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Equilibrium real exchange rates: closed-form theoretical solutions and some empirical evidence

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This paper generates closed-form theoretical solutions for the relationships among the real exchange rate, relative per capita consumption, and relative wealth in a stochastic dynamic general equilibrium model of two countries' representative consumers. The solutions offer insight into the robust cross-sectional relationship between relative per capita GDPs and relative national price levels established in Kravis and Lipsey (1983, 1987, 1988) in a manner consistent with equilibrium exchange rate theories and the productivity-differentials model of Balassa (1964) and Samuelson (1964). Application of panel data from Summers and Heston (1988) to the model's structural equations yields economically-plausible estimates of the elasticity of intertemporal substitution, the relative importance of non-tradables in consumption, and the rate of time preference in several OECD countries relative to that in the United States. (JEL F31). © 1997 Elsevier Science Ltd.

Large and persistent departures from purchasing power parity (PPP) have been documented extensively for real exchange rates. While these departures have

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been observed empirically, one wonders what such departures reveal about the evolution of fundamental differences between countries' economies, as opposed to reflecting transitory monetary or fiscal shocks as in extended Mundell–Fleming approaches (cf., Dornbusch, 1987). Over the past decade and a half, several equilibrium models of exchange rate determination have surfaced that explain PPP departures in terms of relative productivity shocks, cf., Stockman (1980, 1987), Lucas (1982), Helpman and Razin (1982), Stulz (1987), Stockman and Svensson (1987), Hodrick (1989), and Stockman and Dellas (1989). In optimizing frameworks, most of these models employ the 'perfect pooling equilibrium' attributable to Lucas (1982), which precludes any inferences about the relationships over time among countries' relative wealths, consumptions, and real exchange rates.[1] Moreover, most of these models ignore relative taste shocks as a fundamental source of PPP departures, even though calibrated models of two open economies (in the related international real business cycle literature) replicate empirical data better when taste shocks are included alongside technology shocks (cf., Stockman, 1990, and Stockman and Tesar, 1995).

Separately, Kravis *et al.* (1982) note in an extensive cross-country study the large departures of national price levels from even rough parity for 34 countries in 1975. Using data from the United Nations International Comparisons Program (U.N. ICP), some countries' price levels in 1975 were no more than *one-third* the US price level (the numeraire). Summers and Heston (1988) conclude that such departures from absolute PPP have persisted for decades. Kravis and Lipsey (1983, 1987, 1988) show that the cross-country variation in exchange-rate-adjusted national price levels vis-à-vis the US price level — that is, cross-country variation in real exchange rates — could be explained statistically almost entirely by the countries' relative per capita GDPs. As Kravis and Lipsey note, *wealthier* countries tend to have higher general price levels.

In this paper, we present a detailed analysis of the relationships between the distribution of world wealth, relative per capita consumption, and the real exchange rate between pairs of countries with three general purposes in mind. First, we develop a two-country, stochastic, dynamic, general equilibrium model of consumption, wealth, and trade. Unlike earlier studies, the model yields *closed-form* solutions for countries' relative wealths, per capita consumptions, and real exchange rates in terms of relative non-tradables productivities, taste shocks, initial wealths, and rates of time preference. Few studies have characterized the distribution of wealth internationally in terms of exogenous shocks.

Second, empirical studies of this class of models using classical estimation techniques are rare, as noted recently in Taylor (1995, p. 41). Backus and Smith (1993) use the moments of growth rates of relative consumption levels and of real exchange rates to examine the implications from a dynamic, stochastic, two-country equilibrium model. However, their model assumes identical rates of time preference and ignores taste shocks. Kollmann (1995) examines the relationship among per capita consumptions and real exchange rates implied by a similar model, but in the absence of non-tradable goods. The closed-form theoretical solutions in our paper for relative wealths, consumptions, and price levels are potentially estimable and generate insight into the robust empirical relationship between relative per capita real GDPs and real exchange rates established in Kravis and Lipsey (1983, 1987, 1988), in a manner consistent with the Balassa (1964) and Samuelson (1964) productivity-differentials model of PPP departures. Application of panel data from Summers and Heston (1988) to the model's reduced-form and structural equations yields economically-plausible estimates of the relative share of non-tradables in consumption and the elasticity of intertemporal substitution.

Third, the closed-form solutions reveal the difference between the representative consumers' rates of time preference explicitly. Several studies have shown that National Income Product Accounts (NIPA) data, left unadjusted for national differences, yield misrepresentative inferences about the thriftiness of the United States relative to other countries (cf., Lipsey and Kravis, 1987 and Hayashi, 1989). Our model allows estimation of relative rates of time preference directly without resorting to NIPA data.

The remainder of the paper is structured as follows. Section I describes the model. Section II discusses the broader implications of the model. Section III discusses data constraints and econometric issues, and Section IV the empirical results. Section V concludes.

I. A two-country equilibrium model with heterogeneous consumers

In the following, we first describe the model and then derive intertemporal and intratemporal equilibria for specific preferences. Estimable closed-form solutions are obtained for a nested constant relative risk aversion utility function with some symmetry conditions imposed on the preferences of the two representative consumers.

I.A. Description of the model

Following Lucas (1978), we abstract from investment decisions by assuming endowment economies. Stochastic production processes for all goods are owned by the consumers and yield perishable outputs. Each country (foreign variables denoted by *) consists of a tradables production process, a non-tradables production process, and one infinitely-lived representative consumer with a time-additive utility function. The tradables produced in both countries are perfect substitutes.

In the home country and analogously abroad, the representative consumer maximizes the expected present discounted value of the stream of future utilities from consumption:

$$\langle 1 \rangle \qquad \text{Max } E_0 \sum_{t=0}^{\infty} u(c_t, z_t),$$

$$\langle 2 \rangle \qquad c_t = v\left(c_t^T, c_t^N\right),$$

where E_0 represents the expectation conditional on information at time 0; $u(\cdot)$ is a current-period utility function strictly concave in the consumption index c_t and dependent on the state variable z_t which is exogenous to the individual consumer. The consumption index at time t represents an optimally chosen basket of the tradable good, c_t^T, and the non-tradable good, c_t^N.[2] Preferences over both goods embodied in the consumption index are assumed to be homothetic in order that an exact price index may be defined. Without loss of generality, we then may apply a monotonic transformation such that $v(\cdot)$ in equation ⟨2⟩ is homogeneous of degree one and concave in the decision variables.

To obtain closed-form solutions for the real exchange rate and relative consumption, and make the model amenable to empirical research, we assume: (i) a constant relative risk aversion (CRRA) current-period utility function; (ii) a Cobb–Douglas subutility function; and (iii) preferences for the two representative consumers that differ intertemporally in discount factors and taste shocks:[3]

$$\langle 3\rangle \qquad u(c_t, z_t) = z_t (c_t)^{1-\sigma}/(1-\sigma), \qquad \sigma > 0\ (\sigma \neq 1)$$
$$= z_t \ln c_t, \qquad \sigma = 1$$

$$\langle 4\rangle \qquad c_t = \left(c_t^T\right)^{1/(1+\gamma)} \left(c_t^N\right)^{\gamma/(1+\gamma)}, \qquad \gamma > 0$$

$$\langle 5\rangle \qquad z_t = \omega_t \beta^t, \qquad 0 < \beta < 1.$$

The state variable in the CRRA consumption function, z_t, consists of a deterministic discount factor component β^t and a stochastic shock ω_t, which we will refer to subsequently, for simplicity, as a 'taste shock'. To obtain the standard cases, one may set $\omega_t = 1$.

The nested structure of the utility function enables us to separate the representative consumer's decisions into an intratemporal decision concerning the distribution of overall consumption expenditure between the tradable and the non-tradable goods and an intertemporal decision concerning the demand for assets and the overall expenditure on current consumption. Thus, the two equilibrium conditions can be discussed separately.

Assume complete markets so that taste shocks and productivity shocks are insurable.[4] In the absence of price distortions, externalities, and monopoly power, the Second Welfare Theorem implies that the competitive equilibrium outcome will be equivalent to the Pareto optimal allocation as chosen by a social planner. The social planner maximizes the weighted average of the lifetime utilities of the two countries' representative consumers by choosing the distribution of the tradable good subject to the available quantity. Due to the time separability of the lifetime utility functions, the social planner solves the following decision problem for each period:

$$\langle 6\rangle \qquad \underset{c_t^T, c_t^{T*}}{\text{Max}} \left[z_t (c_t)^{1-\sigma} + \alpha z_t^* (c_t^*)^{1-\sigma^*} \right]$$

subject to:

$$\langle 7\rangle \qquad c_t = \left(c_t^T\right)^{1/(1+\gamma)} \left(y_t^N\right)^{\gamma/(1+\gamma)}, \qquad c_t^* = \left(c_t^{T*}\right)^{1/(1+\gamma)} \left(y_t^{N*}\right)^{\gamma/(1+\gamma)},$$

$$\langle 8\rangle \qquad c_t^T + c_t^{T*} = y_t^T + y_t^{T*}.$$

The α denotes the constant weight that the social planner places on the utility of the foreign consumer. The consumption indexes in equation ⟨7⟩ are as in equation ⟨4⟩ but with the consumption of the non-tradable good equal to non-tradable goods production in each country (i.e. market clearing). Equation ⟨8⟩ represents the tradable goods' market-clearing condition reflecting that the countries' tradable goods are perfect substitutes.

To obtain closed-form solutions, some parameters of the utility functions are constrained to be equal; σ equals σ^* and γ equals γ^*. Yet, in any period, ω_t may not equal ω_t^* and β may not equal β^*. Thus, the representative consumers still differ with respect to taste shocks and rates of time preference, as well as initial endowments and consumption opportunities related to the non-tradable goods.

Finally, we define the real exchange rate, x_t, conventionally as:

$$x_t = e_t p_t^* / p_t, \tag{9}$$

where e_t is the nominal exchange rate (expressed in units of domestic currency per unit of foreign currency), p_t is the domestic consumer price level, and p_t^* is the foreign consumer price level.[5]

I.B. Intratemporal equilibrium

The separability of the lifetime utility function implies a budgeting process where the consumer in each period maximizes the (intratemporal) value of the consumption index, c_t in equation ⟨4⟩, subject to a nominal budget constraint, B_t, given by:

$$B_t = p_t^T c_t^T + p_t^N c_t^N, \tag{10}$$

where $p_t^T(p_t^N)$ is the domestic consumer price for the tradable (non-tradable), and similarly for the foreign consumer. Choosing consumption of the tradable good to maximize equation ⟨4⟩ subject to equation ⟨10⟩ yields:

$$\lambda_t = (1+\gamma)^{-1}\left(c_t^T\right)^{-\gamma(1+\gamma)}\left(y_t^N\right)^{\gamma/(1+\gamma)}/p_t^T, \tag{11}$$

and similarly for the foreign country, where λ_t is the Lagrangian multiplier.

Given homotheticity of the subutility function (c_t), one can rewrite equation ⟨10⟩ as:

$$B_t = p_t c_t. \tag{12}$$

Since the Lagrangian multiplier equals the marginal benefit of a unit increase in the budget on the maximum subutility, we also have:

$$\lambda_t = \partial c_t / \partial B_t = 1/p_t. \tag{13}$$

Combining equations ⟨11⟩ and ⟨13⟩, and similarly for the foreign country, and using the definition of the real exchange rate, yields the intratemporal equilibrium condition:

$$x_t = (c_t^*/c_t)^{\gamma}\left(y_t^{N*}/y_t^N\right)^{-\gamma}. \tag{14}$$

I.C. Intertemporal equilibrium

In the intertemporal stage of the budgeting process, the social planner maximizes in each period the weighted average of the two consumers' utilities in equation ⟨6⟩ subject to the quantity constraints in equations ⟨7⟩ and ⟨8⟩. This yields a set of first-order conditions that, by eliminating the Lagrangian multiplier, results in:

$$\langle 15 \rangle \qquad \frac{\left(c_t^T\right)^{-\gamma/(1+\gamma)}\left(y_t^N\right)^{\gamma/(1+\gamma)}}{\left(c_t^{T*}\right)^{-\gamma/(1-\gamma)}\left(y_t^{N*}\right)^{\gamma/(1+\gamma)}} = \frac{\alpha z_t^*(c_t^*)^{-\sigma}}{z_t(c_t)^{-\sigma}}.$$

From the intratemporal equilibrium, the left hand side of equation ⟨15⟩ equals x_t so that the intertemporal equilibrium condition is:

$$\langle 16 \rangle \qquad x_t = \alpha(\omega_t^*/\omega_t)(\beta^*/\beta)^t(c_t^*/c_t)^{-\sigma}.$$

where α and $(\beta^*/\beta)^t$ can be interpreted in the model's context as initial relative wealth and accumulated relative wealth, respectively.

I.D. Interpretation

Figure 1 demonstrates the intratemporal equilibrium locus, GE_t, in logarithmic form with slope γ. It has a positive slope since a higher desired foreign consumption level implies more demand so that the foreign price level and, accordingly, the real exchange rate are higher. Larger γ raises the desire for consumption of the non-tradable and steepens the slope because a given increase in consumption now has a larger non-tradables component, causing a larger increase in the price of non-tradables that determines the real exchange rate. An increase in foreign production of the non-tradable lowers the foreign price level and the real exchange rate, shifting the intratemporal equilibrium locus to the right (point A to point B in Figure 1).

Figure 1 also displays in logarithmic terms the negative relationship at time t between relative consumption and the real exchange rate along the intertemporal equilibrium locus, AE_t, with slope $-\sigma$. A higher relative foreign price level (i.e. real exchange rate) currently provides relatively more incentive for the foreign consumer to defer consumption to the future so that current consumption abroad declines relative to the home country. A larger σ (the inverse of the elasticity of intertemporal substitution) implies more incentive to smooth consumption and a correspondingly steeper negative slope; a given increase in the (relative) price level has little effect in postponing current (relative) consumption. If the foreign consumer has a higher discount factor (lower rate of time preference), $\beta^* > \beta$, then her share of wealth increases over time, shifting the intertemporal equilibrium locus to the right, leading to more demand for the non-tradable as time goes by so that the foreign price level continually rises in relative terms (point A to point C in Figure 1).

In Figure 1, note the four possible quadrants for the equilibrium. Initial equilibrium point A assumes the foreign country has higher per capita consumption and price level than the home country.

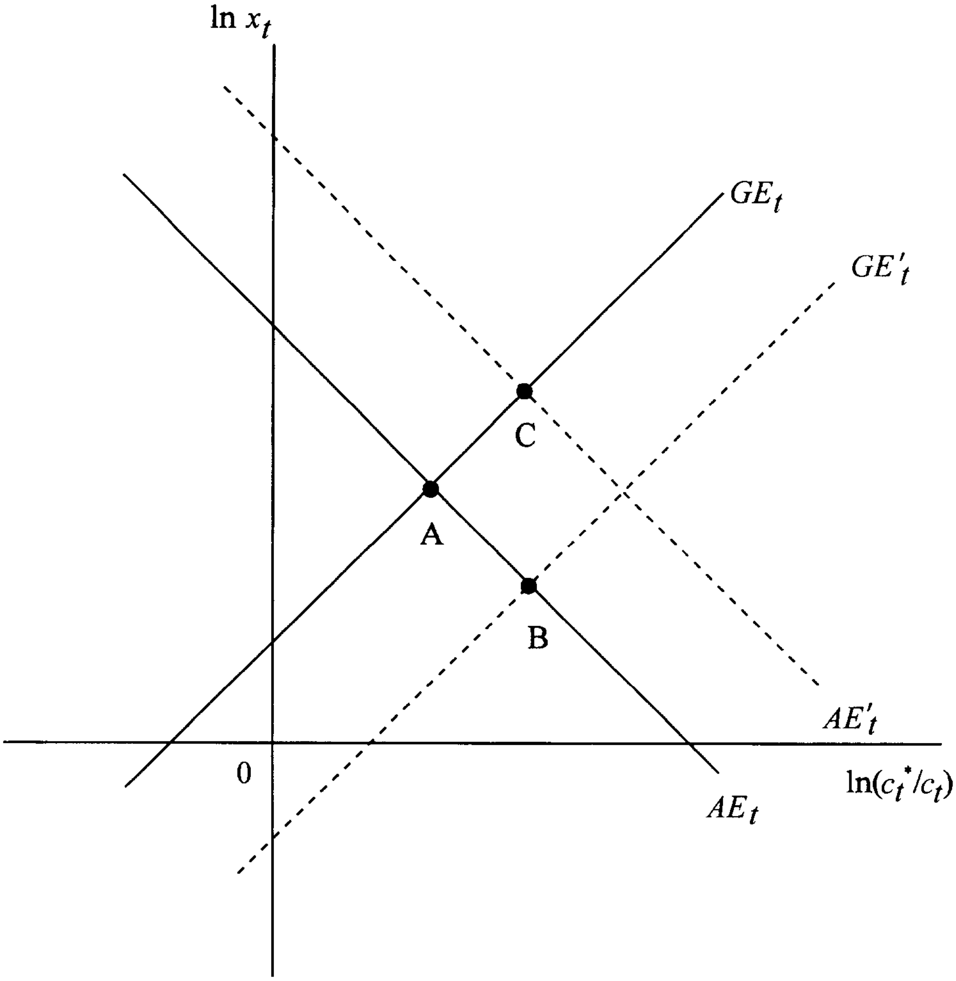

FIGURE 1. Intertemporal and intratemporal equilibrium loci.

II. Implications

II.A. Closed-form solutions for the real exchange rate, relative consumption, and the distribution of world wealth

The model can now be solved explicitly for the real exchange rate and relative consumption expenditure, as suggested by Figure 1. Combining equations ⟨14⟩ and ⟨16⟩ yields reduced forms:

$$\langle 17\rangle \quad x_t = \alpha^{\gamma/(\gamma+\sigma)}(\beta^*/\beta)^{[\gamma/(\gamma+\sigma)]t}\left(y_t^{N*}/y_t^N\right)^{-\gamma\sigma/(\gamma+\sigma)}(\omega_t^*/\omega_t)^{\gamma/(\gamma+\sigma)},$$

$$\langle 18\rangle \quad c_t^*/c_t = (\alpha)^{1/(\gamma+\sigma)}(\beta^*/\beta)^{[1/(\gamma+\sigma)]t}\left(y_t^{N*}/y_t^N\right)^{\gamma/(\gamma+\sigma)}(\omega_t^*/\omega_t)^{1/(\gamma+\sigma)}.$$

Initial wealth, taste shocks, rates of time preference, and non-tradables productivity all affect relative consumption and the real exchange rate.[6] If the domestic consumer's rate of time preference exceeds that of the foreigner ($\beta^* > \beta$), for instance, the larger incentive to save abroad will raise relative foreign consumption and the real exchange rate over time. A similar conclusion is implicit in Helpman and Razin (1982), but their model does not yield the closed-form solutions obtained here.[7]

The perfectly pooled equilibrium models of real exchange rates have precluded by assumption endogenous changes in the distribution of wealth as well as an understanding of how changes in relative wealth are related to equilibrium real exchange rates. In this section, we examine how the closed-form results for relative consumption are related to the distribution of wealth between the two countries' consumers and show that the distribution of wealth in our model is endogenous, variable over time, and systematically related to the equilibrium real exchange rate.

First, the relationship between the home consumer's consumption and wealth (and analogously for the foreign consumer) is well known for the specific preferences assumed here. Current consumption is proportionate to current wealth, consistent with the permanent income hypothesis (cf. the derivation in Ingersoll, 1987, ch. 11):

$$c_t = (1 - b_t)s_t V_t. \tag{19}$$

V_t represents world wealth expressed in terms of the home consumption basket (the before-dividend value of the four production sectors added together) and s_t is the share of world wealth held by the domestic consumer ($s_t + s_t^* = 1$). As wealth is measured in terms of the home consumption basket, $V_t = x_t V_t^*$. The marginal propensity to consume, $(1 - b_t)$, varies over time depending upon the time preference parameters and current and expected future taste and productivity shocks.

Second, the distribution of wealth is endogenous and variable over time. An expression for relative wealth shares, s_t^*/s_t, is obtained by dividing equation ⟨19⟩ by its foreign counterpart to produce:

$$s_t^*/s_t = [(1 - b_t)/(1 - b_t^*)]x_t c_t^*/c_t. \tag{20}$$

Substituting reduced forms ⟨17⟩ and ⟨18⟩ for x_t and c_t^*/c_t, respectively, in ⟨20⟩ yields:

⟨21⟩

$$s_t^*/s_t = [(1 - b_t)/(1 - b_t^*)](\alpha\omega_t^*/\omega_t)^{(1+\gamma)/(\gamma+\sigma)}(\beta^*/\beta)^{((1+\gamma)t)/(\gamma+\sigma)} \times \left(y_t^{N*}/y_t^N\right)^{((1-\sigma)\gamma)/(\gamma+\sigma)}.$$

In contrast to the perfect pooling models, the wealth shares in our model differ between the two countries and vary endogenously, as in Stockman and Dellas (1989).[8] The marginal propensities to consume out of wealth, $1 - b_t$ and $1 - b_t^*$, in general will vary over time also.

Third, consider the special case when utility is logarithmic and taste shocks follow a martingale process. In this case with $\sigma = 1$ and $E_t(\omega_{t+k}) = \omega_t$, $b_t = \beta$

and $b_t^* = \beta^*$ (proof available upon request). A comparison of reduced-form relative consumption equation ⟨18⟩ and relative wealth equation ⟨21⟩ suggests that relative consumption and relative wealth will typically covary positively. With constant relative marginal propensities to save, relative consumption and relative wealth always move in the same direction in response to taste shocks or differences in rates of time preference.

Our model also has implications for the composition of wealth, in particular the domestic consumer's asset portfolio relative to that of the foreign consumer. Both consumers hold portfolios of generally different size but identical composition. The reason is that, in equilibrium, the exchange rate moves to equate across countries the marginal utility per unit of the numeraire, so that consumers with identical wealth and utility functions choose identical portfolios.[9] Moreover, the specific parameterization of our model presumes constant relative risk aversion with the degree of relative risk aversion, σ, equal for both representative consumers; as a result, even consumers with different wealth (and different time preferences or taste shocks that do not affect the degree of relative risk aversion) will hold portfolios of equal composition.

This result contrasts sharply with the portfolio results obtained in Stockman and Dellas (1989). In their paper, the representative consumers hold similar portfolios of the tradable goods, but hold all of the shares of their own non-tradable goods: the portfolios are constructed to let dividends coincide with consumption expenditure. This empirically attractive result is obtained based on utility functions that are additively separable in the tradable and non-tradable goods, and display identical time preference and are equal for the tradable goods. Since the utility from the non-tradable good cannot be affected given endowments and separable preferences, holding all shares of the non-tradable good stabilizes expenditure available for the tradable goods. In addition, perfectly diversifying the tradable-goods assets in proportion to initial wealth stabilizes the relative consumption of tradable-goods over time, which is optimal given identical time preference and identical utility over the tradable goods.

It would, in principle, be possible to obtain such a portfolio result — where each representative consumer holds more of its own country's assets — in our model; but this would involve imposing separability of tradable- and non-tradable-goods utility (which Stockman and Dellas admit is the 'most serious limitation' of their model [1989, p. 288]). As a consequence, we would have to forgo the homotheticity assumption and the simple closed-form solutions.

Since our data and empirical findings can only relate the real exchange rate to relative consumption, the link to relative wealth here is useful for interpreting the empirical results presented later in a broader context.

II.B. The cross-sectional relationship between real exchange rates and relative per capita GDPs

Economists have traditionally examined departures of countries' exchange rates from purchasing power parity using time-series data. However, Kravis *et al.* (1982) and Summers and Heston (1988) investigated systematic departures from purchasing power parity *across pairs of countries and over time*. Using data from the U.N. ICP, some countries' price levels were found to be no more than one-third the US price level, and these departures from absolute purchasing power parity have persisted for decades. Kravis and Lipsey (1983, 1987, 1988) examined systematically determinants of the variation across pairs of countries in their exchange-rate-adjusted national price levels relative to the US price level (the numeraire), that is, cross-country variation in real exchange rates. They found that countries' relative per capita real GDPs explained almost entirely cross-country variation in real exchange rates (in levels or log-levels).

Although Kravis and Lipsey's empirical results are robust, the authors have not explained in a formal theoretical framework their systematic empirical findings.[10] Our structural real exchange rate equations ⟨14⟩ and ⟨16⟩ can be rewritten to readily motivate theoretically the empirical log-linear relationship between relative national price levels — that is, the real exchange rate — and relative per capita GDPs, uncovered by Kravis and Lipsey:

$$\langle 22\rangle \qquad x_t = [(y_t^* - tb_t^*)/(y_t - tb_t)]^{\gamma}(y_t^{N*}/y_t^N)^{-\gamma},$$

$$\langle 23\rangle \qquad x_t = \alpha(\omega_t^*/\omega_t)(\beta^*/\beta)^t[(y_t^* - tb_t^*)/(y_t - tb_t)]^{-\sigma}.$$

Equations ⟨22⟩ and ⟨23⟩ use $c_t = y_t - tb_t$ and its foreign equivalent (which hold because our model does not distinguish between consumption expenditure and aggregate expenditure) where y_t represents per capita GDP and tb_t represents the per capita trade balance, both expressed in terms of the home consumption bundle.

The relationship between relative per capita GDPs and the real exchange rate is more complex than that suggested in Kravis and Lipsey (1983, 1987, 1988). Their explanations are consistent with the positive relationship between relative per capita GDPs and the real exchange rate through the goods market, equation ⟨22⟩. In the context of our model, however, a log-linear regression across pairs of countries of real exchange rates on relative per capita GDPs ignores three issues. First, relative per capita GDPs should be adjusted to reflect relative expenditure (or consumption); this is not a serious problem as in our model total expenditure equals GDP minus the empirically small trade balance. Second, an omitted variables bias is created in estimating ⟨22⟩ when relative per capita non-tradables outputs are ignored; this issue has been addressed in some of Kravis and Lipsey's work by frequent inclusion of the relative shares of non-tradables in the countries' GDPs. Third, a simultaneous equations bias is created by ignoring the relationship between relative per capita GDPs and the real exchange rate through intertemporal equation ⟨23⟩, noting that initial relative wealth (which is unmeasurable) influences the real exchange rate through the asset market and is likely to vary considerably across pairs of countries.

A common explanation for the strong empirical relationship between relative national price levels and relative per capita real GDPs is the productivity-differentials model, attributable to Balassa (1964) and Samuelson (1964). Consider, as in our model, the national price level to be decomposable into non-tradables (mainly services) prices and tradables (mainly commodities) prices. Higher per capita income countries are believed to have absolute productivity advantages in non-tradables and tradables, but a relative productivity advantage in tradables. Thus, the relative price of non-tradables to tradables will be higher in countries with larger per capita incomes. As commodity arbitrage tends to equilibrate tradables' prices across countries, the national price level tends to be higher in rich countries, as their price of non-tradables relative to tradables is higher.

The productivity-differentials theory of Balassa–Samuelson is imbedded in our goods-market equilibrium. In his analysis, Balassa (1964, p. 586) assumes 'invisibles and capital movements do not enter the balance of payments', thus avoiding the relevance of intertemporal equilibrium. It follows that the trade balance must be assumed equal to zero so that not only $y_t^N = c_t^N$ but also $y_t^T = c_t^T$. Combining equations ⟨7⟩ and ⟨14⟩ with the market-clearing conditions produces:

$$x_t = \left[\left(y_t^{T*}/y_t^{N*}\right)/\left(y_t^T/y_t^N\right)\right]^{\gamma/(1+\gamma)}. \tag{24}$$

Equation ⟨24⟩ is the Balassa–Samuelson proposition; the higher the productivity in tradables relative to non-tradables in the foreign relative to the home country, the higher will be the relative national price level in the foreign country — that is, the real exchange rate.

II.C. Is the United States a spendthrift nation?

Numerous studies have questioned the commonly held notion that the United States has a significantly lower savings rate than other industrialized nations, implying that the United States is a 'spendthrift' nation. National Income Product Accounts have become notorious for understating the relative savings propensity of the United States. Lipsey and Kravis (1987), for example, developed a set of adjustments to countries' savings rates to illustrate that the difference between the rate of US capital accumulation and other industrialized countries' rates is smaller than National Income Product Accounts reveal. Similarly, Hayashi (1989) notes that Japanese national accounts value depreciation at historical cost while US national accounts value it at replacement cost. Left unadjusted, the Japanese savings rate between 1978 and 1981 was more than twice the US rate. When the data are made more compatible, the Japanese rate is almost identical to the US rate for those years, although more recently the adjusted Japanese rate has risen relative to the US rate.

Our model yields theoretical conclusions, potentially estimable, concerning the rate of time preference of, say, the United States vis-à-vis other nations. No accounting data would be needed to estimate relative rates of time preference from our model.

III. Econometric issues

In contrast to most previous equilibrium models of real exchange rates, the reduction of our general equilibrium model to closed-form equilibrium conditions ⟨14⟩ and ⟨16⟩ and reduced-form real exchange rate and relative consumption equations ⟨17⟩ and ⟨18⟩, respectively, allows estimation. In this section, we discuss the choice of data and relevant econometric issues.

Given the model's description of equilibrium phenomena, a data set constructed with long-run behavior in mind is optimal. Since our model has implications for both the time-series and cross-sectional behavior of real exchange rates, a panel data set is potentially useful. Summers and Heston (1988) provide annual time series from 1950 to 1985 on relative per capita private consumption expenditures and on relative consumption price levels (or consumption-based real exchange rates) for 130 countries with the United States as the numeraire (US = 100), designed for pooled cross-section time-series investigation.[11] The only shortcoming is the absence of a decomposition of relative consumption expenditures between tradables and non-tradables.

Fortunately, the OECD *National Accounts, 1973–1985* (1987) enables construction of relative shares of consumption expenditures into non-tradables (services) and tradables (commodities) using the same categorization as Kravis *et al.* (1982) for the United States and ten other OECD countries over the years 1973 through 1985.[12] Earlier editions of the *National Accounts* did not contain *systematic* decompositions of consumption expenditures into commodities and services.[13]

The log-linear versions of equations ⟨17⟩ and ⟨18⟩ potentially estimable by ordinary least squares (OLS) are:

$$\langle 25\rangle \qquad \ln x_{it} = \sum_{j=1}^{10} \Phi_j^1 + \sum_{j=1}^{10} \left([\gamma/(\gamma+\sigma)][\ln(\beta_j/\beta)]\right) \text{trend}_{jt} - [\gamma\sigma/(\gamma+\sigma)] \ln\left(y_{it}^N/y_t^N\right) + \epsilon_{it}^1,$$

$$\langle 26\rangle \qquad \ln(c_{it}/c_t) = \sum_{j=1}^{10} \Phi_j^2 + \sum_{j=1}^{10} \left([1/(\gamma+\sigma)][\ln(\beta_j/\beta)]\right) \text{trend}_{jt} + [\gamma/(\gamma+\sigma)] \ln\left(y_{it}^N/y_t^N\right) + \epsilon_{it}^2,$$

where x_{it} is the real exchange rate of country i relative to the United States in year t, c_{it} (c_t) is per capita consumption in country i (the US), and y_{it}^N (y_t^N) is per capita services consumption in country i (the US). Variable trend_{jt} is a time trend when $j = i$ and is 0 for $j \neq i$. β_j/β is the discount rate in country j relative to the US discount rate when $j = i$. The ϵ's are i.i.d. error terms and Φ_j^1 and Φ_j^2 are dummy variables assuming the value 1 when $j = i$ and 0 when $j \neq i$. (For econometric convenience, we include an intercept and 9 dummies for 9 of the 10 country pairs in the estimation of each equation.)

The log-linear versions of equations ⟨14⟩ and ⟨16⟩ potentially estimable by ordinary least squares (OLS) are:

$$\langle 27\rangle \qquad \ln x_{it} = \sum_{j=1}^{10} \Phi_j^3 + \gamma \ln(c_{it}/c_t) - \gamma \ln\left(y_{it}^N/y_t^N\right) + \epsilon_{it}^3,$$

$$\langle 28\rangle \quad \ln x_{it} = \sum_{j=1}^{10} \Phi_j^4 - \sigma[\ln(c_{it}/c_t)] + \sum_{j=1}^{10} \left[\ln(\beta_j/\beta)\right]\text{trend}_{jt} + \epsilon_{it}^4,$$

An econometric issue not yet addressed is the simultaneous equations bias in ⟨27⟩ and ⟨28⟩, owing to the endogeneity of $\ln(c_{it}/c_t)$. Two-stage least squares (2SLS) estimation of structural equations ⟨27⟩ and ⟨28⟩ is used conventionally to obtain consistent estimates of the parameters σ, γ, and β_j/β; this method is used here. In the first stage, we estimate reduced-form relative consumption equation ⟨26⟩ to obtain the predicted values of $\ln(c_{it}/c_t)$. In the second stage, the predicted values of $\ln(c_{it}/c_t)$, denoted $\widehat{\ln(c_{it}/c_t)}$, are used as 'instruments' for $\ln(c_{it}/c_t)$. Hence, regressions ⟨27⟩ and ⟨28⟩ use the predicted values of $\ln(c_{it}/c_t)$ from the first stage.[14]

Finally, since the data set includes times series as well as cross section observations, one needs to address the issue of data stationarity over time. If the individual time series are stationary in log-levels, coefficient estimates in these regressions are consistent and the Student t-distribution can be used to evaluate their statistical significance. However, even if the individual time series are stationary in first-differences of their log-levels, if the series are cointegrated OLS coefficient estimates are consistent but the t-statistics must be adjusted to evaluate the coefficients' statistical significance. Consistent with recent empirical evidence on relative price levels using similar panel data sets, univariate tests suggested that our relative price level, relative per capita consumption, and relative per capita non-tradables consumption variables are stationary in log-levels.[15]

IV. Empirical evidence

In this section, we present first the results of estimating log-linear econometric versions of reduced-form equations ⟨17⟩ and ⟨18⟩. The estimate of reduced-form equation ⟨18⟩ generates the instrument for estimating structural equations ⟨14⟩ and ⟨16⟩.

IV.A. Reduced-form equations

OLS estimation of equations ⟨25⟩ and ⟨26⟩ yields:

⟨29⟩

$$\begin{aligned}\ln x_{it} = {} & \underset{(4.04)}{9.3555} - \underset{(2.04)}{1.2139}\ln\left(y_{it}^N/y_t^N\right) - \underset{(1.17)}{0.0112}\,\text{TrendAus}_{it} \\ & - \underset{(1.44)}{0.0139}\,\text{TrendCan}_{it} - \underset{(3.08)}{0.0288}\,\text{TrendDen}_{it} - \underset{(0.73)}{0.0069}\,\text{TrendFin}_{it} \\ & - \underset{(2.61)}{0.0290}\,\text{TrendGre}_{it} - \underset{(1.78)}{0.0168}\,\text{TrendIta}_{it} + \underset{(1.42)}{0.0147}\,\text{TrendJpn}_{it} \\ & - \underset{(0.56)}{0.0060}\,\text{TrendNor}_{it} - \underset{(3.67)}{0.0415}\,\text{TrendSwe}_{it} - \underset{(0.66)}{0.0065}\,\text{TrendUK}_{it}\end{aligned}$$

$$
\begin{aligned}
&+ \underset{(1.75)}{0.5911}\,\mathrm{DVCan}_{it} + \underset{(3.34)}{0.3719}\,\mathrm{DVDen}_{it} + \underset{(0.28)}{0.0455}\,\mathrm{DVFin}_{it} \\
&- \underset{(1.82)}{0.8617}\,\mathrm{DVGre}_{it} - \underset{(2.24)}{0.5404}\,\mathrm{DVIta}_{it} + \underset{(1.19)}{0.1515}\,\mathrm{DVJpn}_{it} \\
&- \underset{(0.08)}{0.0116}\,\mathrm{DVNor}_{it} + \underset{(3.36)}{0.3963}\,\mathrm{DVSwe}_{it} + \underset{(0.01)}{0.0014}\,\mathrm{DVUK}_{it}
\end{aligned}
$$

$R^2 = 0.72$, $AdjR^2 = 0.67$, S.E.E. $= 0.126$, $n = 130$

⟨30⟩

$$
\begin{aligned}
\ln(c_{it}/c_t) = {} & \underset{(0.03)}{0.0113} + \underset{(11.89)}{1.0639}\ln\left(y_{it}^N/y_t^N\right) - \underset{(0.52)}{0.0007}\,\mathrm{TrendAus}_{it} \\
&- \underset{(2.74)}{0.0040}\,\mathrm{TrendCan}_{it} - \underset{(4.34)}{0.0061}\,\mathrm{TrendDen}_{it} - \underset{(2.32)}{0.0033}\,\mathrm{TrendFin}_{it} \\
&- \underset{(3.50)}{0.0058}\,\mathrm{TrendGre}_{it} + \underset{(2.64)}{0.0037}\,\mathrm{TrendIta}_{it} - \underset{(2.68)}{0.0042}\,\mathrm{TrendJpn}_{it} \\
&- \underset{(2.03)}{0.0033}\,\mathrm{TrendNor}_{it} - \underset{(2.16)}{0.0037}\,\mathrm{TrendSwe}_{it} + \underset{(3.47)}{0.0051}\,\mathrm{TrendUK}_{it} \\
&- \underset{(2.91)}{0.1474}\,\mathrm{DVCan}_{it} + \underset{(6.12)}{0.1023}\,\mathrm{DVDen}_{it} + \underset{(3.57)}{0.0865}\,\mathrm{DVFin}_{it} \\
&+ \underset{(3.23)}{0.2290}\,\mathrm{DVGre}_{it} + \underset{(5.96)}{0.2160}\,\mathrm{DVIta}_{it} - \underset{(12.64)}{0.2414}\,\mathrm{DVJpn}_{it} \\
&+ \underset{(9.51)}{0.2206}\,\mathrm{DVNor}_{it} + \underset{(8.76)}{0.1548}\,\mathrm{DVSwe}_{it} - \underset{(8.06)}{0.1478}\,\mathrm{DVUK}_{it}
\end{aligned}
$$

$R^2 = 0.99$, $AdjR^2 = 0.99$, S.E.E. $= 0.019$, $n = 130$

Absolute values of t-statistics are in parentheses and S.E.E. is the standard error of the regression.

The results indicate that relative per capita non-tradables output has the expected negative relationship with the real exchange rate in ⟨29⟩ and expected positive relationship with relative per capita consumption in ⟨30⟩. As footnote 14 discussed, indirect least squares cannot identify uniquely structural parameters because of overidentification. Equation ⟨30⟩ is used in the first stage of 2SLS to generate the instrument, denoted $\ln(\widehat{c_{it}/c_t})$, used in the next section.

IV.B. Structural equations

Two-stage least squares estimation of equations ⟨27⟩ and ⟨28⟩ yields:

⟨31⟩

$$
\begin{aligned}
\ln x_{it} = {} & \underset{(3.99)}{6.3909} + \underset{(3.59)}{2.8217}\ln\left(\widehat{c_{it}/c_t}\right) - \underset{(4.15)}{3.4819}\ln\left(y_{it}^N/y_t^N\right) \\
&+ \underset{(3.20)}{0.6962}\,\mathrm{DVCan}_{it} + \underset{(0.55)}{0.0426}\,\mathrm{DVDen}_{it} - \underset{(0.52)}{0.0545}\,\mathrm{DVFin}_{it} \\
&- \underset{(3.30)}{1.0003}\,\mathrm{DVGre}_{it} - \underset{(4.50)}{1.0037}\,\mathrm{DVIta}_{it} + \underset{(4.62)}{0.9699}\,\mathrm{DVJpn}_{it}
\end{aligned}
$$

$$- \underset{(2.64)}{0.4375}\,\mathrm{DVNor}_{it} - \underset{(0.45)}{0.0576}\,\mathrm{DVSwe}_{it} + \underset{(3.08)}{0.3013}\,\mathrm{DVUK}_{it}$$

$$R^2 = 0.63,\ AdjR^2 = 0.60,\ \text{S.E.} = 0.138,\ n = 130$$

⟨32⟩

$$\begin{aligned}
\ln x_{it} = {} & \underset{(4.03)}{9.3684} - \underset{(2.04)}{1.1410}\ln\left(\widetilde{c_{it}/c_t}\right) - \underset{(1.27)}{0.0120}\,\mathrm{TrendAus}_{it} \\
& - \underset{(1.76)}{0.0185}\,\mathrm{TrendCan}_{it} - \underset{(3.49)}{0.0358}\,\mathrm{TrendDen}_{it} - \underset{(1.14)}{0.0107}\,\mathrm{TrendFin}_{ir} \\
& - \underset{(3.67)}{0.0357}\,\mathrm{TrendGre}_{it} - \underset{(1.26)}{0.0125}\,\mathrm{TrendIta}_{it} + \underset{(1.04)}{0.0099}\,\mathrm{TrendJpn}_{it} \\
& - \underset{(0.98)}{0.0097}\,\mathrm{TrendNor}_{it} - \underset{(3.63)}{0.0457}\,\mathrm{TrendSwe}_{it} - \underset{(0.07)}{0.0006}\,\mathrm{TrendUK}_{it} \\
& + \underset{(1.63)}{0.4229}\,\mathrm{DVCan}_{it} + \underset{(3.46)}{0.4886}\,\mathrm{DVDen}_{it} + \underset{(0.41)}{0.0532}\,\mathrm{DVFin}_{it} \\
& - \underset{(1.72)}{0.6004}\,\mathrm{DVGre}_{it} - \underset{(2.06)}{0.2939}\,\mathrm{DVIta}_{it} - \underset{(1.01)}{0.1239}\,\mathrm{DVJpn}_{it} \\
& + \underset{(2.28)}{0.2401}\,\mathrm{DVNor}_{it} + \underset{(5.22)}{0.5730}\,\mathrm{DVSwe}_{it} - \underset{(1.57)}{0.1672}\,\mathrm{DVUK}_{it}
\end{aligned}$$

$$R^2 = 0.72,\ AdjR^2 = 0.67,\ \text{S.E.} = 0.126,\ n = 130$$

Absolute values of t-statistics are again in parentheses.

The key coefficient estimates in equations ⟨31⟩ and ⟨32⟩ have the following interpretations. The estimates of γ (2.8217 and 3.4819) are approximately 3; the coefficient estimates of γ are statistically significant at the one percent level (one-tailed t-test). The value of the constrained estimate of γ is 2.98; using an F-statistic, equality of the two coefficients could not be rejected at the 10 percent level ($F[1,18] = 2.73$). This estimate of γ implies a share of non-tradables in the subutility function $[\gamma/(1+\gamma)]$ of 0.75, which is consistent with other empirical evidence that non-tradables (largely services) compose about two-thirds of OECD countries' consumption expenditures.

The estimated inverse of the elasticity of intertemporal substitution (or the coefficient of relative risk aversion), σ, equals 1.14. This estimate is consistent with those in the closed-economy macroeconomics literature; estimates of σ range typically between 0 and 2 (cf., Mehra and Prescott, 1985 and Eichenbaum *et al.*, 1988). This estimate is statistically significantly different from zero at the 2.5 percent level. Moreover, this estimate is not significantly different from unity at the 10 percent significance level, suggesting the plausibility of logarithmic utility.

The estimated values of relative discount rates, β_i/β, in equation ⟨32⟩ range between -5 percent and 1 percent. These estimates suggest that the US rate of time preference is *less* than that of nine other OECD countries; in particular, the results suggest that the United States was economically and statistically significantly 'thriftier' than Denmark, Greece and Sweden. The positive coefficient estimate for the Japan trend variable (1 percent) suggests plausibly that Japan had a lower rate of time preference than the United States (although not statistically significantly lower).[16]

Finally, Taylor (1995) notes that equilibrium exchange rate studies have 'tended to eschew formal econometric analysis' (p. 41). Backus and Smith (1993) and Kollmann (1995) are the exceptions; we comment now on how our empirical results compare with theirs. Backus and Smith found virtually no relationship between relative consumption and real exchange rate *growth rates*. The absence of empirical correlations may be tied to their using first-differences of logarithms of the variables, rather than log-levels. Our panel data tests of stationarity of the variables using more recent techniques (Levin and Lin, 1992) suggested that empirical evaluation of the relationship between log-linear levels of the relevant variables is appropriate.

Moreover, Backus and Smith conclude that their insignificant empirical findings may reflect the absence of taste shocks, a factor important in our framework:

> What features of a more general model might reproduce the patterns evident in figs. 1–3? One possibility would be to admit demand-side shocks (such as taste shocks), in addition to the endowment shocks studied here. Taste shocks lead to a negative correlation between changes in relative consumption and in the real exchange rate in contrast to the positive correlation arising from endowment shocks (pp. 313-4; their real exchange rate is defined as the inverse of ours).

Kollmann (1995) presents an empirical test of an asset-market equilibrium similar to ours. Kollmann finds no evidence of cointegration among per capita consumptions of countries and real exchange rates when assuming complete markets, suggesting the need to explore an incomplete-markets alternative — a useful direction of research. The apparent contrast between the results in Kollmann and our study may be attributed to several considerations. First, Kollmann's study does not incorporate theoretically or empirically non-tradables consumption. Second, Kollmann's analysis tested for stationarity using traditional time-series techniques on logs of individual countries' (not relative) consumption levels and real exchange rates. His findings of non-stationarity for individual consumption levels are plausible. His findings of non-stationarity of real exchange rates are consistent with earlier studies using traditional low-power tests of individual country pairs with short time series. However, our use of more powerful panel tests suggested that real exchange rates and relative consumption levels are stationary. Third, if real exchange rates are stationary (as panel tests and traditional tests using very long time series indicate) and individual countries' consumption levels are non-stationary, one would expect to reject cointegration among real exchange rates and individual consumption levels. By contrast, our study examines the relationship between stationary real exchange rates, relative consumptions, and relative non-tradables outputs. Clearly, more research is warranted to further evaluate the possible complementary importance of non-tradables goods and incomplete markets in explaining real exchange rate behavior.

V. Conclusions

A two-country general equilibrium model was developed to consider the relationship between consumption and real exchange rates across time and across countries. The model yields closed-form solutions for the real exchange rate, relative per capita consumptions, and the distribution of world wealth. The solutions are obtained by considering the relationships between the real exchange rate and relative consumption expenditure that are required for intratemporal and intertemporal equilibrium conditions simultaneously. The real exchange rate obtained depends on relative rates of time preference and on relative non-tradables productivity.

Empirical evaluation of the model using panel data from Summers and Heston (1988) provides some preliminary evidence of parameter estimates using classical statistical methods. Plausible parameter estimates of the elasticity of intertemporal substitution, the relative share of non-tradables in consumption, and relative rates of time preference across countries are found. In particular, we found that the United States appears to have a lower rate of time preference than other OECD countries, with the exception of Japan. Moreover, the theoretical and empirical results are consistent with some stylized facts of large and persistent deviations from purchasing power parity over time (as explained by differences in time preference and endowment shocks over time) and the relationship found by Kravis and Lipsey (1983, 1987, 1988) between relative per capita real GDP and the real exchange rate across pairs of countries, in a manner consistent with the productivity-differentials models in Balassa (1964) and Samuelson (1964).

Appendix: A proof showing that the compositions of foreign and domestic portfolios in the model are identical

In the market economy, the budget constraint for the domestic agent investing in n different assets is:

$$W_{t+1} = \left(\sum_{i=1}^{n} w_t^i R_{t+1}^i \right) (W_t - c_t), \tag{A1}$$

with w_t^i defined as the share of the domestic agent's initial unconsumed wealth invested in asset i and R_{t+1}^i representing the (gross) real return on asset i.

Using ⟨A1⟩ and equation ⟨19⟩, and noting that $W_t = s_t V_t$ in the text, we can write:

$$a_{t+1} \left(\sum_{i=1}^{n} w_t^i R_{t+1}^i \right) = c_{t+1}/c_t, \qquad \text{where} \qquad a_{t+1} \equiv (1 - b_{t+1}) b_t / (1 - b_t). \tag{A2}$$

The utility specification in equations ⟨3⟩ and ⟨5⟩ together with ⟨A1⟩ produce the domestic consumer's first-order conditions for the investment shares in all assets i:

$$\beta E_t [R_{t+1}^i (\omega_{t+1}/\omega_t)(c_{t+1}/c_t)^{-\sigma}] = 1, \qquad \text{for all } i. \tag{A3}$$

Substituting equation ⟨A2⟩ into ⟨A3⟩ implies:

$$\beta E_t \left[R_{t+1}^i (\omega_{t+1}/\omega_t) \left(a_{t+1} \sum_{i=1}^{n} w_t^i R_{t+1}^i \right)^{-\sigma} \right] = 1, \qquad \text{for all } i. \tag{A4}$$

Given the concavity of the utility function and the linearity of the budget constraint equations, ⟨A4⟩ uniquely determines the optimal investment shares w_t^i for all assets (counting assets that are perfect substitutes for each other as one).

Analogously, we obtain the first-order conditions of the foreign consumer:

$$\langle \text{A5} \rangle \qquad \beta^* E_t\left[R_{t+1}^{i*}(\omega_{t+1}^*/\omega_t^*)\left(a_{t+1}^* \sum_{i=1}^{n} w_t^{i*} R_{t+1}^{i*} \right)^{-\sigma} \right] = 1.$$

The law of one price guarantees that nominal returns for all assets are identical in both countries. Converting to real terms then implies for each asset i:

$$\langle \text{A6} \rangle \qquad R_{t+1}^{i*} = R_{t+1}^{i}(x_t/x_{t+1}).$$

Substituting ⟨A6⟩ into ⟨A5⟩ yields:

$$\langle \text{A7} \rangle \qquad \beta^* E_t\left[R_{t+1}^{i}(x_t/x_{t+1})(\omega_{t+1}^*/\omega_t^*)\left(a_{t+1}^* \sum_{i=1}^{n} w_t^{i*} R_{t+1}^{i} x_t/x_{t+1} \right)^{-\sigma} \right] = 1.$$

First-differencing the (log of) equation ⟨16⟩ and substituting equation ⟨A2⟩ and its foreign equivalent (using A6) produces an equation which holds in general equilibrium:

$$\langle \text{A8} \rangle \qquad a_{t+1}\left(\sum_{i=1}^{n} w_t^i R_{t+1}^i \right)(\beta\omega_{t+1}/\omega_t)^{-1/\sigma} = (x_{t+1}/x_t)^{(1-\sigma)/\sigma} a_{t+1}^* \left(\sum_{i=1}^{n} w_t^{i*} R_{t+1}^i \right) \times (\beta^* \omega_{t+1}^*/\omega_t^*)^{-1/\sigma}.$$

Consider next whether the foreign consumer has an incentive to deviate from a general equilibrium outcome in which its portfolio is identical in composition to that of the domestic consumer. In this case ⟨A8⟩ simplifies to:

$$\langle \text{A9} \rangle \qquad a_{t+1}(\beta\omega_{t+1}/\omega_t)^{-1/\sigma} = (x_{t+1}/x_t)^{(1-\sigma)/\sigma} a_{t+1}^*(\beta^*\omega_{t+1}^*/\omega_t^*)^{-1/\sigma}.$$

Employing ⟨A9⟩ to eliminate the terms in equation ⟨A7⟩ yields:

$$\langle \text{A10} \rangle \qquad \beta E_t\left[R_{t+1}^{i}(\omega_{t+1}/\omega_t)\left(a_{t+1} \sum_{i=1}^{n} w_t^{i*} R_{t+1}^{i} \right)^{-\sigma} \right] = 1, \qquad \text{for all } i.$$

Comparison of ⟨A10⟩ and ⟨A4⟩ reveals that the foreign consumer faces the same first-order conditions as the domestic consumer in general equilibrium so that the portfolio allocation with $w_t^{i*} = w_t^i$ for all i is, indeed, an equilibrium.

Notes

1. Helpman and Razin (1982), Stulz (1987), and Stockman and Dellas (1989) do not use the perfect pooling equilibrium; these papers will be discussed separately in later sections.
2. If we define $z_t = \beta^t$, with β representing a standard discount factor, then a conventional dynamic formulation of preferences emerges. For the sake of generality, however, z_t may represent other factors—potentially stochastic—like demographics (if we view the representative consumer as a family unit), fashion, government expenditure, weather, etc. These are all factors that cannot be controlled by individual households.
3. Since we have data that consist of foreign values relative to domestic values, the assumption of identical current-period utility functions proves to be essential for empirical purposes.
4. In Balvers and Bergstrand (1990) and in an appendix that is available from the authors upon request, we present the model in fuller detail using competitive equilibrium and Arrow–Debreu frameworks, respectively. The assumption of complete markets implies that all shocks, including taste shocks, are insurable. Assuming that the taste shocks are systematic (i.e. applying to all households) and exogenous (i.e. not manipulable), the assumed existence of markets that insure against taste shocks is comparable to the

more standard assumption of the existence of markets that insure against productivity shocks. For a study that forms a valuable complement to ours in emphasizing the implications of incomplete markets, see Kollmann (1995).

5. The tradable good serves as the numeraire in both countries. If money is introduced explicitly — for instance, via a binding cash-in-advance constraint, $m_t = p_t\ c_t$ (and analogously abroad) — then the prices of the tradable goods would be determined within the model and the price levels and exchange rate would be nominal.
6. A closed-form solution for consumption in absolute terms can be obtained easily based on the solution in relative terms. In equation ⟨7⟩, set home tradables consumption equal to a fraction, $g(0 < g < 1)$, of world tradables production and foreign tradables consumption equal to $1 - g$ of world tradables production. Dividing foreign consumption by domestic consumption now yields an expression for relative consumption, which, when compared to the solution for relative consumption in equation ⟨18⟩, can be solved for the unknown g. Substituting for g in the transformed version of equation ⟨7⟩ yields the solution for consumption in absolute terms. This solution is not presented here since it is a complex expression of all the stochastic variables in the model.
7. In Stulz (1987), the ultimate effect of a change in the relative per capita non-tradables output on the real exchange rate depends exclusively on the relative importance of non-tradables in expenditures. In Stulz's model, the two representative consumers are assumed to have logarithmic utility functions ($\sigma = 1$), identical time discount factors ($\beta^* = \beta$), identical initial wealths ($\alpha = 1$), and no shocks ($\ln\ \omega_t^* = \ln\ \omega_t = 0$). If these parameters were assumed in our model, reduced-form equation ⟨17⟩ would simplify to:

$$x_t = (y_t^N / y_t^{N*})^{\gamma/(1+\gamma)},$$

which is analogous to the result in Stulz (1987). Further, the specific result in Backus and Smith (1993) can be obtained from our results if $\beta^* = \beta$, and $\ln\ \omega_t^* = \ln \omega_t = 0$ and the result implied by Helpman and Razin (1992) would be obtained if $\sigma = 1$ and $\ln\ \omega_t^* = \ln\ \omega_t = 0$.

8. Stockman and Dellas (1989) avoid the limitations of the perfect pooling approach by allowing non-tradable goods like here. However, relative wealth varies in their model for different reasons than here. In their model, relative wealth varies due to random disturbances in relative non-tradable goods production, as each agent holds all claims to her own country's non-tradable. In our model, the presence of complete markets allows each country's agent to diversify perfectly among the two countries' tradable and non-tradable claims. Thus, relative wealth is not affected by random disturbances to income from non-tradables production *directly*. Rather, relative wealth varies over time in our model due to disturbances in preferences and differences in rates of time preference. Relative non-tradables production *does* influence relative wealth, but only to the extent it alters the relative purchasing power of agents' wealths. In our model, the qualitative effect of relative non-tradables production on relative wealth depends upon the elasticity of intertemporal substitution.
9. A proof of this is in the appendix.
10. Bergstrand (1991) and Falvey and Gemmell (1991, 1996) provide general equilibrium analyses of this systematic cross-country relationship, but address only the goods-market relationship in a single-period framework.
11. The model has been constructed theoretically with two large countries in mind, say, the rest of the world (ROW) as the foreign country and the United States as the home country. Since we could alternatively consider ROW and Japan as the two countries, and then divide appropriate terms to cancel ROW, the structure of the model is basically consistent with bilateral as well as multilateral comparisons.
12. In the work of Kravis, Heston and Summers on the ICP, non-tradables (tradables) are identical to services (commodities) in private final consumption expenditures. The ten other OECD countries are Austria, Canada, Denmark, Finland, Greece, Italy, Japan, Norway, Sweden, and the United Kingdom (denoted $j = 1, \ldots, 10$, respectively). Be-

cause of triangular arbitrage, the results for the real exchange rate, say, between the United States and the United Kingdom and that between the United States and Canada imply a set of results for the (cross) real exchange rate between the United Kingdom and Canada. Higher-frequency (e.g. quarterly) data could provide a longer time series, but such data could not be pooled across countries. Moreover, such higher-frequency data would not be as appropriate for determining long-run equilibrium relationships. On the latter, see Baxter (1994).

13. The constraint that per capita non-tradables could only be constructed reliably for the years 1973 through 1985 from published OECD sources might suggest (erroneously) that our model only applies to flexible exchange rate regimes. The model also applies to fixed exchange rate regimes. Of the countries and years used here, half had essentially 'flexible' exchange rates against the dollar over the entire period while Austria, Denmark, Finland, Norway and Sweden did not; see Baxter and Stockman (1989).
14. Two-stage least squares must be used rather than applying indirect least squares to the reduced-form equations' coefficient estimates because this system of equations is *overidentified.* Indirect least squares is unable in an overidentified system to yield unique values of structural parameters. To illustrate, consider equations ⟨25⟩ and ⟨26⟩. For instance, dividing the coefficient for country 1's time trend in equation ⟨25⟩, $\{[\gamma/(\gamma+\sigma)][\ln(\beta_1/\beta)]\}$, by the coefficient for this time trend in equation ⟨26⟩, $\{[1/(\gamma+\sigma)][\ln(\beta_1/\beta)]\}$, yields identification of γ. However, dividing the coefficient for country 2's (or 3's, etc.) time trend in equation ⟨25⟩, $\{[\gamma/(\gamma+\sigma)][\ln(\beta_2/\beta)]\}$, by the coefficient for this time trend in equation ⟨26⟩, $\{[1/(\gamma+\sigma)][\ln(\beta_2/\beta)]\}$, yields identification of γ also. Thus, using indirect least squares with the reduced forms cannot identify uniquely γ or any of the 10 country pairings' relative discount rates.
15. Levin and Lin (1992) provide theory and evidence of the increased power of testing for unit roots in short time series by pooling cross-sectionally. Recently, three studies (Wu, 1996; Frankel and Rose, 1996; and Oh, 1996) have found evidence that real exchange rates are stationary in log-levels by pooling time-series data across country pairs. Using the techniques in Levin and Lin (1992), we conducted similar tests of stationarity for our relative price, relative consumption and relative non-tradables consumption data. Tests could be conducted for relative prices and relative consumptions for the period 1961–1985 for all ten countries, and for relative non-tradables consumptions for the period 1976–1985 (due to data limitations). For the period 1961–1985, the tests indicated that stationarity could be accepted for relative prices and relative consumptions with or without intercepts, time trends, and/or fixed-year effects. For the period 1976–1985, the tests for relative non-tradables consumptions provided weaker evidence of stationarity, but the test results were comparable to those for relative price levels and relative consumptions over the same abbreviated 10-year period.
16. In the presence of either serial correlation or heteroskedasticity of the error terms, coefficient estimates remain unbiased and consistent. However, the coefficient estimates may be inefficient; that is, their standard errors may be biased. To account for this, we also estimated the two equations using generalized least squares to adjust for first-order serial correlation and heteroskedasticity using standard techniques. The results were consistent with earlier findings. Coefficient estimates remained statistically significant at conventional significance levels.

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