec{H}_2) dA \\
 &= \int \vec{S}_2 \cdot \hat{n} dA
 \end{aligned}$$

$\sim$ B.C.s are a statement of energy conservation!!

Reflection at normal incidence:



plane wave: $\vec{E} = \vec{E}_0 e^{i(kz - \omega t)}$
 $= \vec{E}_0 e^{i\omega(\frac{\tilde{n}_1 z}{c} - t)}$

$z > 0$: $\vec{E} = \vec{E}_t e^{i\omega(\frac{\tilde{n}_1 z}{c} - t)}$

$z < 0$: $\vec{E} = \vec{E}_i e^{i\omega(\frac{n_2 z}{c} - t)} + \vec{E}_r e^{-i(\frac{n_2 z}{c} + t)}$

↑
note change in sign

at the interface: $\boxed{\vec{E}_i + \vec{E}_r = \vec{E}_t}$

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For the magnetic field we also have

$$\vec{H}_i + \vec{H}_r = \vec{H}_t$$

now $\vec{k} \times \vec{E} = \omega \mu \vec{H} \quad + \quad \vec{k}_r = -\vec{k}_i$

$$\Rightarrow \vec{k}_i \times \vec{E}_i - \vec{k}_i \times \vec{E}_r = \vec{k}_t \times \vec{E}_t$$

$$\vec{k} = \frac{\omega \tilde{n}}{c} \hat{e}$$

$$\Rightarrow n_2 (\vec{E}_i - \vec{E}_r) = \tilde{n}_1 \vec{E}_t$$

or $\boxed{\vec{E}_i - \vec{E}_r = \frac{\tilde{n}_1}{n_2} \vec{E}_t} \quad (2)$

Define ~~the~~ reflection & transmission coefficients "r" and "t"

$$\vec{E}_r = r \vec{E}_i$$

$$\vec{E}_t = t \vec{E}_i$$

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from ① $\vec{E}_i + v \vec{E}_i = t \vec{E}_i$

$$\Rightarrow 1 + v = t$$

from ② $\vec{E}_i - v \vec{E}_i = \frac{\tilde{n}_1}{n_2} t \vec{E}_i$

$$\Rightarrow 1 - v = \frac{\tilde{n}_1}{n_2} t = m t \quad m = \frac{\tilde{n}_1}{n_2}$$

add $\Rightarrow 2 = (m+1)t$

$$\Rightarrow t = \frac{2}{m+1}$$

$$\Rightarrow 1 + v = \frac{2}{1+m} \Rightarrow v = \frac{2}{1+m} - 1$$

$$= \frac{1-m}{1+m}$$

Look at Intensities:

$$R = |v|^2 = \left| \frac{1-m}{1+m} \right|^2 \quad m = \frac{n_1 + iK_1}{n_2}$$

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Assume medium 2 is air $\Rightarrow n_2 = 1$

$$\Rightarrow R = \left| \frac{1 - n_1 - iK_1}{1 + n_1 + iK_1} \right|^2$$

$$= \left[\frac{(1 - n_1)^2 + K_1^2}{(1 + n_1)^2 + K_1^2} \right]$$

R is large (≈ 1) if:

(1) K_1 is small and $n_1 \gg 1$

$\Rightarrow$ high contrast in n gives high reflection

(2) $n_1 \approx 1$ and $K \gg 1$

$\Rightarrow$ strongly absorbing materials can be good reflectors.

eg air-glass interface

$$n_1 = 1.54 \quad K_1 \approx 0$$

$$R = \left[\frac{(1 - 1.54)^2}{(1 + 1.54)^2} \right] = 0.04$$

$\sim 4\%$ of light is reflected at
an air glass interface

96% transmitted

2 interfaces $T = (0.96)^2 = 0.92$

4 interfaces $T = 0.85$

2 lenses

8 interfaces $T = 0.72$

eg: high index plastics (eyeglasses)

$$n = 1.7 \Rightarrow R = 0.07$$

$$\Rightarrow T = 0.86 \text{ for 2 surfaces}$$

Metals

eg Au @ $\omega = 1.02 \text{ eV}$ (near-IR)

$$n = 0.35 + 8.145i$$

$$R = 0.98$$

$\sim$ almost completely reflective
v/c at large k .

Ag @ $\omega = 2.13 \text{ eV}$ (visible)

$$n = 0.06 + 4.152i$$

$$R = 0.99$$

$\sim$ metals make good mirrors