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Bonus vetus OLS: A simple method for approximating international trade-cost effects using the gravity equation ☆

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ABSTRACT

Using a Taylor-series expansion, we solve for a simple reduced-form gravity equation revealing a transparent theoretical relationship among bilateral trade flows, incomes, and trade costs, based upon the model in Anderson and van Wincoop [Anderson, James E., and van Wincoop, Eric. "Gravity with Gravitas: A Solution to the Border Puzzle." American Economic Review 93, no. 1 (March 2003): 170–192.]. Monte Carlo results support that virtually identical coefficient estimates are obtained easily by estimating the reduced-form gravity equation including theoretically-motivated exogenous multilateral resistance terms. We show our methodology generalizes to many settings and delineate the economic conditions under which our approach works well for computing comparative statics and under which it does not.

1. Introduction

For nearly a half century, the gravity equation has been used to explain econometrically the *ex post* effects of economic integration agreements, national borders, currency unions, immigrant stocks, language, and other measures of "trade costs" on bilateral trade flows. Until recently, researchers typically focused on a simple specification akin to Newton's Law of Gravity, whereby the bilateral trade flow from region i to region j was a multiplicative (or log-linear) function of the two countries' gross domestic products (GDPs), their bilateral distance, and an array of bilateral dummy variables assumed to reflect the bilateral trade costs between that pair of regions; we denote this the "traditional" gravity equation.

However, the traditional gravity equation has come under scrutiny, partly because it ignores that the volume of trade from region i to region j should be influenced by trade costs between regions i and j *relative* to those of the rest-of-the-world (ROW), and the economic sizes of the ROW's regions (and prices of their goods) matter as well. While two early formal theoretical foundations for the gravity equation with trade costs — first Anderson (1979) and later Bergstrand (1985) — addressed the role of "multilateral prices," Anderson and van Wincoop (2003) refined the theoretical foundations for the gravity equation to emphasize the importance of accounting properly for the endogeneity of prices. Two major conclusions surfaced from the seminal Anderson and van Wincoop (henceforth, A-vW) study, "Gravity with *Gravitas*." First, traditional cross-section empirical gravity equations have been misspecified owing to the omission of theoretically-motivated endogenous multilateral (price) resistance terms for exporting and importing regions. Second, to estimate properly the general equilibrium comparative statics of a national border or an EIA, one needs to estimate these multilateral resistance (MR) terms for any two regions with *and* without a border, in a manner consistent with theory. Due to the nonlinearity of the structural relationships, A-vW applied a custom nonlinear least squares (NLS) program to account for the endogeneity of prices and estimate the general equilibrium comparative statics.

Another — and computationally less taxing — approach to estimate unbiased gravity equation coefficients, which also acknowledges the influence of theoretically-motivated MR terms, is to use region-

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specific fixed effects, as noted by A-vW, Eaton and Kortum (2002), and Feenstra (2004). An additional benefit is that this method avoids the measurement error associated with measuring regions' "internal distances" for the MR variables. Indeed, van Wincoop himself — and nearly every gravity equation study since A-vW — has employed this simpler technique of fixed effects for determining gravity-equation parameter estimates, cf., Rose and van Wincoop (2001) and Baier and Bergstrand (2007a). Yet, without the structural system of nonlinear equations, one still cannot generate region- or pair-specific general equilibrium (GE) comparative statics; fixed effects estimation precludes estimating MR terms with *and* without EIAs. Empirical researchers can use fixed effects to obtain the key gravity-equation parameter estimates, and then simply construct a system of nonlinear equations to estimate multilateral price terms with and without the "border." But they don't.

Consequently, the empirical researcher faces a tradeoff. A customized NLS approach can potentially generate consistent, efficient estimates of gravity-equation coefficients *and* comparative statics, but it is computationally burdensome relative to ordinary least squares (OLS) and subject to measurement error associated with internal distance measures. Fixed-effects estimation uses OLS and avoids internal distance measurement error for MR terms, but one cannot retrieve the multilateral price terms necessary to generate quantitative comparative-static effects without also employing the structural system of equations. Is there a third way to estimate gravity equation parameters using *exogenous* measures of multilateral resistance and "good old" ("*bonus vetus*") OLS and/or compute region-specific resistance terms that can be used to approximate MR terms for comparative statics or other purposes (to be discussed later) without using a nonlinear solver? This paper suggests a method that may be useful.

Following some background, this paper has three major parts: theory, estimation, and comparative statics. First, we suggest a method for "approximating" the MR terms based upon theory. We use a simple first-order log-linear Taylor-series expansion of the MR terms in the A-vW system of equations to motivate a reduced-form gravity equation that includes theoretically-motivated exogenous MR terms that can be estimated potentially using OLS. However — unlike fixed-effects estimation — this method can *also* generate theoretically-motivated general equilibrium comparative statics without using a system of nonlinear equations or assuming symmetric bilateral trade costs.

Second, we show that our first-order log-linear approximation method provides *virtually identical* coefficient estimates for gravity-equation parameters to those in A-vW. For tractability, we apply our technique first to actual trade flows using the same context and Canadian–U.S. data sets as used by McCallum (1995), A-vW, and Feenstra (2004). However, the insights of our paper have the potential to be used in numerous contexts, especially estimation of the effects of tariff reductions and free trade agreements on world trade flows — the most common usage of the gravity equation in trade. Using Monte Carlo techniques, we show that the linear approximation approach works in the context of regional (intra-continental) and world (intra- and inter-continental) trade flows.

Third, we demonstrate the economic conditions under which our approximation method works well to calculate comparative-static effects of key trade-cost variables… and when it does not. We compare the comparative statics generated using our approach versus those using A-vW's approach both for the Canadian–U.S. context and for world trade flows using Monte Carlo simulations. We find that the largest comparative static changes in multilateral price terms (and largest approximation errors) tend to be among — not just small GDP-sized economies (and consequently those with large trading partners) as emphasized in A-vW — but small countries that are *physically close*. Using a fixed-point iterative matrix manipulation, the approximation errors can be eliminated using an $N \times N$ matrix of GDP shares *relative to* bilateral distances, that is, measures of economic "density." Since our approximation method can generate MR terms even when trade costs are bilaterally asymmetric, our method can yield lower average absolute biases of comparative statics than the A-vW method (which only addresses average border barriers under asymmetry).

The remainder of the paper is as follows. Section 2 reviews the A-vW analysis. Section 3 uses a first-order log-linear Taylor-series expansion to motivate a simple reduced-form gravity equation. In Section 4, we apply our estimation technique to the McCallum–A-vW–Feenstra data set and compare our coefficient estimates to these papers' findings. Section 5 examines the economic conditions under which our approach approximates the comparative statics of trade-cost changes well and under which it does not. Section 6 concludes.

2. Background: the gravity equation and prices

2.1. The A-vW theoretical model

To understand the context, we initially describe a set of assumptions to derive a gravity equation; for analytical details, see A-vW (2003). First, assume a world endowment economy with N regions and N (aggregate) goods, each good differentiated by origin. Second, assume consumers in each region j have identical constant-elasticity-of-substitution (CES) preferences. Maximizing utility subject to a budget constraint yields a set of first-order conditions that can be solved for the demand for the nominal bilateral trade flow from i to j (X_{ij}):

$$X_{ij} = \left(\frac{p_i t_{ij}}{P_j}\right)^{1-\sigma} Y_j \tag{1}$$

where p_i is the exporter's price of region i's good, t_{ij} is the gross trade cost (one plus the *ad valorem* trade cost) associated with exports from i to j, Y_j is GDP of country j, and P_j is the CES price index given by:

$$P_j = \left[\sum_{i=1}^{N} \left(p_i t_{ij}\right)^{1-\sigma}\right]^{1/(1-\sigma)}. \tag{2}$$

A third assumption of market-clearing and some algebraic manipulation yields:

$$X_{ij} = \left(\frac{Y_i Y_j}{Y^T}\right)\left(\frac{t_{ij}}{\Pi_i P_j}\right)^{1-\sigma} \tag{3}$$

where

$$\Pi_i = \left[\sum_{j=1}^{N} \left(\theta_j / t_{ij}^{\sigma-1}\right) P_j^{\sigma-1}\right]^{1/(1-\sigma)} \tag{4}$$

$$P_j = \left[\sum_{i=1}^{N} \left(\theta_i / t_{ij}^{\sigma-1}\right) \Pi_i^{\sigma-1}\right]^{1/(1-\sigma)} \tag{5}$$

Y^T denotes total income of all regions, which is constant across region pairs, and θ_i (θ_j) denotes Y_i / Y^T (Y_j / Y^T). It will be useful now to define the term "economic density." For i, the bilateral "economic density" of a trading partner j is the amount of economic activity in j relative to the cost of trade between i and j, or $\theta_j / t_{ij}^{\sigma-1}$.

To solve this system of equations, A-vW employed a strong fourth assumption: trade costs are symmetric bilaterally, implying $t_{ij} = t_{ji}$. Under this fourth assumption, their model simplifies to a system of N^2 equations in $N(N-1)$ endogenous trade flows and N endogenous price terms (P).

2.2. The A-vW econometric model

As is common to this literature, for an econometric model we assume the log of the observed trade flow ($\ln \boldsymbol{X}_{ij}$) is equal to the log of the true trade flow ($\ln X_{ij}$) plus a log-normally distributed error term (ε_{ij}). Y_i can feasibly be represented empirically by observable $\mathbf{GDP}_i$. However, the world is not so generous as to provide observable measures of bilateral trade costs t_{ij}. Following the literature, a fifth assumption is that the gross trade cost factor is a log-linear function of (boldfaced) *observable* variables, such as bilateral distance ($\mathbf{DIS}_{ij}$) and dummy variable $e^{-\alpha\mathbf{EIA}_{ij}}$:

$$t_{ij} \equiv \mathrm{DIS}_{ij}^{\rho} e^{-\alpha \mathrm{EIA}_{ij}} \tag{6}$$

where equals $e^{-\alpha\mathbf{EIA}_{ij}}$ equals $e^{-\alpha}$ (<1) if the two regions are in an economic integration agreement (assuming $\alpha>0$). One could also include a language dummy, an adjacency dummy, etc.; for brevity, we ignore these.

In the McCallum–A-vW–Feenstra context of Canadian provinces and U.S. states, $\mathbf{EIA}_{ij}=1$if the two regions are in the same country, and 0 otherwise. A-vW describe one customized nonlinear procedure for estimating Eqs. (3)–(6) to generate unbiased estimates in a two-country world with 10 Canadian provinces, 30 U.S. states and an aggregate rest-of-U.S. (the other 20 states plus the District of Columbia), or 41 regions total. A-vW also estimate a multicountry model; we discuss that later. This procedure minimized the sum-of-squared residuals of:

$$\ln\left[\mathbf{X}_{ij}/(\mathrm{GDP}_i\mathrm{GDP}_j)\right] = a_0 + a_1 \ln \mathbf{DIS}_{ij} + a_2\mathbf{EIA}_{ij} - \ln P_i^{1-\sigma} - \ln P_j^{1-\sigma} + \varepsilon_{ij} \tag{7}$$

subject to the 41 market-equilibrium conditions ($j=1,\ldots, 41$):

$$P_j^{1-\sigma} = \sum_{k=1}^{41} P_k^{\sigma-1}\left(\mathrm{GDP}_k/\mathrm{GDP}^T\right)e^{a_1 \ln \mathrm{DIS}_{kj} + a_2\mathrm{EIA}_{kj}} \tag{8}$$

to estimate a_0, a_1, and a_2 where $a_0=-\ln\mathbf{GDP}^T$, $a_1=-\rho(\sigma-1)$, $a_2=-\alpha(\sigma-1)$, and trade costs are bilaterally symmetric.

2.3. Estimating comparative-static effects

MR terms $P_i^{1-\sigma}$ and $P_j^{1-\sigma}$ are "critical" to understanding the impact of border barriers on bilateral trade. Once estimates of a_0, a_1, and a_2 are obtained, one can retrieve estimates of $P_j^{1-\sigma}$ for all $j=1,\ldots,41$ regions in the presence and absence of a national border. Let $\mathbf{P}_i^{1-\sigma}$ ($\mathbf{P}{*}_i^{1-\sigma}$) denote the estimate of the MR region i with (without) an EIA following NLS estimation of Eqs. (7) and (8). A-vW and Feenstra (2004) both show that the ratio of bilateral trade between any two regions with an EIA ($\mathbf{X}_{ij}$) and without an EIA ($\mathbf{X}_j$) is given by:

$$\mathbf{X}_{ij}/\mathbf{X}_{ij}^{*} = e^{a_2\mathrm{EIA}_{ij}}\left(\mathbf{P}_i^{*1-\sigma}/\mathbf{P}_i^{1-\sigma}\right)\left(\mathbf{P}_j^{*1-\sigma}/\mathbf{P}_j^{1-\sigma}\right). \tag{9}$$

However, while fixed effects can determine gravity equation parameters consistently, as A-vW note, estimation of country-specific border effects still requires construction of the structural system of nonlinear price equations to distinguish MR terms with *and* without borders. Moreover, the procedure works only under the case of bilaterally symmetric trade costs. We demonstrate in this paper a simple technique that yields virtually identical estimates of gravity-equation parameters *and* (in many instances) the comparative statics by applying a Taylor-series expansion to the theory, allowing asymmetric bilateral trade costs ($t_{ij}\neq t_{ji}$).

3. Theory

In this section, we apply a first-order log-linear Taylor-series expansion to the system of price equations $\prod_i$ and P_j in Eqs. (4) and (5) above to generate a reduced-form gravity equation — including theoretically-motivated exogenous multilateral resistance (MR) terms — that can be estimated using OLS. A first-order Taylor-series expansion of any function $f(x_i)$, centered at x, is given by $f(x_i)=f(x)+[f'(x)](x_i-x)$.[2] Since the solution to a Taylor-series expansion is sensitive to how it is centered, we use in our static trade context the natural choice of an expansion "centered" around a world with symmetric trade frictions ($t_{ij}=t$).[3] However, we note that our method will yield approximations of $\prod_i$ and P_j as functions of asymmetric GDP shares and asymmetric bilateral trade costs.

We begin with N Eq. (4) from Section 2.1. It will be useful to divide both sides of Eq. (4) by a constant $t^{1/2}$, yielding:

$$\begin{aligned} \Pi_i/t^{1/2} &= \left[\sum_{j=1}^{N}\theta_j\left(t_{ij}/t^{1/2}\right)^{1-\sigma}/P_j^{1-\sigma}\right]^{1/(1-\sigma)} \\ &= \left[\sum_{j=1}^{N}\theta_j\left(t_{ij}/t\right)^{1-\sigma}/\left(P_j/t^{1/2}\right)^{1-\sigma}\right]^{1/(1-\sigma)}. \end{aligned} \tag{10}$$

Define $\tilde{\prod}_i=\prod_i/t^{1/2}$, $\tilde{P}_j=P_j/t^{1/2}$ and $\tilde{t}_{ij}=t_{ij}/t$. Substituting these expressions into Eq. (10) yields:

$$\tilde{\Pi}_i = \left[\sum_{j=1}^{N}\theta_j\left(\tilde{t}_{ij}/\tilde{P}_j\right)^{1-\sigma}\right]^{1/(1-\sigma)} \tag{11}$$

for $i=1,\ldots, N$. It will be useful for later to rewrite Eq. (11) as:

$$e^{(1-\sigma)\ln\tilde{\Pi}_i} = \sum_{j=1}^{N} e^{\ln\theta_j}e^{(\sigma-1)\ln\tilde{P}_j}e^{(1-\sigma)\ln\tilde{t}_{ij}} \tag{12}$$

where e is the natural logarithm operator. In a world with symmetric trade costs ($t>0$), $t_{ij}=t$, implying $\tilde{t}_{ij}=1$. In this world, the latter implies:

$$\tilde{\Pi}_i^{1-\sigma} = \sum_{j=1}^{N}\theta_j\tilde{P}_j^{\sigma-1} \tag{13}$$

for all $i=1,\ldots, N$. Multiplying both sides of Eq. (13) by $\tilde{\prod}_i^{\sigma-1}$ yields:

$$1 = \sum_{j=1}^{N}\theta_j\left(\tilde{\Pi}_i\tilde{P}_j\right)^{\sigma-1}. \tag{14}$$

As noted in Feenstra (2004, p. 158, footnote 11), the solution to Eq. (14) is $\tilde{\prod}_i=\tilde{P}_j=\tilde{P}_j=1$. Hence, under symmetric trade costs ($t_{ij}=t$), $\tilde{t}_{ij}=\tilde{\prod}_i=\tilde{P}_j=1$ and it follows that $\prod_i=P_j=t^{1/2}$.

A first-order log-linear Taylor-series expansion of Eq. (12) and its analogue for $\tilde{P}_j$, centered at $\tilde{t}=\tilde{\prod}=\tilde{P}=1$ (and $\ln\tilde{t}=\ln\tilde{\prod}=\ln\tilde{P}=0$) yields $2N$ equations:

$$\ln\Pi_i = -\sum_{j=1}^{N}\theta_j\ln P_j + \sum_{j=1}^{N}\theta_j\ln t_{ij} \tag{15}$$

and

$$\ln P_j = -\sum_{i=1}^{N}\theta_i\ln\Pi_i + \sum_{i=1}^{N}\theta_i\ln t_{ij} \tag{16}$$

using $d[e^{(1-\sigma)\ln\tilde{x}}]/d(\ln\tilde{x})=(1-\sigma)e^{(1-\sigma)\ln\tilde{x}}$ and some algebraic manipulation (which generates an expression using the original "pre-transformed" variables). A solution to this system of equations (normalizing $P_1=1$) is:

$$\ln\Pi_i = \sum_{j=1}^{N}\theta_j\ln t_{ij} + \sum_{k=1}^{N}\theta_k\ln t_{k1} - \sum_{k=1}^{N}\sum_{m=1}^{N}\theta_k\theta_m\ln t_{km}, \quad i=2,\ldots,N \tag{17}$$

$$\ln P_j = \sum_{i=1}^{N}\theta_i\ln t_{ij} - \sum_{k=1}^{N}\theta_k\ln t_{k1}, \quad j=2,\ldots,N. \tag{18}$$

See Appendix A for derivations.

Eqs. (17) and (18) are critical to understanding this analysis; they are clearly "multilateral resistance" terms. Consider first Eq. (18). The

[2] We find using a Monte Carlo robustness analysis that a first-order Taylor series works well for estimating gravity equation coefficients. Higher-order terms are largely unnecessary for estimation. However, such terms are relevant for subsequent comparative statics; we address this later in Section 5.

[3] We are grateful to a referee that suggested this center that collapsed two cases into one more general case.

first term on the RHS is a GDP-share-weighted (geometric) average of the gross trade costs facing importer j across all exporters i. The higher this average, the greater overall multilateral resistance in importer j. Holding constant bilateral determinants of trade, the larger is j's multilateral resistance, the lower are bilateral trade costs relative to multilateral trade costs. Hence, the larger the bilateral trade flow from i to j will be. The second term on the RHS (which is constant across all P_j, $j=2,\ldots, N$) scales each P_j to account for normalizing country 1's "inward" multilateral price, $P_1=1$.

Consider now Eq. (17). The first term on the RHS is a GDP-share-weighted (geometric) average of the gross trade costs facing exporter i across all importers j. The higher this average, the greater overall multilateral resistance facing exporter i. Holding constant bilateral determinants of trade, the larger is i's multilateral resistance, the lower are bilateral trade costs relative to multilateral trade costs. Hence, the *larger* the bilateral trade flow from i to j will be. The second term on the RHS scales each $\prod_i$ to account for country 1's inward multilateral resistance term being normalized, as for Eq. (17). The third term on the RHS of "outward" multilateral resistance Eq. (17) (which is also constant across all $\prod_i$, $i=2,\ldots, N$) scales each $\prod_i$ to account for the normalization applying to an "inward" price, P. While the second and third and the second terms on the RHS of Eqs. (17) and (18), respectively, are constant in estimation, under asymmetry comparative statics will be sensitive to the choice of numeraire.

Under the assumption of bilaterally symmetric trade costs, a solution to Eqs. (15) and (16) is easier to derive. Under symmetry, the solution is:

$$\ln P_i = \left[\sum_{j=1}^{N} \theta_j \ln t_{ij} - (1/2) \sum_{k=1}^{N} \sum_{m=1}^{N} \theta_k \theta_m \ln t_{km}\right] \tag{19}$$

and:

$$\ln P_j = \left[\sum_{i=1}^{N} \theta_i \ln t_{ij} - (1/2) \sum_{k=1}^{N} \sum_{m=1}^{N} \theta_k \theta_m \ln t_{km}\right] \tag{20}$$

where the multilateral prices are normalized by (the square root of) GDP-weighted average trade costs, cf., Baier and Bergstrand (2006, 2007b).

In the remainder of this paper, we focus on coefficient estimation and comparative statics under the case of bilaterally symmetric trade costs, which implies $\prod_i=P_i$. In the context of the theory just discussed and bilaterally symmetric trade costs, we can obtain consistent estimates of the gravity equations' coefficients — accounting for the endogenous multilateral price variables — by estimating *using OLS* reduced form:

$$\begin{aligned}\ln \mathbf{X}_{ij} = {} & \beta_0' + \ln \mathbf{GDP}_i + \ln \mathbf{GDP}_j - (\sigma-1) \ln t_{ij} \\ & + (\sigma-1)\left[\left(\sum_{k=1}^{N} \theta_k \ln t_{ik}\right) - \frac{1}{2}\left(\sum_{k=1}^{N} \sum_{m=1}^{N} \theta_k\theta_m \ln t_{km}\right)\right] \\ & + (\sigma-1)\left[\left(\sum_{k=1}^{N} \theta_k \ln t_{kj}\right) - \frac{1}{2}\left(\sum_{k=1}^{N} \sum_{m=1}^{N} \theta_k\theta_m \ln t_{km}\right)\right]\end{aligned} \tag{21}$$

where $\beta_0'=-\ln Y^T$ is a constant across country pairs, as is $\sum_{k=1}^{N} \sum_{m=1}^{N} \theta_k\theta_m \ln t_{km}$. Thus, in the context of the theoretical model, the influence of the endogenous multilateral price variables can be accounted for — once we have measures of t_{ij} — using these theoretically-motivated *exogenous* multilateral resistance variables.

We close this section noting that it is useful to exponentiate Eq. (21). After some algebra, this yields:

$$\frac{X_{ij}}{Y_i Y_j / Y^T} = \left(\frac{t_{ij}}{t_i(\theta) t_j(\theta) / t^T(\theta)}\right)^{-(\sigma-1)} \tag{22}$$

where $t_i(\theta)=\prod_{k=1}^{N} t_{ik}^{\theta_k}$, $t_j(\theta)=\prod_{k=1}^{N} t_{kj}^{\theta_k}$, $t^T(\theta)=\prod_{k=1}^{N}\prod_{m=1}^{N} t_{km}^{\theta_k\theta_m}$ and recall $\theta_i=Y_i/Y^T$ and $t_{ij}=t_{ji}$ (by assumption). Our use of the Taylor-series expansion simplifies further the "significantly simplified" gravity equation implied by A-vW's Eqs. (7)–(9), cf., A-vW (2003, p. 176). Eq. (22) is a simple reduced-form equation capturing the theoretical influences of bilateral and multilateral trade costs on bilateral trade (relative to GDPs). Given data on bilateral trade flows, national incomes, and bilateral trade costs, Eq. (21) can be estimated by "good old" OLS, noting the possible endogeneity bias introduced by GDP-share weights in RHS variables.[4]

4. Estimation

The goal of this section is to show that one can generate virtually identical gravity equation coefficient estimates ("partial" effects) to those generated using the technique in A-vW but using instead OLS with exogenous multilateral-resistance terms suggested in the previous section. While the approach should work in numerous contexts, we apply it first empirically in Section 4.1 to McCallum's U.S.–Canadian case, a popular context. In Section 4.2, to avoid measurement and specification biases, we provide Monte Carlo analyses for two contexts: Canadian–U.S. flows and world trade flows among 88 countries. In both cases, we assume symmetric bilateral trade costs; we discuss another case of asymmetry later.[5]

Before implementing Eq. (21) econometrically, one issue needs to be addressed. We need to replace the unobservable theoretical trade-cost variable t_{ij} in Eq. (21) with an *observable* variable. Eq. (6) earlier suggests two typical observable variables likely influencing unobservable t_{ij} — bilateral distance ($\mathbf{DIS}_{ij}$) and a dummy representing the presence or absence of an economic integration agreement ($\mathbf{EIA}_{ij}$). We define a dummy variable, $\mathbf{BORDER}_{ij}$, which assumes a value of 1 if regions i and j are *not* in the same nation; hence, $\mathbf{EIA}_{ij}\equiv 1-\mathbf{BORDER}_{ij}$.[6] Taking the logarithms of both sides of Eq. (6) and then substituting the resulting equation for $\ln t_{ij}$ into Eq. (21) yields:

$$\begin{aligned}\ln \mathbf{x}_{ij} = {} & \beta_0' - \rho(\sigma-1) \ln \mathbf{DIS}_{ij} - \alpha(\sigma-1)\mathbf{BORDER}_{ij} + \rho(\sigma-1)\mathbf{MRDIS}_{ij} \\ & + \alpha(\sigma-1)\mathbf{MRBORDER}_{ij} + \varepsilon_{ij}\end{aligned} \tag{23}$$

where

$$\begin{aligned}\mathbf{MRDIS}_{ij} = {} & \left[\left(\sum_{k=1}^{N} \theta_k \ln \mathbf{DIS}_{ik}\right) + \left(\sum_{m=1}^{N} \theta_m \ln \mathbf{DIS}_{mj}\right)\right. \\ & \left. - \left(\sum_{k=1}^{N} \sum_{m=1}^{N} \theta_k\theta_m \ln \mathbf{DIS}_{km}\right)\right],\end{aligned}$$

$$\begin{aligned}\mathbf{MRBORDER}_{ij} = {} & \left[\left(\sum_{k=1}^{N} \theta_k \mathbf{BORDER}_{ik}\right) + \left(\sum_{m=1}^{N} \theta_m \mathbf{BORDER}_{mj}\right)\right. \\ & \left. - \left(\sum_{k=1}^{N} \sum_{m=1}^{N} \theta_k\theta_m \mathbf{BORDER}_{km}\right)\right],\end{aligned}$$

and $\mathbf{x}_{ij}\equiv \mathbf{X}_{ij}/\mathbf{GDP}_i\mathbf{GDP}_j$. To conform to our theory, coefficient estimates for $\ln$**DIS** (**BORDER**) and **MRDIS** (**MRBORDER**) are restricted to have identical but oppositely-signed coefficient values.

As readily apparent, Eq. (23) can be estimated using OLS, once data on trade flows, GDPs, bilateral distances, and borders are provided. We

[4] We ignore here the possibility of "zero" trade flows. Such issues have been dealt with by various means; see, for example, Felbermayer and Kohler (2006) and Santos Silva and Tenreyro (2006).

[5] Note that since the second RHS terms in Eqs. (17)–(20) (and the third RHS term in Eq. (18)) are constants in estimation, the MR approximation terms are effectively identical under symmetric or asymmetric bilateral trade costs.

[6] It will be useful now to distinguish "regions" from "countries." We assume that a country is composed of regions (which, for empirical purposes later, can be considered states or provinces). We will assume N regions in the world and n countries, with $N>n$. Our theoretical model applies to a two-country or multi-country ($n>2$) world. We will assume $n\geq 2$. A "border" separates countries. Also, we use **BORDER** rather than **EIA** so that the coefficient estimates for **DIS** and **BORDER** are both negative and therefore are consistent with A-vW (2003) and Feenstra (2004). The model is isomorphic to being recast in a monopolistically-competitive framework.

Table 1
Estimation results: Canada–U.S

Parameters	(1) OLS w/o MR terms	(2) A-vW NLS-2	(3) A-vW NLS-3	(4) OLS with MR terms	(5) Fixed effects	(6) A-vW NLS-2-a	(7) OLS with MR terms-a
$-\rho(\sigma-1)$ for distance	−1.06 (0.04)	−0.79 (0.03)	−0.82 (0.03)	−0.82 (0.04)	−1.25 (0.04)	−0.92 (0.03)	−1.02 (0.03)
$-\alpha(\sigma-1)$ for border	−0.71 (0.06)	−1.65 (0.08)	−1.59 (0.08)	−1.11 (0.07)	−1.54 (0.06)	−1.65 (0.07)	−1.24 (0.07)
Avg. error terms							
US–US	−0.21	0.06	0.06	0.39	0.00	0.05	0.27
CA–CA	1.95	−0.17	−0.02	−0.34	0.00	−0.22	−0.23
US–CA	0.00	−0.05	−0.04	−0.50	0.00	−0.04	−0.35
R^2	0.42	n.a.	n.a.	0.36	0.66	n.a.	0.60
No. of obs.	1511	1511	1511	1511	1511	1511	1511

Numbers in parentheses are standard errors of the estimates. n.a. denotes not applicable.

note that the inclusion of these additional MR terms appears reminiscent of early attempts to include — what A-vW term — "atheoretical remoteness" variables, typically GDP-weighted averages of each country's distance from all of its trading partners. However, there are two important differences here. First, our additional (the last two) terms are motivated by theory. Second, previous atheoretical remoteness measures included only multilateral *distance*, ignoring other multilateral "border" variables (such as adjacency, language, etc.).

4.1. Estimation using the McCallum–A-vW–Feenstra data set for actual Canadian–U.S. trade flows

We follow the A-vW procedure (for the two-country model) of estimating the gravity equation for trade flows among 10 Canadian provinces and 30 U.S. states. As in A-vW, we do not include trade flows internal to a state or province. We calculate the distance between the aggregate U.S. region and the other regions in the same manner as A-vW. We also compute and use the internal distances as described in A-vW for **MRDIS**. Some trade flows are zero and, as in A-vW, these are omitted. As in A-vW and Feenstra (2004), we have 1511 observations for trade flows from year 1993 from Robert Feenstra's website.

Table 1 provides the results. For purposes of comparison, column (1) of Table 1 provides the benchmark model (McCallum) results estimating Eq. (23) except *omitting* **MRDIS** and **MRBORDER**. Columns (2) and (3) provide the model estimated using NLS as in A-vW for the two-country and multi-country cases, respectively. Column (4) provides the results from estimating Eq. (23). For completeness, column (5) provides the results from estimating Eq. (23), but using region-specific fixed effects instead of **MRDIS** and **MRBORDER**.

Column (1)'s coefficient estimates for the basic McCallum regression, ignoring multilateral resistance terms, are biased, as expected. This specification can be compared with Feenstra (2004, Table 5.2, column 3), since it uses US–US, CA–CA, and US–CA data for 1993. Note, however, that Feenstra did not constrain the GDP elasticities to be unity and we report the border dummy's coefficient estimate ("Indicator border") whereas Feenstra reports instead the implied "Country Indicator" estimates.[7] Columns (2) and (3) in Table 1 report the estimates (using *GAUSS*) of the A-vW benchmark coefficient estimates; these correspond exactly to those in A-vW's Table 2 and (for the two-country case) Feenstra's Table 5.2, column (4). The coefficient estimates from our OLS specification (23) are reported in column (4) of Table 1. Both our coefficient estimates in column (4) and the NLS estimates in columns (2) and (3) differ from the estimates using fixed effects in column (5). Recall that — as both A-vW and Feenstra note — fixed effects should provide unbiased coefficient estimates of the bilateral distance and bilateral border effects, accounting fully for multilateral-resistance influences in estimation. Our column (5) estimates match exactly those in A-vW and Feenstra (2004).[8]

We now address the difference between coefficient estimates in columns (2)–(5). As A-vW (2003, p. 188) note, the bilateral distance coefficient estimate using their NLS program is quite sensitive to the calculation of "internal distances." In their sensitivity analysis, they provide alternative coefficient estimates when the internal distance variable values are doubled (or, 0.5 minimum capitals' distance). These are reported in column (6) of our Table 1; note that the absolute value of the distance coefficient increases. These results confirm A-vW's suspicion that the NLS estimation technique is sensitive to measurement error in internal distances.

However, adjustment for internal distance measurement error cannot explain entirely the difference between coefficient estimates in columns (2) and (3) and those in column (5). As A-vW (2003, p. 180) note, potential specification error can also bias the coefficient estimates in columns (2)–(4). In particular the trade-cost function from Eq. (6) may be misspecified. Classic omitted variables bias may exist. As noted above, with GDPs on both the LHS and RHS of Eq. (23), endogeneity bias may arise. When GDP coefficients are not constrained to unity, the coefficient estimates in all the specifications change; column (7) reports the estimates just for the distance and border coefficients using our Eq. (23) allowing unconstrained GDP elasticities.

The differences between the coefficient estimates in Table 1 suggest that an alternative approach is needed to compare the consistency of estimates using A-vW's method, our approach, and fixed effects. Is there a way to compare the estimation results of A-vW and our approach *excluding* mis-measurement and specification biases?

4.2. Monte Carlo analyses

In this section, we employ a Monte Carlo approach to show that our OLS method yields estimates of border and distance coefficient estimates that are *virtually identical* to those using A-vW's NLS method when we know the "true" model. To do this, in Section 4.2.1 we construct the "true" bilateral international trade flows among 41 regions using the theoretical model of A-vW described in Section 2. We assume the world is described precisely by Eqs. (7) and (8), assuming various arbitrary values for α, ρ, and σ. Using Canadian–U.S. province and state data on GDPs and bilateral distances and dummy variables for borders, we can compute the "true" bilateral trade flows *and* "true" multilateral resistance terms associated with these economic characteristics for given values of parameters α, ρ, and σ. We then assume that there exists a log-normally distributed error term for each trade flow equation. We make 1000 draws for each trade equation and run various regression specifications 1000 times.[9] We use *GAUSS* in all estimates. In Section 4.2.2, to show that this approach works in the more traditional context of world trade flows, we employ the same Monte Carlo approach and provide the results.

4.2.1. Monte Carlo analysis #1: Canada–U.S.

We consider five specifications. Specification (1) is the basic gravity model ignoring multilateral resistance terms, as used by McCallum. In

[7] In Feenstra's Table 5.2, column 3, he does not report the actual dummy variable's coefficient estimate (comparable to our estimate of 0.71). Instead, he reports only the implied "Indicator Canada" and "Indicator US" estimates of 2.75 and 0.40, respectively. The implied Indicator Canada and Indicator US estimates from our regression are 2.66 and 0.48, respectively; the difference is that we restrict the GDP elasticities to unity. When we relax the constraints on GDP elasticities, our estimates match those in Feenstra's Table 5.2, column 3 and A-vW's Table 1 exactly.

[8] The coefficient estimates from the fixed-effects regression in A-vW's Table 6, column (viii) are not reported. However, they were generously provided by Eric van Wincoop in e-mail correspondence, along with the other coefficient estimates associated with their Table 6. A-vW's Distance (Border) coefficient estimate using fixed effects was − 1.25 (− 1.54).

[9] The error terms' distribution is such that the R^2 (and standard error of the estimate) from a regression of trade on GDP, distance, and borders using a standard gravity equation is similar to that typically found (an R^2 of 0.7 to 0.8).

Table 2
Monte Carlo simulations: Canada–U.S

Specification	Coefficient estimate average	Standard deviation	Fraction within 2 standard errors of true value
(1) McCallum			
Border	−0.799	0.054	0.000
Distance	−0.569	0.036	0.000
(2) OLS with atheoretical remoteness terms			
Border	−1.139	0.057	0.000
Distance	−0.958	0.042	0.042
(3) A-vW			
Border	−1.648	0.061	0.955
Distance	−0.791	0.028	0.954
(4) Fixed effects			
Border	−1.648	0.069	0.953
Distance	−0.793	0.045	0.937
(5) OLS with MR approximations			
Border	−1.639	0.069	0.962
Distance	−0.850	0.042	0.744

True border coefficient = −1.65.
True distance coefficient = −0.79.

the context of the theory, we should get biased estimates of the true parameters since we intentionally omit the true multilateral price terms or fixed effects. Specification (2) is the basic gravity model augmented with "atheoretical remoteness" terms ($\mathbf{REMOTE}_i$ and $\mathbf{REMOTE}_j$), as in McCallum (1995) and Helliwell (1998). Eq. (7) would include $\mathbf{REMOTE}_i$ and $\mathbf{REMOTE}_j$, instead of P_i and P_j, where $\mathbf{REMOTE}_i = \ln\Sigma_j^N(\mathbf{DIS}_{ij}/\mathbf{GDP}_j)$ and analogously for $\mathbf{REMOTE}_j$. In the context of the theory, we should get biased estimates of the true parameters since we are using atheoretical measures of remoteness. This specification also ignores other multilateral trade costs. For Specification (3), we take the system of equations described in Eq. (8) to generate the "true" multilateral resistance terms associated with given values of $-\rho(\sigma-1)$ and $-(\sigma-1)$. We then estimate the regression (7) using the true values of the multilateral resistance terms. In the presence of the true MR terms, we expect the coefficient estimates to be virtually identical to the true parameters. Specification (4) uses region-specific fixed effects. As discussed earlier, region-specific fixed effects should also generate unbiased estimates of the coefficients. Specification (5) is our OLS Eq. (23). If our hypothesis is correct, these parameter estimates should be virtually identical to those estimated using Specifications 3 and 4.

We run these five specifications for values for $a_1=-\rho(\sigma-1)=-0.79$ and $a_2=\alpha(\sigma-1)=-1.65$ (the values obtained empirically in A-vW using NLS). We report three statistics in Table 2. First, we report the average coefficient estimates for a_1 and a_2 from the 1000 regressions for each specification. Second, we report the standard deviation of these 1000 estimates. In the last column, we report the fraction of times (from the 1000 regressions) that the coefficient estimate for a variable was within two standard errors of the true coefficient estimate.[10]

Table 2 reports the estimated values for the five specifications under this scenario in columns (2)–(4). There are two major results worth noting. First, the first two specifications provide biased estimates of the border and distance coefficient estimates, as expected. Second, both fixed effects and our approximation method provide estimates very close to those using Specification 3, as expected. While the approximation method's average coefficient estimates depart slightly from the average A-vW estimates, 96 (74) percent of the border (distance) coefficient estimates in Specification 5 are within two standard errors of true values.

4.2.2. Monte Carlo analysis #2: gravity equations for world trade flows

Of course, the gravity equation has been used over the past four decades to analyze economic and political determinants of a wide range of aggregate "flows." However, the most common usage of the gravity equation has been for explaining *world* (intra- and inter-continental) bilateral trade flows. The issues raised in A-vW (2003) and in this paper have potential relevance for estimating effects of EIAs and of tariff rates on world trade flows.

We constructed a set of "artificial" aggregate bilateral world trade flows among 88 countries for which data on the exogenous RHS variables discussed above were readily available. Three exogenous RHS variables that typically explain world trade flows are countries' GDPs, their bilateral distances, and a dummy representing the presence (0) or absence (1) of a common land border ("NoAdjacency"). We then estimate the relationship among bilateral trade flows, national incomes, bilateral distances and NoAdjacency among 88 countries using our OLS method. We simply redo Section 4.2.1's Monte Carlo simulations.[11] In short, the main finding is that the estimation biases for coefficients remain small; results are omitted for brevity.

A full multi-country empirical analysis of world trade flows using our approach is beyond the space constraints of this paper. However, two recent empirical gravity equation studies examining determinants of world trade flows have found that the coefficients estimated using our approach are *virtually identical* to the respective coefficient estimates using fixed effects, cf., Adam and Cobham (2007) and Dolman (2008).

5. Comparative statics

In this section, we examine the potential usefulness of our approximation approach for conducting comparative statics.[12] In Section 5.1, our linearization method implies we can compute the comparative statics analytically and provide intuition for why the approach provides a "good approximation" of the comparative-static (overall) country effects for Canada and the United States provided in A-vW (2003), but a poor one of region-pair-specific (such as Alberta–Alabama) comparative statics. In Section 5.2, we move to the most common context — gravity equations of international trade flows among large numbers of countries with symmetric bilateral trade costs — to examine under what conditions the approximation method works well for comparative statics — and when it does not. We find that the approximation method works best the smaller is the comparative-static effect, as would be expected from any linear Taylor-series expansion of a nonlinear equation. However, extending A-vW, the effects of trade costs on multilateral price terms are not simply the greatest for economically small countries (with consequently large trading partners), but for small countries *that are close* in distance. Using the concept of economic "density," or $\theta_j/t_{ij}^{\sigma-1}$

[10] Note that the standard deviation refers to the square root of the variance of all the coefficient estimates for a specification. We also calculated the standard errors of each coefficient estimate. The last column in each table refers to the fraction of the 1000 regressions that the estimated coefficient is within two standard errors of the true value.

[11] Naturally, we could also introduce in this exercise an array of other typical bilateral dummies, such as common language, common EIA, etc. However, this would have no bearing on the generality of our results.

[12] Other potential uses exist also. For instance, recent analyses of the effects of free trade agreements (FTAs) on trade using *nonparametric* (matching) econometric techniques require theory-based indexes of multilateral resistance, cf., Egger et al. (2008) and Baier and Bergstrand (2009); an approach such as ours is needed. Also, recent estimation of economic and political determinants of EIAs between country pairs using probit models of the likelihoods of FTAs requires theory-based (exogenous) measures of multilateral resistance, cf., Mansfield and Reinhardt (2003), Baier and Bergstrand (2004), and Mansfield et al. (2008).

from Eqs. (4) and (5), we show that a simple fixed-point iterative matrix manipulation can eliminate the approximation errors of our multilateral resistance approximations.[13] Finally, since our approximation method was derived allowing bilaterally asymmetric trade costs, our approach may generate lower biases in comparative statics (relative to "true" values) than the A-vW method under bilaterally asymmetric trade costs, as the latter method only addresses "average" border effects (cf., A-vW, 2003, Footnote 12), and we discuss one implementation of our approach under bilateral asymmetry in Section 5.3.

5.1. Analytical estimates of country-specific comparative statics using the approximation approach

Our procedure allows one to estimate the country-specific border effects without employing the nonlinear system of equations. Recall Eq. (9) to calculate (region-specific) border effects for $\mathbf{x}_{ij}$, using its log-linear form:

$$\mathbf{BB}_{ij} = \ln \mathbf{x}_{ij} - \ln \mathbf{x}^*_{ij} = a_2 - \ln \mathbf{P}_i^{1-\sigma} + \ln \mathbf{P}_i^{*1-\sigma} - \ln \mathbf{P}_j^{1-\sigma} + \ln \mathbf{P}_j^{*1-\sigma} \qquad (24)$$

where $\mathbf{x}_{ij} = \mathbf{X}_{ij} / \mathbf{Y}_i \mathbf{Y}_j$, a_2 is the estimate of $-\alpha(\sigma-1)$, and $a_2 < 0$. We substitute Eq. (6) into Eqs. (19) and (20) to find the MR terms with *and* without national borders. Substituting these results into Eq. (24) yields:

$$\mathbf{BB}_{ij} = \ln \mathbf{x}_{ij} - \ln \mathbf{x}^*_{ij} = a_2 \left\{ \left[1 - \left(\sum_{k=1}^{N} \theta_k \mathbf{BORDER}_{ik} \right) - \left(\sum_{m=1}^{N} \theta_m \mathbf{BORDER}_{mj} \right) + \left(\sum_{k=1}^{N} \sum_{m=1}^{N} \theta_k \theta_m \mathbf{BORDER}_{km} \right) \right] \right\} \qquad (25)$$

where $\mathbf{BORDER}_{ij} = 1$ if regions *i* and *j* are not in the same nation and 0 otherwise (distance components cancel out).[14]

For the simple Canadian–U.S. case, Eq. (25) can be calculated analytically once we have data on Canadian province and U.S. state GDPs and an estimate of a_2; we use $a_2 = -1.65$. Given the definition of $\mathbf{BORDER}_{ij}$, it turns out that the second term in the large brackets on the RHS in Eq. (25) is simply Canada's share of Canadian and U.S. GDPs (0.08) and the third term in the brackets is simply the U.S. share of Canadian and U.S. GDPs (0.92). Consequently, the sum of these terms cancels out the 1 and the effect is −1.65 times the last term in the brackets; the last term simplifies to 2×0.08×0.92, or 0.147. Hence, the general equilibrium comparative static effect of the national border on the trade between a Canadian province and U.S. state, using our linear approximation method, is −1.65×0.147 = −0.243, implying that the ratio of trade with the barrier (BB) to trade without the barrier (NB) is 0.78 ($= e^{-0.243}$). This is larger than the A-vW multi-country (two-country) estimate of 0.56 (0.41).

While our approximation method can generate border-effect estimates similar to those reported in the recent "border-puzzle" debate, a more demanding test of the method is to evaluate the comparative statics for *specific pairs* of regions. In this particular context, our method provides only a crude approximation, since Canada's share and the U.S. share of the two countries' joint GDP are identical for every region-pair. Consequently, the "country-wide" border effects are identical to the region-pair border effects. However, using A-vW's NLS system, the region-pair border effects vary from 0.32 to 0.49 with an average of 0.41 (using the A-vW two-country technique). Consequently, for particular pairs of Canadian provinces and U.S. states, our method cannot capture the aspect of A-vW that regions within smaller countries face larger multilateral resistance than regions within larger countries.

5.2. Comparative statics using world trade flows and symmetric bilateral trade costs

A-vW motivated the importance of estimating appropriate comparative statics in the context of one specific case: McCallum's Canadian–U.S. "border puzzle." However, for nearly half a century, the gravity equation in international trade has been used most commonly to analyze bilateral aggregate international trade flows and — in particular — the effects of free trade agreements (FTAs) on such flows.

In this section we use the same Monte Carlo approach used earlier for our 88-country world (see Section 4.2). We calculated the "true" trade flows using the A-vW NLS specification including real GDPs, bilateral distance, an adjacency dummy, a language dummy, and a dummy variable representing the presence or absence of an EIA. To keep the approach similar to the literature, we define "NoEEA" as one if the two countries are not members of the European Economic Area (EEA), and zero if they are; the ratios calculated are then interpreted similar to the effects of "border barriers" discussed earlier. We calculated the comparative statics by pairs of countries of NoEEA using A-vW. We then calculated the same comparative statics using our (GDP-share-weighted) approximation method.

5.2.1. The European economic area

The most important economic integration agreement in post-WWII history has been European economic integration. Consequently, an important context to evaluate the approximation method's accuracy is measuring the trade-cost effects of removing the "European Economic Area," or "NoEEA." Among our 88 countries, the potential number of country-pairs that are directly affected by EEA include 165 of the 3872 country-pairs in our sample. Reporting the results for all 165 pairs — much less the *other* 3707 pairs is prohibitive in terms of space. Consequently, we can only summarize the results.

The most notable result from this Monte Carlo experiment is that *74%* of the comparative statics using the approximation method are within 5% of the "true" (here, A-vW-method-determined) comparative statics. Another 9% of the comparative statics using the approximation method have biases between 5 and 10% of the A-vW values. Ninety two percent of the approximation-method comparative statics are within 20% of the A-vW values. As expected using a Taylor approximation, the largest biases are for country pairs with the largest changes in their MR terms (and hence in the comparative statics).

Eight percent of the approximation-method comparative statics differ from the A-vW values by more than 20%. The largest error is 38%. Yet, *every single one* of the country-pairs with a bias greater than 20% includes either Austria, Belgium, Denmark, Ireland, or Switzerland in the pair. Moreover, every single pair where the approximation method performs poorly involves economically small EEA countries that are *close* to one another (and to large trading partners), that is, where countries' multilateral economic densities ($\theta_j / t_{ij}^{\sigma-1}$) are large, cf., Footnote 13.[15]

We find in our general setting of world trade flows that our approximation method for computing pair-specific general equilibrium comparative statics is accurate within 10% of the A-vW values in *83%* of our 3872 country pairings. This result — demanding only OLS — is clearly an improvement over simply using the coefficient estimate (or "partial" effect) of an FTA dummy variable, as is typically done.

[13] Economic density refers, in general, to the amount of economic activity for a given physical area; a large literature exists on its measurement, cf., Ciccone and Hall (1996). In the trade context, a country's multilateral economic density is high when there is a strong negative correlation between partners' sizes and bilateral distances. For instance, Switzerland has a very high multilateral economic density; its largest trading partners are quite close.

[14] In reality, GDP shares will change also. However, as in A-vW, such changes are very small. Consequently, these GDP changes are ignored here.

[15] Since Switzerland is in EFTA, which has an FTA with the EU, it is considered in the EEA here.

5.2.2. A method to eliminate the approximation errors

The Monte Carlo analysis above indicated that the largest MR term changes (from trade costs) were not just for the economically small countries (with consequently large trading partners) as A-vW emphasized, but for small countries that are *physically close*.[16] We discuss a simple fixed-point iteration procedure that eliminates the approximation errors without using NLS estimation or a higher-order Taylor expansion (which, as for modern dynamic macroeconomic models, is very difficult and outside the paper's scope). A simple "fixed-point" iteration on a matrix equation can generate precise estimates of the A-vW MR terms. The key matrix in the equation is an $N\times N$ matrix of GDPs *scaled* by bilateral trade costs, $\theta_j/t_{ij}^{\sigma-1}$, that is, a measure of multilateral economic "densities."[17] We summarize the process briefly, referring the reader to a technical appendix for details (available on request). First, calculate initial estimates of every $P_i^{1-\sigma}$ ($P_i^{*1-\sigma}$) using OLS, denoted $P_i{}^{1-\sigma}{}_0$ ($P_i^{*1-\sigma}{}_0$), for every region (i=1,…, N); * denotes "without borders." Denote the $N\times 1$ vector of these MR terms V_0 (V_0*) and the $N\times 1$ vector of the inverses of each of these MR terms V_0^- (V_0^-*). Second, define an $N\times N$ matrix of GDP-share-weighted trade costs, B, where each element, b_{ij}, equals $\theta_j/t_{ij}^{\sigma-1}$. Third, compute V_{k+1} (with borders) according to:

$$V_{k+1} = zBV_k^- + (1-z)V_k \tag{26}$$

starting at k=0 until successive approximations are less than a predetermined value of ε (say, 1×10^{-9}), where $\varepsilon = \max|V_{k+1}-V_k|$ and z is a dampening factor with $z\in(0,1)$, and analogously for V_k* (without borders). Given the initial estimates of $P_i^{1-\sigma}$ ($P_i^{*1-\sigma}$) using OLS (i=1,…, N), this fixed-point iteration process will converge to the set of multilateral price terms identical to those generated using A-vW's NLS quasi-Newton program (which uses a second-order Taylor series).[18] We have run this set of matrix calculations and the correlation coefficient between our MR terms (using fixed-point iteration) and A-vW's MR terms (using NLS) is 1.0 (reported to seven decimal places) in both the Canadian–U.S. context and the 88-country context.

5.3. Comparative statics using world trade flows and asymmetric bilateral trade costs

Finally, even though previous cases assume bilaterally symmetric trade costs, our MR approximations were derived under the more general case of bilaterally asymmetric trade costs. In reality, many trade costs are bilaterally asymmetric, such as tariff rates (cf., Bergstrand et al., 2007) and preferential trade agreements (such as the Generalized System of Preferences). As A-vW (2003, Footnote 12) note, in the case of bilaterally asymmetric trade costs, one must interpret the border barriers in their approach as an average of the barriers in both directions. Consequently, comparative static computations will reflect this constraint.

Since our MR approximations were derived allowing for bilaterally asymmetric trade costs, it is possible that general equilibrium comparative static estimates using our approach could be more accurate than those computed using A-vW's technique under the case of bilateral asymmetry. There is a trade-off: while our method is a first-order approximation of the MR terms, the average absolute errors in estimates may be lower than those generated using A-vW's method because the latter approach constrains the barrier to be an average of bilaterally asymmetric barriers.

Space constraints prevent a full Monte Carlo analysis of this issue. However, in a separate Monte Carlo study, Bergstrand et al. (2007, Table 2) computed similar comparative statics in a setting with asymmetric bilateral trade costs using our method alongside the A-vW method (assuming either symmetry *or* asymmetry). In their analysis, the "true" values were computed from a fully-specified general equilibrium model. That analysis confirms that the average absolute errors from the "true" values using our approximation method are lower than those using the A-vW method (assuming symmetry or asymmetry). Thus, while A-vW's technique outperforms our approach for computing comparative statics under bilaterally symmetric trade costs, one context in which our method can generate lower average absolute errors is under bilaterally asymmetric trade costs.

6. Conclusions

Six years ago, theoretical foundations for the gravity equation in international trade were enhanced to recognize the systematic bias in coefficient estimates of bilateral trade-cost variables from omitting theoretically-motivated endogenous "multilateral (price) resistance" (MR) terms. This paper has attempted to make four potential contributions. First, we have demonstrated that a first-order log-linear Taylor series expansion of the system of nonlinear price equations suggests an alternative OLS log-linear specification that introduces theoretically-motivated *exogenous* MR terms. Second, Monte Carlo simulations suggest that the method yields virtually identical coefficient estimates to fixed effects and NLS estimation. Third, we have shown using Monte Carlo simulations that — in the case of symmetric bilateral trade costs — the comparative statics associated with our approximation method have a bias of no more than 5% in 74% of the 3872 country pairings of 88 countries examined; moreover, we identified the size of countries relative to their bilateral trade costs as the key economic variable explaining the approximation errors. Finally, in the case of asymmetric bilateral trade costs, the linear approximation method can actually have lower average absolute comparative-static errors (compared to the "true" values) than the A-vW method, when the latter assumes either symmetric or asymmetric bilateral trade costs.

Appendix A

Because of the complexity of allowing asymmetric bilateral trade costs and the matrix inversion needed, we assume a three-country world (i=1, 2, 3). In such a world, the system of 2N Eqs. (15) and (16) simplifies to:

$$\begin{bmatrix} \theta_1 & \theta_2 & \theta_3 & 1 & 0 & 0 \\ \theta_1 & \theta_2 & \theta_3 & 0 & 1 & 0 \\ \theta_1 & \theta_2 & \theta_3 & 0 & 0 & 1 \\ 1 & 0 & 0 & \theta_1 & \theta_2 & \theta_3 \\ 0 & 1 & 0 & \theta_1 & \theta_2 & \theta_3 \\ 0 & 0 & 1 & \theta_1 & \theta_2 & \theta_3 \end{bmatrix} \begin{bmatrix} \ln P_1 \\ \ln P_2 \\ \ln P_3 \\ \ln \Pi_1 \\ \ln \Pi_2 \\ \ln \Pi_3 \end{bmatrix} = \begin{bmatrix} ext_1 \\ ext_2 \\ ext_3 \\ imt_1 \\ imt_2 \\ imt_3 \end{bmatrix} \tag{A1}$$

where we define $ext_i \equiv \sum_{j=1}^{N}\theta_j \ln t_{ij}$ and $imt_j \equiv \sum_{i=1}^{N}\theta_i \ln t_{ij}$. To solve the system of equations for ln P_1, ln P_2, ln P_3, ln $\prod_1$, ln $\prod_2$ and ln $\prod_3$, we need to invert the LHS 6×6 matrix. However, the determinant of this matrix is zero, implying a redundant equation (because $\theta_1+\theta_2+\theta_3=1$ is imposed). We set P_1 (=1) as the numeraire and also eliminate one equation, $\theta_1 \ln P_1 + \theta_2 \ln P_2 + \theta_3 \ln P_3 + \ln \prod_1 = ext_1$ leaving a system of 5 equations in 5 unknowns. Since the determinant of the LHS 5×5

[16] A-vW's emphasis on relative size of trading partners (θ_j), of course, is not inconsistent with our emphasis on $\theta_j/t_{ij}^{\sigma-1}$.

[17] One advantage of the fixed-point method is that it is computationally much less resource-intensive than used by A-vW, as it does not require computation of the Jacobian of the system of equations, nor does it even require that the inverse of the Jacobian exists.

[18] We use Eq. (6) to measure $t_{ij}^{\sigma-1}$ using bilateral distance and a coefficient estimate. The results are robust to alternative initial values of $P_i^{1-\sigma}$ ($P_i^{*1-\sigma}$).

matrix is nonzero (and equal to θ_1), we can invert this matrix. Consequently, we find:

$$\begin{bmatrix} \ln P_2 \\ \ln P_3 \\ \ln \Pi_1 \\ \ln \Pi_2 \\ \ln \Pi_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ -\theta_2/\theta_1 & -\theta_3/\theta_1 & 2-\theta_1 & \theta_2(\theta_2+\theta_3)/\theta_1 & \theta_3(\theta_2+\theta_3)/\theta_1 \\ 1 & 0 & \theta_2+\theta_3 & -\theta_2 & -\theta_3 \\ 0 & 1 & \theta_2+\theta_3 & -\theta_2 & -\theta_3 \end{bmatrix} \begin{bmatrix} ext_2 \\ ext_3 \\ imt_1 \\ imt_2 \\ imt_3 \end{bmatrix}. \tag{A2}$$

Since good 1 is the numeraire, we ignore ln P_1 and ln $\prod_1$. Focusing on countries 2 and 3, we have:

$$\ln P_2 = imt_2 - imt_1 = \sum_{i=1}^{3} \theta_i \ln t_{i2} - \sum_{i=1}^{3} \theta_i \ln t_{i1} \tag{A3}$$

$$\ln P_3 = imt_3 - imt_1 = \sum_{i=1}^{3} \theta_i \ln t_{i3} - \sum_{i=1}^{3} \theta_i \ln t_{i1} \tag{A4}$$

$$\ln \Pi_2 = ext_2 + \theta_2(imt_1 - imt_2) + \theta_3(imt_1 - imt_3) \tag{A5}$$
$$= \sum_{j=1}^{3} \theta_j \ln t_{2j} + \sum_{i=1}^{3} \theta_i \ln t_{i1} - \sum_{i=1}^{3} \sum_{j=1}^{3} \theta_i \theta_j \ln t_{ij}$$

$$\ln \Pi_3 = ext_3 + \theta_2(imt_1 - imt_2) + \theta_3(imt_1 - imt_3) \tag{A6}$$
$$= \sum_{j=1}^{3} \theta_j \ln t_{3j} + \sum_{i=1}^{3} \theta_i \ln t_{i1} - \sum_{i=1}^{3} \sum_{j=1}^{3} \theta_i \theta_j \ln t_{ij}.$$

It is clear that this will generalize to N countries, such that for any two countries i and j ($i, j \neq 1$):

$$\ln \Pi_i = \sum_{j=1}^{N} \theta_j \ln t_{ij} + \sum_{k=1}^{N} \theta_k \ln t_{k1} - \sum_{k=1}^{N} \sum_{m=1}^{N} \theta_k \theta_m \ln t_{km}, \quad i = 2, \ldots, N \tag{17}$$

$$\ln P_j = \sum_{i=1}^{N} \theta_i \ln t_{ij} - \sum_{k=1}^{N} \theta_k \ln t_{k1}, \quad j = 2, \ldots, N \tag{18}$$

where t_{ij} need not equal t_{ji}, and the second and third RHS terms in Eq. (17) and second RHS term in Eq. (18) are constant across the N countries.

References

Adam, Christopher, Cobham, David, 2007. Modelling multilateral trade resistance in a gravity model with exchange rate regimes. Oxford University working paper. August.

Anderson, James E., 1979. A theoretical foundation for the gravity equation. American Economic Review 69 (no. 1), 106–116 March.

Anderson, James E., van Wincoop, Eric, 2003. Gravity with gravitas: a solution to the border puzzle. American Economic Review 93 (no. 1), 170–192 March.

Baier, Scott L., Bergstrand, Jeffrey H., 2004. Economic determinants of free trade agreements. Journal of International Economics 64 (no. 1), 29–63 October.

Baier, Scott L., Bergstrand, Jeffrey H., 2006. *Bonus vetus* OLS: a simple approach for addressing the "border puzzle" and other gravity-equation issues. working paper. March.

Baier, Scott L., Bergstrand, Jeffrey H., 2007a. Do free trade agreements actually increase members' international trade? Journal of International Economics 71 (no. 1), 72–95 March.

Baier, Scott L., Bergstrand, Jeffrey H., 2007b. *Bonus vetus* OLS: a simple method for approximating international trade-cost effects using the gravity equation. working paper. October.

Baier, Scott L., Bergstrand, Jeffrey H., 2009. Estimating the Effects of Free Trade Agreements on International Trade flows using Matching Econometrics. Journal of International Economics 77, 63–76.

Bergstrand, Jeffrey H., 1985. The gravity equation in international trade: some microeconomic foundations and empirical evidence. Review of Economics and Statistics 67 (no. 3), 474–481 August.

Bergstrand, Jeffrey H., Egger, Peter, Larch, Mario, 2007. Gravity *redux*: structural estimation of gravity equations with asymmetric bilateral trade costs. working paper.

Ciccone, Antonio, Hall, Robert E., 1996. Productivity and density of economic activity. American Economic Review 86 (no. 1), 54–70 March.

Dolman, Ben, 2008. Migration, trade and investment. Australian Government Productivity Commission Staff Working Paper. February, http://ssrn.com/abstract=1154910.

Eaton, Jonathan, Kortum, Samuel, 2002. Technology, geography, and trade. Econometrica 70 (no. 5), 1741–1779 September.

Egger, Harmut, Egger, Peter, Greenaway, David, 2008. The Trade Structure Effects of Endogenous Regional Trade Agreements. Journal of International Economics 74 (no. 2), 278–298.

Feenstra, Robert C., 2004. Advanced International Trade: Theory and Evidence. Princeton Univ Press, Princeton, NJ.

Felbermayer, Gabriel J., Kohler, Wilhelm, 2006. Exploring the intensive and extensive margins of world trade. Review of World Economics 142, 642–674.

Helliwell, John F., 1998. How Much Do National Borders Matter? Brookings Institution, Washington, DC.

Mansfield, Edward D., Reinhardt, Eric, 2003. Multilateral determinants of regionalism: the effects of GATT/WTO on the formation of preferential trading arrangements. International Organization 57 (no. 4), 829–862 Fall.

Mansfield, Edward D., Milner, Helen, C., Pevehouse, Jon C., 2008. Democracy, Veto Players, and the Depth of Regional Integration. The World Economy 31 (no. 1), 67–96.

McCallum, John, 1995. National borders matter: Canada–U.S. regional trade patterns. American Economic Review 85 (no. 3), 615–623 (June).

Rose, Andrew, van Wincoop, Eric, 2001. National money as a barrier to international trade: the real case for currency union. American Economic Review Papers and Proceedings 91 (no. 2), 386–390 May.

Silva Santos, J.M.C., Tenreyro, Silvana, 2006. The log of gravity. Review of Economics and Statistics 88 (no. 4), 641–658 November.