



# Gravity Redux: Estimation of gravity-equation coefficients, elasticities of substitution, and general equilibrium comparative statics under asymmetric bilateral trade costs

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## ABSTRACT

A large class of models with CES utility and iceberg trade costs are now known to generate isomorphic “gravity equations.” Economic interpretations of these gravity equations vary in terms of two basic elements: the exporter’s “mass” variable and the elasticity of trade with respect to true ad valorem “trade costs.” In this paper, we offer three potential contributions. First, we formulate and estimate a structural gravity equation based on the standard Krugman model of monopolistic competition and increasing returns. In the context of this model, a key parameter, the elasticity of substitution in consumption ( $\sigma$ ), can be estimated precisely – even without observable true ad valorem trade-cost measures – using exporter’s population and observable variables that influence trade costs. Second, in the empirical context of the well-known McCallum Canadian–U.S. “border puzzle,” our approach – allowing estimation of  $\sigma$  – yields considerably different general equilibrium comparative static trade-flow and economic welfare effects than those in an Armington endowment economy and assumed values of  $\sigma$ . Moreover, our predicted trade flows and GDPs are highly correlated with their respective observed values in the case of bilaterally symmetric or asymmetric Canadian–U.S. border effects. Third, a Monte Carlo analysis confirms unbiased and precise estimates of all coefficients, the elasticity of substitution, and comparative statics using our approach.

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## 1. Introduction

For half a century, the “gravity equation” has been used to estimate econometrically the ex post partial (or direct) effects of economic integration agreements, national borders, currency unions, language, and other measures of trade costs on bilateral international trade flows (cf., Anderson, 2011 and Bergstrand and Egger, 2011 for recent surveys). While two early formal theoretical foundations for the gravity equation with trade costs – first Anderson (1979) and later Bergstrand (1985) – addressed the role of “multilateral prices,” Anderson and van Wincoop (2003) refined the theoretical foundations for the gravity equation to emphasize the importance of accounting properly for the endogeneity of prices in a structural gravity model. Eaton and Kortum (2002), Melitz (2003), Helpman et al. (2008), and Chaney (2008) refined the theoretical foundations further for firm heterogeneity in productivity

and zero trade flows. As Eaton and Kortum (2002) and Arkolakis et al. (2012) note, there is a large class of models with constant-elasticity-of-substitution (CES) preferences, iceberg trade costs, and complete specialization that generate isomorphic gravity equations.

In Anderson and van Wincoop (2003), or AvW, a complete derivation of a standard Armington (“conditional,” in AvW terms) general equilibrium endowment-economy model of bilateral trade in a multi-region ( $N > 2$ ) setting with one good per region and iceberg trade costs suggests that traditional cross-section empirical gravity equations have been misspecified owing to the omission of theoretically motivated nonlinear multilateral price terms for exporting and importing regions. Their model yields the bilateral trade “structural” gravity model allowing asymmetric bilateral trade costs (ABTC):

$$X_{ij} = \frac{Y_i Y_j}{Y_W} \frac{t_{ij}^{1-\sigma}}{\Pi_i^{1-\sigma} P_j^{1-\sigma}}, \quad (1)$$

$$\text{where } \Pi_i^{1-\sigma} = \sum_{j=1}^N \frac{Y_j}{Y_W} \frac{t_{ij}^{1-\sigma}}{P_j^{1-\sigma}}, \quad P_j^{1-\sigma} = \sum_{i=1}^N \frac{Y_i}{Y_W} \frac{t_{ij}^{1-\sigma}}{\Pi_i^{1-\sigma}},$$

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where  $X_{ij}$  is the nominal trade flow from region  $i$  to region  $j$ ,  $Y_i$  ( $Y_j$ ) is the nominal GDP in  $i$  ( $j$ ),  $Y_W$  is world GDP,  $t_{ij}$  is one plus the iceberg trade costs (the latter expressed as an ad valorem rate) on goods exported from  $i$  to  $j$ , and  $\sigma$  is the elasticity of substitution in consumption. In the special case of symmetric bilateral trade costs (SBTC),  $t_{ij} = t_{ji}$ , the system of Eq. (1) reduces to:

$$X_{ij} = \frac{Y_i Y_j}{Y_W} \frac{t_{ij}^{1-\sigma}}{P_i^{1-\sigma} P_j^{1-\sigma}}, \text{ where } P_i^{1-\sigma} = \sum_{j=1}^N \frac{Y_j}{Y_W} \frac{t_{ij}^{1-\sigma}}{P_j^{1-\sigma}}. \quad (2)$$

Owing to the nonlinearities in both sets of structural relationships, AvW employ a nonlinear least squares (NLS) program for estimation, focusing on Eq. (2) with SBTC. In the absence of observable measures of  $t_{ij}$ , AvW assume  $t_{ij}^{1-\sigma} = d_{ij}^{(1-\sigma)\rho} e^{(1-\sigma)lnb_{US, CA} Border_{ij}}$  where  $d_{ij}$  is bilateral distance,  $Border_{ij}$  is a dummy variable with a value of 1 (0) if the two regions are separated by a national border,  $\sigma$ ,  $\rho$ , and  $lnb_{US, CA}$  are unknown parameters, and  $e$  is the natural log base. Consequently, the system of equations does not permit identification of  $\sigma$  separately from  $\rho$  and  $lnb_{US, CA}$ .<sup>1</sup> Hence, for general equilibrium comparative static estimates, they assume values of  $\sigma$ . Moreover, all estimation of parameters and calculation of comparative statics were conducted under the assumption of SBTC, even though many bilateral trade costs are asymmetric. For instance, based on bilateral tariff data from the Global Trade Analysis Project (GTAP) on 67 countries in 2001, there is a large heterogeneity bilaterally in tariff rates. Fifty-eight percent of the bilateral tariff rates are asymmetric; moreover, the asymmetry can be as large as 150%.<sup>2</sup>

Eaton and Kortum (2002), or EK, introduced an alternative Ricardian framework to generate a structural gravity equation where the key parameter,  $\theta$ , governs the heterogeneity in firms' productivities (or comparative advantages):

$$X_{ij} = \frac{T_i Y_j (w_i t_{ij})^{-\theta}}{\sum_{k=1}^N T_k (w_k t_{kj})^{-\theta}}, \quad (3)$$

where  $T_i$  is an index of country  $i$ 's "state of technology" and  $w_i$  is labor's wage rate.<sup>3</sup> While preferences are also modeled using CES utility, the elasticity of substitution in consumption ( $\sigma$ ) does not have a role in determining equilibrium bilateral trade flows. The structure of the model implies that the key parameter for estimation (and subsequently for comparative statics) is the supply-side measure of firm heterogeneity ( $\theta$ ). To identify  $\theta$ , EK used two alternative approaches, one using retail price data and another using wage data.<sup>4</sup>

However, as Arkolakis et al. (2012) note, the perfect competition models in AvW and EK and the monopolistic competition models in Krugman (1980) and Melitz (2003) are all in a broad class of models sharing Dixit–Stiglitz preferences, one factor of production, linear cost functions, complete specialization, and iceberg trade costs. Arkolakis et al. (2012) show that if three "macro-level" restrictions hold, then all four models will share a common estimator of the gains from trade – which depends only on the import-penetration ratio and a

gravity-equation-based estimate of the "trade-cost" elasticity of trade flows (of which  $\sigma$  is a measure of in many models).

In this paper, we first formulate a structural gravity equation based on Krugman (1980) monopolistic competition and increasing returns to scale (MC-IR) model as an alternative framework to AvW and EK for estimating gravity-equation coefficients, the elasticity of substitution in consumption ( $\sigma$ ), and general equilibrium comparative statics, allowing ABTC.<sup>5</sup> We show in the context of this model that  $\sigma$  (the key parameter for welfare analysis) can be identified precisely – even without observable ad valorem trade-cost measures. The reason is that the exporter's absolute factor endowment – related to the number of varieties produced – helps identify (or "pin down") individual exporters' price levels ( $p_i$ ), which allows identification of  $\sigma$  (not possible in the AvW framework). In reality, unlike in the AvW framework with one good per region, most regions produce a variety of products and evidence suggests that the number of varieties/producers – that is, the extensive margin of varieties produced per region – is related to the absolute factor endowment size of the region (cf., Bernard et al., 2009).<sup>6</sup>

Second, in the empirical context of the well-known "McCallum border puzzle," we apply our approach for estimating gravity-equation coefficients, the elasticity of substitution, and general equilibrium comparative statics. Since our estimated  $\sigma$  differs from the one assumed in AvW, our general equilibrium comparative statics differ significantly from those in AvW. However, our estimate of approximately 7 is in the middle of the range of typical estimates for  $\sigma$  of 5–10 discussed in Anderson and van Wincoop (2004).

Third, we use a Monte Carlo analysis to confirm precise and unbiased estimates of all coefficients, the elasticity of substitution, and general equilibrium comparative statics using our approach, even in small samples. We also demonstrate using this analysis that the comparative static effects on trade flows of a given trade-cost change can be very sensitive to the elasticity of substitution, with such effects for  $\sigma = 10$  more than 40 times those for  $\sigma = 3$ . We show that our approach and the AvW approach under ABTC can also be used for general equilibrium comparative statics for real economic variables, such as economic welfare. However, these results are very sensitive – even qualitatively – to  $\sigma$ , providing further motivation for finding a technique that identifies  $\sigma$  precisely.

The remainder of this paper is as follows. Section 2 presents the well-known Krugman MC-IR model to derive a structural gravity model that allows estimation of the elasticity of substitution (given consistently estimated gravity equation parameters) and of comparative static effects. Section 3 provides an empirical analysis of our approach and compares it to the results from other approaches, including AvW, in the well-known context of the McCallum Canadian–U.S. border puzzle. Section 4 presents Monte Carlo results demonstrating – in the absence of specification and measurement error – that we can obtain precise and unbiased estimates of the elasticity of substitution and of comparative statics using our approach. Section 5 concludes.

<sup>5</sup> As noted above, AvW could only assume values for  $\sigma$ . Waugh (2007) also notes the importance of asymmetric bilateral trade costs for explaining observed bilateral trade flow patterns and relative price and real per capita income differences between countries.

<sup>6</sup> An earlier related paper to ours is Lai and Trefler (2002), which examines using panel data the MC-IR model. With regard to model specification issues, Lai and Trefler find that the MC-IR model works well, explaining about 78% of the variation in bilateral (aggregate manufacturing) trade flows and they also use their model to estimate welfare effects of tariff liberalizations. Our approach also has parallels to Redding and Venables (2004), where the authors derive a gravity equation based upon the MC-IR model. They estimate the model and the predicted trade flows are then used to construct market-access and supply-access variables, which are then used (as generated regressors) in a wage (per capita income) equation to explain sources of wage variation. They do not address comparative statics, estimation of any nonlinear price terms, nor estimation of the elasticity of substitution.

<sup>1</sup> Of course, this issue generalizes for any number of determinants of  $t_{ij}^{1-\sigma}$ .

<sup>2</sup> Balistreri and Hillberry (2007) first showed that allowing for asymmetric bilateral trade costs influences the results of estimating the Canadian–U.S. "border effect."

<sup>3</sup> For conciseness, we ignore intermediate goods in this specification, a component of the original EK model.

<sup>4</sup> In one approach, they employed cross-country disaggregate retail price level data to approximate bilateral ad valorem trade costs ( $t_{ij}$ ). Assuming commodity price arbitrage, the maximum difference between two countries' prices for similar goods bounds ad valorem bilateral trade costs. Second, they used wage rate data (and alternatively instruments for  $w_i$ ) to estimate  $\theta$ . Using wage rate data (wage instruments), EK found an estimate for  $\theta$  of 2.86 (3.60). Using the disaggregated price data and the commodity-price-arbitrage condition, EK found estimates of  $\theta$  ranging between 2.44 and 12.86 depending upon OLS or two-stage least squares estimation.

## 2. Methodology

In Section 2.1, we summarize the Krugman MC-IR model and describe the structural gravity equation it implies. In Section 2.2, we show how to use the approach to estimate the elasticity of substitution in consumption, general equilibrium comparative static trade-flow effects of a reduction in trade costs, and economic welfare effects of trade-cost reductions (i.e., the “gains from trade”).

### 2.1. The monopolistic competition-increasing returns model

#### 2.1.1. Aggregate bilateral trade flows

Following Krugman (1980), we assume there exists a single industry where preferences are constant-elasticity-of-substitution (CES). We assume iceberg transport costs and symmetric firms within each region, and hence all products in region  $i$  sell at the same price,  $p_i$ . As in Krugman (1980) and Feenstra (2004), the value of aggregate bilateral exports from region  $i$  to region  $j$ ,  $X_{ij}$ , equals  $n_i p_i t_{ij} c_{ij}$ , where  $n_i$  is the number of firms (varieties produced) in  $i$ ,  $t_{ij} \geq 1$  are ad valorem iceberg trade costs, and  $c_{ij}$  is demand in  $j$  for output of each firm in  $i$ . As standard,  $X_{ij}$  is determined by:

$$X_{ij} = n_i \left( \frac{p_i t_{ij}}{P_j} \right)^{1-\sigma} Y_j, \text{ where } P_j = \left[ \sum_{i=1}^N n_i (p_i t_{ij})^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \quad (4)$$

which is identical to Eq. (5.26) in Feenstra (2004, p. 153). We assume the  $t_{ij}$ s are the true – but empirically unobservable – ad valorem trade costs.

#### 2.1.2. Production

The assumption of a monopolistically competitive market with increasing returns to scale in production (internal to the firm) and a single factor (labor) is sufficient to identify the exporting region’s number of varieties. The representative firm in region  $i$  is assumed to maximize profits subject to the workhorse linear cost function:

$$l_i = \alpha + \phi y_i, \quad (5)$$

where  $l_i$  denotes labor used by the representative firm in country  $i$ ,  $y_i$  denotes the output of the firm,  $\alpha$  denotes the labor requirement necessary to setup a firm, and  $\phi$  denotes the amount of labor necessary to produce one unit of output. We assume that these labor requirements are homogeneous across all countries, as standard in the Krugman MC-IR model.

Two conditions characterize equilibrium in this class of models. First, profit maximization ensures that prices are a markup over marginal costs:

$$p_i = \frac{\sigma}{\sigma-1} \phi w_i, \quad (6)$$

where  $w_i$  is the wage rate in country  $i$  and  $\phi w_i$  determines the marginal cost of production.<sup>7</sup> Second, under monopolistic competition, zero economic profits in equilibrium ensure:

$$y_i = \frac{\alpha}{\phi} (\sigma-1) = y, \quad (7)$$

so that the output of each firm in each country is the same ( $y_i = y$ ). An assumption of full employment of labor in each region ensures that the size of the exogenous factor endowment,  $L_i$ , determines the number of firms/varieties (the extensive margin), using Eq. (7):

$$n_i = \frac{L_i}{\alpha + \phi_i y} = \frac{L_i}{\alpha \sigma}. \quad (8)$$

#### 2.1.3. The gravity equation

We can now formulate an (estimable) gravity equation. First, the trade flow from  $i$  to  $j$  is a function of GDPs, labor endowments, and trade costs. With labor being the only factor of production,  $Y_i = w_i L_i$  or  $w_i = Y_i/L_i$ . Using Eqs. (6) and (8), we can substitute  $\sigma \phi w_i / (\sigma-1)$  for  $p_i$  and  $L_i / (\alpha \sigma)$  for  $n_i$  in Eq. (4) and substitute  $Y_i/L_i$  for  $w_i$  in the resulting equation. Allowing bilateral trade flows to be measured with multiplicative error  $\epsilon_{ij}$ , this yields:

$$X_{ij} = Y_i Y_j \frac{(Y_i/L_i)^{-\sigma} t_{ij}^{1-\sigma}}{\sum_{k=1}^N Y_k (Y_k/L_k)^{-\sigma} t_{kj}^{1-\sigma}} \epsilon_{ij} \Rightarrow \frac{X_{ij} Y_W}{Y_i Y_j} = \frac{Y_W (Y_i/L_i)^{-\sigma} t_{ij}^{1-\sigma}}{\sum_{k=1}^N Y_k (Y_k/L_k)^{-\sigma} t_{kj}^{1-\sigma}} \epsilon_{ij}. \quad (9)$$

Second, the deterministic part of stochastic Eq. (9) is identical to the gravity equation in Feenstra (2004, p. 154) with GDPs and prices, after noting that  $Y_i/L_i$  is proportional to the producer price  $p_i$ .<sup>8</sup> However, a missing element in Eq. (9) is the market-clearance (or multi-lateral trade-balance) condition, as in AvW or EK. Market-clearance is ensured by assuming  $N$  equations:

$$Y_i = \sum_{j=1}^N X_{ij} \quad i = 1, \dots, N. \quad (10)$$

Hence, our structural gravity model is based on Eq. (9) subject to Eq. (10). The latter allows solving for  $N$  endogenous GDPs,  $Y_i$ ; because of this, we can later assess how well the solutions for  $Y_i$  in the model ( $\hat{Y}_i$ ) “match” the corresponding observed data on GDP ( $Y_i$ ).<sup>9</sup>

Finally, we note three results. First, the system of Eq. (9) subject to Eq. (10) allows ABTC. Second, all endogenous variables ( $X_{ij}$ ,  $Y_i$ , and  $Y_i/L_i$ ) have observable values for initial conditions. Third, since  $n_i$  is proportional to observable  $L_i$  and  $w_i$  is pinned down by  $Y_i/L_i$ , then  $p_i$  (proportional to  $w_i$ ) is identified, which was not possible in the AvW model.

### 2.2. Estimating the elasticity of substitution in consumption and general equilibrium comparative statics

While AvW focused on structural estimation of Eq. (2) using an NLS program, the literature since then has adopted as a norm (in the absence of zeros in trade flows) the estimation of a log-linear form of their equation for  $X_{ij}$  in Eq. (2) using region-specific fixed effects for the nonlinear multilateral price terms to avoid omitted variables bias and ensure consistent gravity equation coefficient estimates. In the context of our model above, replacing  $Y_i (Y_i/L_i)^{-\sigma}$  and  $\frac{Y_j}{\sum_{k=1}^N Y_k (Y_k/L_k)^{-\sigma} t_{kj}^{1-\sigma}}$  in Eq. (9) with exporter and importer fixed effects, respectively, yields the log-linear gravity equation:

$$\ln X_{ij} = \lambda + \eta_i + \zeta_j + (1-\sigma) \ln t_{ij} + u_{ij}, \quad (11)$$

where  $u_{ij} = \ln \epsilon_{ij}$  and one would substitute observable bilateral trade-cost variables to proxy for the unobservable bilateral ad valorem trade-cost variable  $t_{ij}$ .<sup>10</sup>

Few have gone further to compute general equilibrium (GE) comparative statics. The two notable studies that estimated GE comparative

<sup>8</sup> Moreover, this equation is isomorphic to Eq. (3) above from EK.

<sup>9</sup> We are grateful to a referee for this suggestion. In our empirical work later using bilateral trade flows among 10 Canadian provinces and 30 U.S. states, we note now that our 40 observed (or actual) GDPs ( $Y_i$ ) will not match our 40 endogenous predicted GDPs ( $\hat{Y}_i$ ) owing to multilateral trade imbalances. In the spirit of Dekle et al. (2007), we will also generate later endogenous predicted GDPs and trade flows (including predicted trade flows with ROW) to calculate a correction for these imbalances to demonstrate whether or not our general equilibrium comparative static estimates for trade flows and economic welfare are sensitive to such imbalances.

<sup>10</sup> For instance, as noted earlier, AvW assumed  $(1-\sigma) \ln t_{ij} = (1-\sigma) \rho \ln d_{ij} + [(1-\sigma) \ln b_{US, CA}] \text{Border}_{ij}$ .

<sup>7</sup> The wage rate in country 1 serves as the numéraire.

statics are AvW and EK. AvW did not provide a method to solve for  $\sigma$  in their framework; for comparative statics they simply assumed a range for  $\sigma$  from 2 to 20. EK instead estimated their key structural parameter,  $\theta$ , a Ricardian index of product heterogeneity on the supply side, using alternatively price and wage rate data discussed earlier. EK found estimates of  $\theta$  ranging from 2 to 13.

2.2.1. Estimating  $\sigma$

There are three alternative ways to estimate  $\sigma$  here, and we denote them as Methods 1, 2, and 3:

2.2.1.1. *Method 1.* If ad valorem bilateral tariff rates and cif-fob ratios were correctly measured,  $\sigma$  could be estimated directly as a parameter on the true observable ad valorem bilateral trade costs ( $t_{ij}$ ), even with exporter and importer fixed effects.<sup>11</sup> However, most gravity models do not use such measures. Moreover, it is well known that ad valorem bilateral cif-fob ratios and tariff rates are prone to considerable measurement error; see Anderson and van Wincoop (2004) and Hummels and Lugovsky (2006) on the former and Fisman and Wei (2004) and Javorcik and Narciso (2008) on the latter.

2.2.1.2. *Method 2.* In a structural version of our model's Eqs. (9) and (10),  $\sigma$  may be estimated as a parameter on potentially observable  $Y_i/L_i$  without observable values of  $t_{ij}$ . However, given Eq. (9) proper estimation would require using observed values of  $Y_i/L_i$  to estimate  $\sigma$  in the first step of an iterative process, then updating endogenous  $Y_i$  from solving Eq. (10), estimating  $\sigma$  again, updating  $Y_i$  again, etc., until convergence is achieved.<sup>12</sup> Yet, such a procedure may yield biased estimates of  $\sigma$  owing to correlation of the error terms with observed  $Y_i/L_i$ . To ensure consistent estimation of parameters in the first stage, one would need exporter and importer fixed effects. However, in that case,  $Y_i/L_i$  would be subsumed in the exporter's fixed effect, precluding estimation of  $\sigma$ .

2.2.1.3. *Method 3.* We suggest an approach that allows unbiased and precise estimates of  $\sigma$  (and then of general equilibrium comparative statics) allowing ABTC. Estimation of Eq. (11) using observable determinants of bilateral trade costs generates estimates  $\widehat{t_{ij}^{1-\sigma}}$ .<sup>13</sup> We can then utilize the structure of the model ex post to generate an estimate of  $\sigma$ . Substituting  $\widehat{t_{ij}^{1-\sigma}}$  in Eq. (9) to generate  $\widehat{X_{ij}}$ ,  $\widehat{t_{mj}^{1-\sigma}}$  in its analog to generate  $\widehat{X_{mj}}$ , and taking the resulting equations' ratio implies:

$$\frac{\widehat{X_{ij}}}{\widehat{X_{mj}}} = \frac{Y_i}{Y_m} \left( \frac{Y_i/L_i}{Y_m/L_m} \right)^{-\sigma} \frac{\widehat{t_{ij}^{1-\sigma}}}{\widehat{t_{mj}^{1-\sigma}}} \tag{12}$$

$$\Rightarrow \hat{\sigma} = - \left[ \ln \left( \frac{\widehat{X_{ij}}}{\widehat{X_{mj}}} \frac{Y_m}{Y_i} \frac{\widehat{t_{mj}^{1-\sigma}}}{\widehat{t_{ij}^{1-\sigma}}} \right) / \ln \left( \frac{Y_i/L_i}{Y_m/L_m} \right) \right],$$

where  $Y_i$ ,  $Y_m$ ,  $L_i$  and  $L_m$  are observable also.<sup>14</sup> We can then calculate  $N^2(N-1)$  such values of  $\sigma$  by using all combinations  $i, j$ , and  $m$  ( $m \neq i$ ). In our empirical investigation later using the Canadian–U.S. border-puzzle data set with  $N=40$  regions, this results in  $40^2(40-1)=62,400$  values. As a measure of central tendency, we use the median value of  $\sigma$  as our (summary) estimate, since the distribution of  $\sigma$  estimates is

skewed to the right in that context (and the mean is slightly higher than the median). Standard errors for  $\sigma$  are obtained via bootstrapping.<sup>15</sup>

The advantage of Method 3 over Methods 1 and 2 is that  $\widehat{t_{ij}^{1-\sigma}}$  is estimated without bias from its possible correlation with region-specific omitted variables as the estimates are obtained from Eq. (11) with  $i$ -specific and  $j$ -specific fixed effects. In the remainder of the paper, we use Method 3 for computing  $\hat{\sigma}$ .<sup>16</sup>

2.2.2. Comparative statics methodology

In the next section, we will estimate gravity-equation coefficients, elasticities of substitution, and two comparative static effects, the GDP-scaled nominal trade flow  $\left( \frac{X_{ij}Y_W}{Y_iY_j} \right)$  and the economic welfare ( $EV_i$ ) effects of a change in trade costs. For our “Suggested Model” (MC-IR), the comparative static effect on GDP-scaled nominal trade flows in percent where  $c$  denotes counter-factual values is:

$$\Delta \frac{X_{ij}Y_W}{Y_iY_j} = 100 \left[ \frac{[Y_W^c(Y_i/L_i)^{-\hat{\sigma}}(\widehat{t_{ij}^{1-\sigma}})^c] / [\sum_{k=1}^N Y_k^c(Y_k/L_k)^{-\hat{\sigma}}(\widehat{t_{kj}^{1-\sigma}})^c]}{[Y_W(Y_i/L_i)^{-\hat{\sigma}}(\widehat{t_{ij}^{1-\sigma}})] / [\sum_{k=1}^N Y_k(Y_k/L_k)^{-\hat{\sigma}}(\widehat{t_{kj}^{1-\sigma}})]} - 1 \right]. \tag{13}$$

where we let  $\Delta \frac{X_{ij}Y_W}{Y_iY_j}$  denote a *percentage* change (deleting henceforth “hat” notation for  $\widehat{X_{ij}}$  and  $Y_i$ ). The corresponding equivalent variation for country  $i$  ( $EV_i$ , in percentage change) is defined as:

$$EV_i = 100 \left[ \frac{Y_i^c / [\sum_{k=1}^N Y_k^c(Y_k/L_k)^{-\hat{\sigma}}(\widehat{t_{ki}^{1-\sigma}})^c]^{\frac{1}{1-\hat{\sigma}}}}{Y_i / [\sum_{k=1}^N Y_k(Y_k/L_k)^{-\hat{\sigma}}(\widehat{t_{ki}^{1-\sigma}})]^{\frac{1}{1-\hat{\sigma}}}} - 1 \right]. \tag{14}$$

3. Empirical evidence

We now apply our technique to actual data on trade flows, populations, GDPs, bilateral distances, and dummy variables for national borders. We consider a well-known empirical context: the U.S.–Canadian “border puzzle” case as examined in AvW. We will address it in the presence of either symmetric or asymmetric bilateral trade costs. In Section 3.1, we re-estimate the same specifications addressed in that literature, initially assuming SBTC (as assumed there). We show that if coefficient estimates are identical from the first stage (using fixed-effects parameter estimation) then our approach and that of AvW lead to identical comparative static effects if the elasticity of substitution is assumed to be the same. However, if one uses the estimated elasticity generated using Method 3 in our approach, significantly different comparative statics will result. In Section 3.2, we allow bilaterally asymmetric (direct) border effects for Canada and the United States, resulting in some different findings relative to those under SBTC.

3.1. Symmetric Canadian–U.S. national border barriers

In this section, we present the results of re-evaluating the Canadian–U.S. “border puzzle” empirical analysis retaining the assumption of SBTC but using alternative estimation techniques: traditional ordinary least squares (OLS), the Baier and Bergstrand (2009, 2010) linear approximation methods (BV-OLS-1 and BV-OLS-2), AvW using non-linear least squares (NLS), AvW using fixed effects in estimation, and our “Suggested (MC-IR) Model” (also using fixed effects in estimation and following Method 3 for estimating  $\sigma$ ). All data on Canadian–U.S.

<sup>11</sup> The term “cif-fob” ratios refer to bilateral trade flows measured “cost-insurance-freight” and “free-on-board.”

<sup>12</sup> This procedure can be done with or without the correction for multilateral trade imbalances.

<sup>13</sup> For instance, in the AvW context,  $\widehat{t_{ij}^{1-\sigma}}$  would be determined by the exponentiated value of  $[(1-\sigma)p] \ln d_{ij} + [(1-\sigma)\ln b_{i,j,c}] \text{Border}_{ij}$ .

<sup>14</sup> Alternatively, we can use the predicted  $Y$ s from the model. We show in our empirical results that the correlation coefficient between observed and predicted  $Y$ s is 0.992.

<sup>15</sup> Standard errors for all other parameters are the analytical standard errors of the corresponding models. However, the bootstrapped standard errors of those parameters are very similar to the analytical ones.

<sup>16</sup> The Monte Carlo analysis later will demonstrate in the absence of measurement and specification errors that all parameter estimates and comparative statics using Method 3 are unbiased and precise.

trade flows, GDPs, bilateral distances, and border dummies are from James Anderson's website. The only other variable needed was populations of Canadian provinces and U.S. states; these data came from Statistics Canada and the U.S. Census Bureau, respectively. We assume that  $t_{ij}^{1-\sigma} = d_{ij}^{(1-\sigma)\rho} e^{[(1-\sigma)lnb_{US,CA}]Border_{ij}}$  where  $d_{ij}$  is symmetric bilateral distance between the economic centers of regions  $i$  and  $j$ ,  $Border_{ij}$  is a dummy variable with a value of 1 (0) if the two regions are separated by a national border, and  $\sigma, \rho$  and  $lnb_{US,CA}$  are unknown parameters.

Hence, our OLS specification refers to:

$$\ln \frac{X_{ij} Y^W}{Y_i Y_j} = \lambda + [(1-\sigma)\rho] \ln d_{ij} + [(1-\sigma)lnb_{US,CA}] Border_{ij} + u_{ij}, \quad (15)$$

with all variables and parameters defined earlier. *BV-OLS-1* and *BV-OLS-2* refer to the linear approximation method introduced for estimating gravity equations with multilateral price terms in Baier and Bergstrand (2009). In this method, linear approximations of the (exogenous components of the) nonlinear multilateral price terms in AvW are introduced as exogenous RHS variables in estimation, and general equilibrium comparative statics of trade flows can be approximated allowing for multilateral influences. Specifications using *BV-OLS-1* employ simple averages of the exogenous bilateral resistance terms, whereas specifications using *BV-OLS-2* employ GDP-weighted averages of the bilateral resistance terms. As explained in Baier and Bergstrand (2010), parameter estimates are expected to be less biased using *BV-OLS-1* than *BV-OLS-2* because the simple-weighted averages in *BV-OLS-1* work similar to region fixed effects. However, comparative static estimates using *BV-OLS-2* are likely to approximate non-linear estimates better due to the GDP weights, as discussed in Baier and Bergstrand (2009, 2010). SBTC-AvW NLS refers to the nonlinear estimation of Eq. (2), after substituting in bilateral distance and the border dummy, as in AvW (2003). SBTC-AvW fixed effects refers to a specification similar to Eq. (15) above, but including fixed effects terms  $\eta_i$  and  $\xi_j$  as in Eq. (11) earlier; multilateral prices  $P_i$  are calculated as in AvW using a nonlinear solver (following Eq. (2)). Finally, for our suggested (MC-IR) model, we also estimate parameters first using the same fixed-effects specification as above;

predicted trade flows, GDPs, prices, and the elasticity of substitution are calculated for our MC-IR approach as described earlier.

Table 1 presents four panels with the alternative estimation techniques described across columns (2)–(7). In the four panels, we provide coefficient (parameter) estimates of the gravity equation, correlation coefficients of observed and predicted trade flows and GDPs (for our approach), GDP-scaled nominal trade-flow comparative statics, and – for the AvW and MC-IR approaches – (real) economic welfare comparative statics. In the top panel, we have estimates of the coefficients for bilateral distance  $((1-\sigma)\rho)$ , the border dummy  $((1-\sigma)lnb_{US,CA})$ , and – only for our MC-IR model in column (7) – of  $\sigma$ . We start with the results in column (2). As expected, the coefficient estimates for OLS in column (2) are biased, owing to the absence of multilateral price terms (or fixed effects). Coefficient estimates using *BV-OLS-1* in column (3) are essentially the same as those using fixed effects estimations for AvW in column (6) and for our model in column (7) as expected. Also as expected, coefficient estimates using *BV-OLS-2* are biased. And those for AvW using NLS are biased – as in AvW (2003) – whenever trade costs are correlated with the region-specific error components. In column (7), the estimate of  $\sigma$  is 6.982. This value is in the middle of the range of typical estimates of  $\sigma$  of 5–10, as suggested in Anderson and van Wincoop (2004). Note the  $R^2$  values are highest in columns (6) and (7) using fixed effects and are lowest using OLS and AvW with NLS due to correlated errors.

The second panel reports correlation coefficients between observed and model-predicted (GDP-scaled) nominal trade flows and between observed and predicted GDPs for our approach. Two results are worth noting. First, the highest correlation coefficient estimate between observed and predicted trade flows is for our suggested MC-IR model at 0.808. Interestingly, the next highest correlations are for the two *BV-OLS* approaches. Second, only our MC-IR method allows GDPs (and per capita GDPs) to be endogenous in the benchmark equilibrium so that we can report a correlation coefficient between observed and predicted values for our model. As noted in column (7), the correlation coefficient for observed and predicted GDPs from our model is extremely high at 0.992, providing strong

**Table 1**  
Estimation results for the AvW data-set assuming symmetric border barrier effects.

Estimates	SBTC-AvW					
	OLS	<i>BV-OLS-1</i>	<i>BV-OLS-2</i>	NLS	Fixed effects	Suggested model
(1)	(2)	(3)	(4)	(5)	(6)	(7)
<i>Parameters</i>						
$(1-\sigma)\rho$	–1.057 (0.039)	–1.261 (0.043)	–1.191 (0.043)	–0.788 (0.032)	–1.252 (0.037)	–1.252 (0.037)
$(1-\sigma)lnb_{US,CA}$	–0.714 (0.058)	–1.526 (0.069)	–1.317 (0.068)	–1.646 (0.077)	–1.551 (0.059)	–1.551 (0.059)
$\sigma$	–	–	–	–	–	6.982 (0.048)
$R^2$	0.425	0.513	0.480	0.435	0.664	0.664
$\sigma^2$	1.148	0.972	1.038	1.062	0.841	0.841
<i>Correlation (corr.) of baseline predictions with data</i>						
$\frac{X_{ij} Y^W}{Y_i Y_j}$ (corr.)	0.324	0.716	0.788	0.374	0.684	0.808
$Y_i$ (corr.)	–	–	–	–	–	0.992
<i>Comparative static trade effects of border barrier abolition (average change in percent; <math>\Delta \frac{X_{ij} Y^W}{Y_i Y_j}</math>)</i>						
Intra-US trade	0.000	–17.363	–3.827	8.173	13.027	–5.140
Intra-CA trade	0.000	–82.030	–85.484	22.476	55.186	–52.248
Inter-trade	104.136	77.207	39.475	112.823	155.505	217.972
<i>Comparative static welfare effects of border barrier abolition (average equivalent variation in percent; <math>EV_i</math>)</i>						
US	–	–	–	1.428	1.179	0.973
CA	–	–	–	39.654	23.105	12.577

**Table 2**  
Estimation results for the AvW data-set allowing for asymmetric border barrier effects.

Estimates	ABTC-AvW						
	OLS	BV-OLS-1	BV-OLS-2	NLS	Fixed effects	Suggested model	Suggested model, MTI
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
<i>Parameters</i>							
$(1 - \sigma)\rho$	-1.058 (0.039)	-1.252 (0.037)	-1.191 (0.043)	-0.788 (0.008)	-1.252 (0.037)	-1.252 (0.037)	-1.252 (0.037)
$(1 - \sigma)lnb_{US}$	-0.540 (0.072)	-0.470 (0.046)	-0.151 (0.300)	-0.264 (0.173)	-0.470 (0.046)	-0.470 (0.046)	-0.470 (0.046)
$(1 - \sigma)lnb_{CA}$	-0.891 (0.073)	-0.825 (0.047)	-2.485 (0.301)	-3.028 (0.104)	-0.825 (0.047)	-0.825 (0.047)	-0.825 (0.047)
$\sigma$	-	-	-	-	-	7.101 (0.087)	7.101 (0.087)
$R^2$	0.431	0.664	0.485	0.435	0.664	0.664	0.664
$\sigma^2$	1.137	0.841	1.028	1.062	0.841	0.841	0.841
<i>Correlation (corr.) of baseline predictions with data</i>							
$\frac{X_i Y_w}{Y_i Y_w}$ (corr.)	0.321	0.574	0.786	0.380	0.684	0.810	0.642
$Y_i$ (corr.)	-	-	-	-	-	0.992	1.000
<i>Comparative static trade effects of border barrier abolition (average change in percent; <math>\Delta \frac{X_i Y_w}{Y_i Y_w}</math>)</i>							
Intra-US trade	0.000	-7.774	-3.831	85.796	-2.099	-2.548	-1.745
Intra-CA trade	0.000	-51.729	-85.510	33.540	-6.687	-36.486	-26.719
Inter-trade	107.227	27.479	41.183	160.630	28.218	60.624	119.291
<i>Comparative static welfare effects of border barrier abolition (average equivalent variation in percent; EV<sub>i</sub>)</i>							
US	-	-	-	-1.183	-0.394	0.519	0.409
CA	-	-	-	28.667	10.917	6.764	2.417

Notes: the model in the last column controls for multilateral trade imbalances.

evidence of the relevance of our model, even without adjustment yet for multilateral trade imbalances.<sup>17</sup>

The third panel reports the estimates of the (GDP-scaled) nominal trade-flow comparative statics from all the models. The most interesting result from this panel concerns the “Intra-US trade” and “Intra-Canadian trade” comparative statics. Our prior would be that the elimination of the border would decrease intra-national trade and increase international trade. In Table 1, only our MC-IR approach and the two BV-OLS approaches predict decreases in intra-national trade and increases in international trade. Assuming SBTC, both AvW using NLS and using fixed effects predict increases in international trade but increases in intra-national trade, suggesting that the income effects of trade liberalization in these approaches dominate the price effects (captured by  $P_i$  and  $P_j$ ).

The fourth panel reports the estimates of the (real) economic welfare comparative static effects of removing the border. The difference in welfare effects between the two AvW techniques is attributable to different parameter estimates, not to different  $\sigma$ s (assumed equal to 5 in both). The AvW NLS parameter estimates are likely biased by correlated errors, whereas the AvW fixed effects parameter estimates are not. However, the difference between the AvW fixed-effects welfare effects and the MC-IR model's welfare effects is due primarily to differing  $\sigma$ s. Our model uses a higher estimated elasticity of about 7, whereas the AvW fixed-effects model assumed  $\sigma = 5$ . Consequently, as expected, the welfare effects estimated by our model are lower.<sup>18</sup>

### 3.2. Asymmetric Canadian–U.S. national border effects

In this section, all specifications allow the direct (or partial) effect of the national border to be bilaterally asymmetric. To accomplish this, we introduce separate dummy variables and coefficients for the border's effect on U.S. imports from Canada and on Canadian imports from the United States. Other than this change, Table 2's format is the same as Table 1's, except that we add a column (8) of numbers associated with estimating the robustness of our results to allowing for multilateral trade imbalances in our sample, which we discuss later.

The top panel in Table 2 reports the estimates of the coefficients for bilateral distance  $((1 - \sigma)\rho)$ , the impact of the border on U.S. imports from Canada  $((1 - \sigma)lnb_{US})$ , and the impact of the border on Canadian imports from the United States  $((1 - \sigma)lnb_{CA})$ . The first result is that the border's direct effect is considerably asymmetric; this notable difference holds across all specifications. The direct trade-reducing effect of the Canadian–U.S. border is much larger for Canadian imports from the United States relative to U.S. imports from Canada. However, the distance coefficient estimates are identical to those in Table 1.

The second panel reports estimates of correlation coefficients between observed and predicted (GDP-scaled) nominal trade flows and between observed and predicted GDPs for our model. First, we find that the highest correlation coefficient estimate between observed and model-predicted trade flows is for the suggested MC-IR model in column (7) at 0.810, consistent with the finding in Table 1. As before, the next highest correlation coefficient is for BV-OLS-2. Second, as in Table 1, the correlation coefficient estimate is very high for GDPs using our model (0.992), shown in column (7). By design, GDPs as predicted by the model in column (8) are perfectly correlated with observed GDPs (in the benchmark equilibrium) when accounting for multilateral trade imbalances; we discuss this in more detail shortly.

The third panel reports the estimates of the (GDP-scaled) nominal trade-flow comparative statics from all the models. The most interesting result from this panel again concerns the “Intra-US trade”

<sup>17</sup> The low values for the trade-flow correlation coefficients for the AvW approach are attributable to non-linearities embedded in the calculations of  $P_i$  and  $P_j$ .

<sup>18</sup> Note in Table 1 that the higher U.S. and Canadian welfare effects for model 6 relative to model 7 are due to a lower  $\sigma$  in model 6. However, the higher U.S. and Canadian welfare effects for model 5 relative to model 6 are due to a larger direct (border) effect in model 5.

and “Intra-Canadian trade” comparative statics. Our prior is that the elimination of the border would decrease intra-national trade and increase international trade. In Table 2, our MC-IR approach, the two BV-OLS approaches, and the AvW approach using fixed effects predict increases in international trade and decreases in intra-national trade. Only the AvW approach using NLS predicts increases in intra-national trade. However, the quantitative differences between the estimates of comparative static effects of the preferred AvW model (in this case, AvW using fixed effects) and the MC-IR model remain economically quite significant.

The fourth panel reports the estimate of the (real) economic welfare comparative static effects of removing the border. The difference in welfare effects between the two AvW techniques is attributable to different parameter estimates, not to different  $\sigma$ s (assumed equal to 5 in both). The AvW NLS parameter estimates are as before likely compromised by correlated errors, whereas the AvW fixed effects parameter estimates are not. However, the difference between the AvW fixed-effects welfare effects and our model's welfare effects is due solely to differing  $\sigma$ s. Our model under ABTC estimates an elasticity of approximately 7.1, whereas the AvW fixed-effects model assumed  $\sigma=5$ . We discuss this in more detail below.

Finally, we note one more specification in Table 2, labeled (8). In reality, observed GDPs of any province or state  $i$  would not likely equal observed internal absorption plus exports, due to multilateral trade imbalances (MTIs). Hence, market-clearing condition (10) is likely violated in reality, which potentially influences all comparative statics. Ideally, it would be simple to account for this influence using observed MTIs for each region  $i$  ( $MTI_i$ ). However, as AvW note, although data exist for Canadian provinces' trade flows with ROW implying observable MTIs for the provinces, no such data exists for U.S. states trade with the ROW, precluding observed MTIs for U.S. states.<sup>19</sup>

Hence, we adjust for MTIs in the following way to generate a set of comparative statics where predicted GDPs equal exactly observed GDPs (satisfying market-clearing). Since we have data on GDPs and populations and estimated values of  $\hat{\sigma}$  and  $t_{ij}^{1-\sigma}$  for all region-pairs including province and state trade flows with ROW, we can predict  $X_{ij}$  for all region-pairs including every region's trade with ROW using Eq. (9). Using these  $\hat{X}_{ij}$ , we can compute estimates of  $MTI_i$  using  $\hat{MTI}_i = \sum_j \hat{X}_{ij} - \sum_j \hat{X}_{ji}$ . We then add  $\hat{MTI}_i$  to each region  $i$ 's market-clearing condition (10) to eliminate any violations (in the benchmark equilibrium).<sup>20</sup> We then re-estimate the (GDP-scaled) nominal trade flows for the benchmark equilibrium and all trade-flow and economic welfare comparative statics. We note three results in column (8). First, the predicted scaled trade flows in the second panel of Table 2 are not as highly correlated with observed scaled trade flows as in our benchmark model, column (7). However, by design, predicted GDPs are perfectly correlated with observed GDPs. Second, there are some minor changes quantitatively in the comparative static trade-flow effects of eliminating the Canadian–U.S. border with the adjustment relative to those in column (7), but the results are qualitatively the same. Third, comparative static effects for welfare of eliminating the border are also qualitatively the same in columns (7) and (8), but lower in column (8) than in column (7).

We summarize now the key findings from our empirical analysis under ABTC. First, coefficient estimates for specifications (6), (7), and (8) in Table 2 are unbiased owing to the use of the exporter and importer fixed effects. By allowing ABTC, the (partial) effects of a national border are different for Canadian province imports from the United States relative to U.S. state imports from Canada. Specification (7) is preferred because our MC-IR model allows estimation of

the key parameter, the elasticity of substitution ( $\sigma$ ).<sup>21</sup> Another notable finding is that the estimate of  $\sigma$  is higher than assumed in the AvW models and only slightly higher than in column (7) of Table 1, where we assumed SBTC. We will use a Monte Carlo analysis in the next section to show that our methodology generates unbiased and precise estimates of  $\sigma$ .

Second, the MC-IR model in column (7) (or (8)) allows real GDPs to be estimated in the benchmark and counterfactual equilibria, whereas the other approaches use observed real GDPs in the benchmark. Nevertheless, our model's predicted real GDPs are highly correlated with observed real GDPs (0.992). Moreover, the results hold up qualitatively when adjusted for multilateral trade imbalances.

Third, in the presence of ABTC, all the specifications in Table 2, with the exception of AvW using NLS, yield economically plausible trade-flow changes in responses to elimination of the Canadian–U.S. border. Intra-national trade flows decline in both countries (except using AvW-NLS), more in Canada than in the United States as expected. International trade flows increase in all specifications and by plausible magnitudes.

Fourth, even more interesting is the difference between the MC-IR model and the two AvW models in terms of the economic welfare effects. Specifications (6) and (7) share common (first-stage) gravity-equation coefficient estimates, since both are estimated using exporter and importer fixed effects. However, our approach, based on the MC-IR model, allows estimation of  $\sigma$ . There result two substantive differences between the two specifications' welfare effects in Table 2. For Canada, eliminating the U.S. border improves welfare in Canada by much less using the MC-IR model with an estimated  $\sigma$  of 7.1 than using the AvW model with an assumed  $\sigma=5$ . This result is because, in the AvW model with a lower elasticity of substitution, there is a greater “love-of-variety.” Consequently, there is a larger welfare gain for Canada from lowering trade costs due to importing a larger volume of highly differentiated goods from the United States.

The other substantive difference concerns the relative U.S. and Canadian economic welfare effects. The differences can be explained by examining the equivalent variation (EV) comparative-static formula in Eq. (14). The change in welfare from abolishing the border barrier is the ratio of the change in nominal income to the change in the price index. A fall in iceberg trade costs creates excess labor supply, tending to lower nominal wages (relative to the numéraire). However, with lower trade costs real income is higher, tending to raise demand, prices ( $p$ ), and wages ( $w$ ). Since Canada is a much smaller economy than the United States and consequently relies much more on traded goods, Canada's wage rate ( $w_{CA}$ ) rises on net and its price index ( $P_{CA}$ ) falls, relative to their U.S. counterparts. Accordingly, both  $w_{CA}$  and  $P_{CA}$  react much more to abolishing the border than their counterparts in the United States ( $w_{US}$  and  $P_{US}$ ); smaller countries benefit relatively more from trade liberalization. This effect is exacerbated with larger partial trade cost effects in Canada than in the United States [ $(1-\hat{\sigma})lnb_{CA} - (1-\hat{\sigma})lnb_{US} < 0$ ]. Moreover, the discrepancy of welfare gains in Canada relative to the U.S. is larger the smaller is  $\sigma$ , due to the stronger love of variety and less elastic demand in that case. For that reason, the difference in welfare gains of Canada relative to the United States is larger in model 6 with an assumed  $\sigma=5$  than in model 7 with an estimated  $\sigma=7.1$ .<sup>22</sup>

Finally, it would be useful to know if the economic welfare changes using the two alternative approaches – fixed-effects AvW and our suggested approach – would be the same if the elasticities were identical. However, that exercise requires comparing the two

<sup>21</sup> Specification (8) is slightly less preferred due to the lower correlation between predicted and observed scaled trade flows, relative to specification (7).

<sup>22</sup> Of course, whether the comparative static effect on any endogenous variable is positive or negative in one country on average (or in a given state or province) depends on the choice of the numéraire. Of interest effectively is the relative difference in changes between U.S. and Canadian variables.

<sup>19</sup> See the discussions in footnote 22 on page 181 and Appendix A of AvW.

<sup>20</sup> Note that this approach has the additional advantage that the world trade balance is zero as  $\sum_i \sum_j \hat{X}_{ij} - \sum_i \sum_j \hat{X}_{ji} = 0$  when summed over all 41 regions including titROW.

welfare estimates in a “laboratory” setting. In the next section, we use a Monte Carlo analysis to show that estimates of  $\sigma$ , comparative static trade-flow changes, and economic welfare comparative static effects are unbiased and precise using our approach. Moreover, in this setting we show that – when both approaches use identical elasticities of substitution – the welfare effects from trade-cost reductions are indeed identical.

#### 4. Monte Carlo analysis

Although fixed effects ensure unbiased estimates of gravity equation parameters, we need a Monte Carlo analysis to confirm unbiased and precise estimates for  $\sigma$  using Method 3 and the associated general equilibrium (GE) comparative statics using the MC-IR model in comparison to some other models. The Monte Carlo analysis is able also to shed light on whether comparative static effects (of bilateral trade flows or welfare) are sensitive to  $\sigma$ , in the absence of measurement and specification error, for alternative models.

Among numerous results, we show that Method 3 provides unbiased and precise estimates of  $\sigma$  and GE comparative statics. While the empirical findings of AvW suggested that nominal trade-flow comparative statics were insensitive to the value of  $\sigma$ , our Monte Carlo analysis demonstrates that this is not generally true. Trade-flow and economic welfare comparative static estimates appear highly sensitive to the value of the elasticity of substitution, motivating the importance of a method providing unbiased and precise estimates of  $\sigma$ . In fact, an incorrect value of  $\sigma$  in AvW can suggest even the wrong direction of economic welfare estimates. Finally, irrespective of SBTC or ABTC, comparative static effects of trade costs are biased when using linearized versions of AvW’s model, but the bias declines as the number of countries in the world increases and the value of  $\sigma$  decreases.

##### 4.1. Monte Carlo design

We use alternative sets of parameter values described below to generate sets of all endogenous variables in the MC-IR model –  $X_{ij}$ ,  $Y_i$ , and  $Y_i/L_i$  – as functions of exogenous endowments  $L_i$ , exogenous bilateral trade costs  $g_{ij}$ , and the model’s parameters in the baseline equilibrium. This also determines the endogenous variables in the AvW system in a baseline equilibrium. We then change exogenous trade costs holding constant the model’s parameters and all endowments to obtain counterfactual values for all endogenous variables in a “laboratory” setting.

We specify nine different configurations (or scenarios) of the world economy for robustness. We consider three alternative values for  $\sigma \in \{3, 5, 10\}$  to allow us to study the role of “curvature” for estimation of parameters, elasticities, and comparative statics.<sup>23</sup> Moreover, we consider three alternative sizes of the number of countries in the world,  $N \in \{10, 20, 40\}$ , to study the performance of alternative techniques for estimation and comparative statics as sample size increases. This gives nine parameter configurations.

For each of these configurations, we use 10 different draws from a set of empirical observations for populations (as a proxy for endowments,  $L_i$ ) for 207 countries from *World Development Indicators* (2003) and from measurable cif-fob ratios (as a proxy for  $t_{ij}$ ), from the International Monetary Fund’s *Direction of Trade Statistics* (2003). We assume true – but unobservable – bilateral trade costs,  $t_{ij}$ , are determined as  $t_{ij}^{1-\sigma} = g_{ij}^{(1-\sigma)\rho}$ , where  $g_{ij}$  is a cif-fob factor and assume arbitrarily  $\rho = 2$  (without loss of generality). Hence, employing  $g_{ij}$  does not allow for direct estimation of  $1 - \sigma$ .<sup>24</sup> Bilateral cif-fob factors  $g_{ij}$

are drawn from empirical realizations of the cif-fob factors in the 25th–75th percentiles of the empirical distribution for the same 207 countries.<sup>25</sup> The 10 draws from these data along with the nine parameter combinations generate 90 alternative baseline equilibria (9 scenarios  $\times$  10 draws per scenario) of  $X_{ij}$  and  $Y_i$  (and, hence,  $Y_i/L_i$ ) in the MC-IR model and of  $X_{ij}$ ,  $Y_i$ ,  $\Pi^{1-\sigma}$ , and  $P_i^{1-\sigma}$  in the AvW model consistent with general equilibrium (before stochastic error terms are introduced). For each of those 90 equilibria we determine a counterfactual equilibrium which corresponds to an arbitrary alternative draw of  $g_{ij}$  (under SBTC or ABTC).

To each of the obtained 90 baseline configurations and equilibria, we add a stochastic error to the bilateral trade equation (as in empirical gravity-equation settings). We do so independently 2000 times assuming random disturbances which are uncorrelated with the determinants of aggregate nominal bilateral trade in the model and, alternatively, 2000 times assuming disturbances which are correlated with trade costs. We can then estimate parameters and comparative static effects in Monte Carlo simulations using 4000 runs per scenario, treating the 90 parameter configurations and equilibria as fixed in repeated samples (totaling 360,000 simulations).

##### 4.2. Error structure

In Eq. (9) we introduced a multiplicative stochastic term  $\epsilon_{ij}$ , and in Eq. (11) defined the log-additive disturbance term  $u_{ij} \equiv \ln \epsilon_{ij}$ . The two alternative error structures in the Monte Carlo simulations are specified as follows. In general, we assume that the error terms  $u_{ij}$  in the log-linearized bilateral trade equations are given by  $u_{ij} = \mu_i + \nu_j + \xi_{ij}$ .<sup>26</sup> In all cases,  $\xi_{ij}$  is drawn from a normal distribution with  $\mathcal{N}(0, s_\xi^2)$  and  $s_\xi = 0.35s_x$ , where  $s_x$  denotes the standard deviation of true log bilateral exports.<sup>27</sup> First, we assume that the error terms ( $u_{ij}$ ) are uncorrelated with the right-hand-side variables. In the tables, this error structure is labeled “uncorrelated.” In this case,  $\mu_i$  and  $\nu_j$  are each distributed as  $\mathcal{N}(0, s_\mu^2)$  with  $s_\mu = 0.15s_x$ . Then, AvW’s iterative non-linear least squares (NLS) estimation approach is consistent, as is fixed effects estimation. We made 2000 draws for the error terms under this structure. Second, we also consider an error structure where we know the  $u_{ij}$  are correlated with the observable bilateral trade cost variable,  $g_{ij}$ .<sup>28</sup> In the tables, this error structure is labeled “correlated.” In the latter case, AvW’s iterative NLS approach is inconsistent, but fixed effects is consistent. We made 2000 draws for the error terms under this structure also.

##### 4.3. Monte Carlo results assuming symmetric bilateral trade costs

Table 3a reports information about true and estimated parameters from a gravity equation, where we report the mean, standard deviation, and mean absolute error (MAE) of each parameter estimate under the assumption that SBTC holds.<sup>29</sup> The MAEs are reported as

<sup>25</sup> In the interquartile range, the average cif-fob ratio is  $\frac{1}{N(N-1)} \sum_i^N \sum_{j \neq i} g_{ij} = 1.196$ , the standard deviation of that measure is 0.067, and the corresponding minimum and maximum are 1.010 and 1.455, respectively.

<sup>26</sup> An additive log-linear error term is conventional to the general-equilibrium-based literature on gravity-model estimation (cf. Anderson and van Wincoop, 2003). In particular, it seems to be a suitable assumption in the absence of zero trade flows, as in our application. We have chosen to add the stochastic error term in only the trade flow equation. GDP could potentially have measurement error as well. However, because we estimate the trade flow equation with region-specific fixed effects, region-specific measurement error or correlation of trade costs with the region-specific error components will not bias our parameter estimates.

<sup>27</sup> We scale the variance of the error term ( $\xi$ ) to a fraction of the variance of exports (0.35) in order for the Monte Carlo simulations of the gravity equations to have  $R^2$  values of approximately 0.65.

<sup>28</sup> To do this, we added ten times the average exporter-specific trade cost variable ( $g_i \equiv (1/N) \sum_{j=1}^N g_{ij}$ ) to the respective  $\mu_i$  and ten times the average importer-specific trade cost variable ( $g_j \equiv (1/N) \sum_{i=1}^N g_{ij}$ ) to the respective  $\nu_j$ .

<sup>29</sup> Recall that one cannot solve for an estimate of  $\sigma$  in the AvW approach or in reduced forms.

<sup>23</sup> We set both  $\alpha$  (the fixed cost in the production function) and  $\phi$  (the labor unit input coefficient) to unity, without loss of generality.

<sup>24</sup> The reason we chose this particular distribution is that it includes both bilaterally symmetric observations as well as bilaterally asymmetric ones, which is not possible if we used instead bilateral distance (because  $d_{ij} = d_{ji}$ ).



**Table 3a**  
Monte Carlo results for gravity-equation parameters in the case of a  $\sigma=5$  and symmetric trade costs.

Estimates	True	OLS		BV-OLS-1		BV-OLS-2		AvW		Suggested model	
		Uncorr.	Corr.	Uncorr.	Corr.	Uncorr.	Corr.	Uncorr.	Corr.	Uncorr.	Corr.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
10-country-world, $\sigma=5$											
$\rho(1-\sigma)$											
Mean	-8	-7.6241	-6.8319	-8.0214	-8.0354	-7.7444	-7.8193	-8.0149	-7.5514	-8.0214	-8.0354
Std.	-	0.5502	0.6785	0.3232	0.3205	0.6343	0.6395	0.4738	0.5197	0.3232	0.3205
MAE	-	6.7748	14.7083	3.2207	3.2069	6.6943	6.5789	4.6685	6.7944	3.2207	3.2069
$\sigma$											
Mean	5	-	-	-	-	-	-	-	-	5.0177	5.0264
Std.	-	-	-	-	-	-	-	-	-	0.2487	0.2467
MAE	-	-	-	-	-	-	-	-	-	3.7163	3.7071
20-country-world, $\sigma=5$											
$\rho(1-\sigma)$											
Mean	-8	-7.7905	-7.206	-8.0107	-8.0087	-7.8510	-7.7691	-8.0107	-7.6167	-8.0107	-8.0087
Std.	-	0.2655	0.3237	0.1558	0.1559	0.2712	0.2235	0.2570	0.2333	0.1558	0.1559
MAE	-	3.5705	9.9623	1.5518	1.5545	3.0960	3.2839	2.5415	4.8903	1.5518	1.5545
$\sigma$											
Mean	5	-	-	-	-	-	-	-	-	5.0092	5.0078
Std.	-	-	-	-	-	-	-	-	-	0.1249	0.1248
MAE	-	-	-	-	-	-	-	-	-	1.8199	1.8232
40-country-world, $\sigma=5$											
$\rho(1-\sigma)$											
Mean	-8	-7.7949	-7.4301	-8.0029	-8.0026	-7.7537	-7.7207	-8.0106	-7.8565	-8.0029	-8.0026
Std.	-	0.1205	0.1547	0.0764	0.0776	0.1944	0.2150	0.1359	0.1305	0.0764	0.0776
MAE	-	2.5981	7.1239	0.7633	0.7733	3.2803	3.5994	1.3514	2.0033	0.7633	0.7733
$\sigma$											
Mean	5	-	-	-	-	-	-	-	-	5.0008	5.0003
Std.	-	-	-	-	-	-	-	-	-	0.0780	0.0789
MAE	-	-	-	-	-	-	-	-	-	1.0884	1.1039

Notes: the mean absolute error (MAE) is expressed as a percent of the true value.

a percent of the true parameter value. Since  $\sigma=5$  for all results in Table 3a, the findings pertain to 2000 draws for “world” sizes of  $N \in \{10, 20, 40\}$  each.

Table 3b reports information about general equilibrium (GE) comparative static estimates. This table provides information about the true and estimated percent changes of (GDP-scaled nominal bilateral) trade flows and of real economic welfare (equivalent variation), in percent. As with parameter estimates, we report the means, standard deviations, and MAEs of the true and estimated (GE) comparative static effects across all country-pairs, Monte Carlo runs, and  $N$ -configurations at  $\sigma=5$ , corresponding to Table 3a. Since the true comparative static effects are already expressed in percent, the MAEs in Table 3b are measured as an average absolute percentage point deviation from the true comparative static percentage change. Both Tables 3a and 3b are divided horizontally into three blocks of estimates, each corresponding to a different scenario of  $N$ . Supplemental tables for values of  $\sigma \in \{3, 10\}$  are reported in the Appendix Tables A1a, A1b and A2a, A2b.

Tables 3a and 3b have 12 columns each. In Table 3a, column (1) lists the two parameters,  $(1-\sigma)\rho$  and  $\sigma$ . In Table 3b, column (1) lists the two endogenous variables relevant for comparative static estimates, the trade flow and the equivalent variation (EV). Column (2) specifies the true values of the parameters assumed in the simulations,  $(1-\sigma)\rho=8$  and  $\sigma=5$  in Table 3a, and the true values of the comparative static effects,  $\Delta(X_{ij}Y_W/Y_iY_j)$  and  $EV_i$ , in Table 3b. Columns (3)–(12) provide estimates of the parameters and comparative statics using various estimation techniques which we will describe.<sup>30</sup> For brevity, we discuss only the bottom third of each table, that is, the 40-country-world.

<sup>30</sup> Since only the structural MC-IR model can estimate  $\sigma$ , we leave the corresponding discussion until columns (11) and (12) of Tables 3a and 3b.

Columns (3) and (4) in Tables 3a and 3b present the estimates using traditional OLS gravity equations *ignoring* entirely the role of endogenous prices; hence, this specification follows Eq. (2) omitting region fixed effects for the  $P_i$ s and  $P_j$ s, as in McCallum (1995).<sup>31</sup> Columns (3) and (4) present the OLS results assuming uncorrelated and correlated errors, respectively. The purpose of these specifications is to confirm the extent of bias of traditional gravity-equation parameter estimates when ignoring multilateral price terms as suggested by AvW, or even regions' fixed effects. The parameter-estimate bias is large under both error structures, and especially large under correlated errors. Not surprisingly, in Table 3b the means of the trade-flow comparative static effects indicate considerable bias and the MAEs are quite large.

Columns (5)–(8) provide estimates using linear approximations for  $\ln P_i^{1-\sigma}$  and  $\ln P_j^{1-\sigma}$  in the log-transformed model of AvW as derived in Baier and Bergstrand (2009). In this method, linear approximations of the (exogenous components of the) nonlinear multilateral resistance terms are introduced as exogenous RHS variables and consequently comparative statics of trade flows can be conducted, respecting multilateral resistances. Specifications (5) and (6) – labeled BV-OLS-1 – employ simple averages of bilateral trade costs for the multilateral price terms, whereas specifications (7) and (8) – labeled BV-OLS-2 – employ GDP-weighted averages of bilateral trade costs for the multilateral terms, the latter used in Baier and Bergstrand (2009). As explained in Baier and Bergstrand (2010), coefficient estimates are expected to be less biased using BV-OLS-1 than BV-OLS-2 in Table 3a because the simple-weighted averages work similar to region fixed effects. Trade-flow comparative static estimates are biased in the BV-OLS specifications in Table 3b because comparative statics are based on linear approximations of the nonlinear price terms in

<sup>31</sup> Consequently, there will be no EV comparative static estimates for the OLS results.

**Table 3b**

Monte Carlo results for predicted trade flow and welfare comparative statics in the case of a  $\sigma=5$  and symmetric trade costs.

Estimates	True	OLS		BV-OLS-1		BV-OLS-2		AvW		Suggested model	
		Uncorr.	Corr.	Uncorr.	Corr.	Uncorr.	Corr.	Uncorr.	Corr.	Uncorr.	Corr.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
<b>10-country-world, <math>\sigma=5</math></b>											
$\Delta \frac{X_{ij} Y_{ij}}{Y_i Y_j}$											
Mean	15.6348	21.2083	16.9485	14.6863	14.7388	20.9057	21.2467	15.7720	13.7475	15.7586	15.8197
Std.	67.7946	73.8767	61.3019	63.0199	63.1647	81.6642	82.1467	68.4578	62.1077	68.2929	68.4372
MAE	-	26.4978	24.1339	19.0619	19.0723	22.7052	22.8996	2.5982	3.7948	1.8482	1.8396
$EV_i$											
Mean	1.7030	-	-	-	-	-	-	1.7038	1.6365	1.7011	1.6972
Std.	9.1959	-	-	-	-	-	-	9.1918	9.0527	9.1915	9.1818
MAE	-	-	-	-	-	-	-	0.1483	0.2054	0.2234	0.2213
<b>20-country-world, <math>\sigma=5</math></b>											
$\Delta \frac{X_{ij} Y_{ij}}{Y_i Y_j}$											
Mean	23.3920	24.3130	20.7552	19.0991	19.0883	26.1341	25.4988	23.4934	21.2079	23.4703	23.4569
Std.	90.5346	84.9751	75.3124	80.1990	80.1646	96.5163	94.8488	90.9789	83.7935	90.8372	90.7944
MAE	-	22.2170	21.7635	17.9046	17.9053	14.7770	14.2352	1.8146	3.3970	1.1651	1.1674
$EV_i$											
Mean	0.8988	-	-	-	-	-	-	0.8997	0.8486	0.8984	0.8986
Std.	7.2242	-	-	-	-	-	-	7.2294	7.0372	7.2194	7.2202
MAE	-	-	-	-	-	-	-	0.0903	0.1692	0.0788	0.0789
<b>40-country-world, <math>\sigma=5</math></b>											
$\Delta \frac{X_{ij} Y_{ij}}{Y_i Y_j}$											
Mean	23.8155	18.9762	17.0660	19.2797	19.2779	25.0291	24.7589	23.8913	22.9440	23.8488	23.8473
Std.	89.7503	78.7336	73.0796	78.7351	78.7310	92.7942	91.6696	90.0318	87.0564	89.8462	89.8410
MAE	-	23.5040	23.5699	20.6474	20.6487	11.3734	11.2878	0.9867	1.4566	0.7160	0.7260
$EV_i$											
Mean	-0.0880	-	-	-	-	-	-	-0.0878	-0.0941	-0.0906	-0.0909
Std.	7.7841	-	-	-	-	-	-	7.7914	7.6844	7.7875	7.7880
MAE	-	-	-	-	-	-	-	0.0587	0.0878	0.0658	0.0668

Notes: the mean absolute error (MAE) is expressed in percentage points of the true value (as scaled trade flows and EVs are in percentage points already).

**Table 4a**

Monte Carlo results for gravity-equation parameters in the case of a  $\sigma=5$  and asymmetric trade costs.

Estimates	True	OLS	BV-OLS-1	BV-OLS-2	SBTC-AvW, FE	ABTC-AvW, FE	Suggested model, FE
		Uncorr.	Uncorr.	Uncorr.	Uncorr.	Uncorr.	Uncorr.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
<b>10-country-world, <math>\sigma=5</math></b>							
$\rho (1-\sigma)$							
Mean	-8	-7.3749	-8.0382	-7.7583	-8.0382	-8.0382	-8.0382
Std.	-	0.6668	0.3156	0.54048	0.3156	0.3156	0.3156
MAE	-	8.8907	3.1563	5.9763	3.1563	3.1563	3.1563
$\sigma$							
Mean	5	-	-	-	-	-	5.0203
Std.	-	-	-	-	-	-	0.1939
MAE	-	-	-	-	-	-	2.8983
<b>20-country-world, <math>\sigma=5</math></b>							
$\rho (1-\sigma)$							
Mean	-8	-7.7019	-8.0098	-7.7381	-8.0098	-8.0098	-8.0098
Std.	-	0.3093	0.1640	0.3348	0.1640	0.1640	0.1640
MAE	-	4.5091	1.6493	4.2320	1.6493	1.6493	1.6493
$\sigma$							
Mean	5	-	-	-	-	-	5.0063
Std.	-	-	-	-	-	-	0.0951
MAE	-	-	-	-	-	-	1.5143
<b>40-country-world, <math>\sigma=5</math></b>							
$\rho (1-\sigma)$							
Mean	-8	-7.7564	-8.0002	-7.7129	-8.0002	-8.0002	-8.0002
Std.	-	0.1234	0.0738	0.3207	0.0738	0.0738	0.0738
MAE	-	3.0832	0.7190	3.8906	0.7190	0.7190	0.7190
$\sigma$							
Mean	5	-	-	-	-	-	5.0003
Std.	-	-	-	-	-	-	0.0472
MAE	-	-	-	-	-	-	0.7339

Notes: the mean absolute error (MAE) is expressed as a percent of the true value.

**Table 4b**  
Monte Carlo results for predicted trade flow and welfare changes in the case of a  $\sigma=5$  and asymmetric trade costs.

Estimates	True	OLS	BV-OLS-1	BV-OLS-2	SBTC-AvW, FE	ABTC-AvW, FE	Suggested model, FE
		Uncorr.	Uncorr.	Uncorr.	Uncorr.	Uncorr.	Uncorr.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
10-country-world, $\sigma=5$							
$\Delta \frac{X_{ij} Y_{ij}}{Y_i Y_j}$							
Mean	20.6721	15.5437	16.2140	25.6395	19.9104	20.9153	20.9538
Std.	87.9681	73.5495	66.1239	99.9547	85.6286	89.0451	89.1178
MAE	–	30.4533	23.7172	17.2115	7.0729	2.0564	2.1426
$EV_i$							
Mean	–0.0872	–	–	–	0.6541	–0.0872	–0.0913
Std.	7.1420	–	–	–	10.3313	7.1578	7.1357
MAE	–	–	–	–	4.2611	0.1145	0.1286
20-country-world, $\sigma=5$							
$\Delta \frac{X_{ij} Y_{ij}}{Y_i Y_j}$							
Mean	23.8479	17.135	18.7285	27.9349	23.2814	23.9096	23.9158
Std.	91.909	77.1394	73.4616	99.3701	90.3398	92.1191	92.1374
MAE	–	30.9097	27.6675	15.4569	4.4997	1.1872	1.2932
$EV_i$							
Mean	–0.0894	–	–	–	0.8445	–0.0890	–0.0900
Std.	7.1531	–	–	–	10.5091	7.1519	7.1522
MAE	–	–	–	–	4.4064	0.0639	0.0522
40-country-world, $\sigma=5$							
$\Delta \frac{X_{ij} Y_{ij}}{Y_i Y_j}$							
Mean	21.8075	18.1371	19.3053	22.1782	21.9812	21.8084	21.8109
Std.	88.5205	77.9766	80.7079	88.7955	88.1429	88.5378	88.5429
MAE	–	23.7442	20.5455	11.0263	2.3084	0.5078	0.5361
$EV_i$							
Mean	–0.0144	–	–	–	–0.6080	–0.0138	–0.0142
Std.	5.2335	–	–	–	7.0521	5.2333	5.2341
MAE	–	–	–	–	3.7975	0.0210	0.0182

Notes: The mean absolute error (MAE) is expressed in percentage points of the true value (as scaled trade flows and EVs are in percentage points already).

AvW. However, the bias is small in the 40-country case using GDP-weighted averages in the linear approximations (BV-OLS-2) compared to simple averages (BV-OLS-1), and the bias of BV-OLS-2 in particular diminishes as the number of countries in the world ( $N$ ) increases.<sup>32</sup>

Columns (9) and (10) in Tables 3a and 3b present estimates using the NLS estimation procedure used in AvW (2003), assuming uncorrelated errors in column (9) and correlated errors in column (10). Since we are assuming SBTC, we estimate the AvW model as in (2) using NLS. Under uncorrelated errors, column (9) reveals that the AvW coefficient estimates and comparative statics are unbiased. The purpose of estimating the same model with correlated errors is to verify that the iterative SBTC-AvW procedure actually used in AvW (2003) yields biased gravity-equation coefficient estimates and comparative statics in the presence of such errors. Thus, if there exists any possibility in an empirical application of correlated errors, one would be better off using region fixed effects in estimation and then subsequently using a nonlinear solver for estimating comparative statics.

The final specifications in columns (11) and (12) are estimates using our (“Suggested”) structural MC-IR model. First, parameters are estimated using fixed effects. Consequently, regardless of uncorrelated or correlated errors, these coefficient estimates are unbiased. Second, as discussed previously, only the MC-IR approach generates an estimate of  $\sigma$ ; columns (11) and (12) report unbiased and precise estimates of  $\sigma$  using Method 3. Third, as shown in Table 3b, we find that both AvW’s (uncorrelated errors) approach and our approach deliver unbiased and precise estimates of comparative statics. Of course, in AvW, we have assumed knowledge of the true  $\sigma=5$  whereas our approach estimates  $\sigma$ . In sum, both AvW’s

<sup>32</sup> The biases of BV-OLS-2 for trade-flow comparative statics also diminish as  $\sigma$  decreases.

and our approach provide unbiased and precise parameter and comparative static estimates, and ours also provides unbiased and precise estimates of  $\sigma$ .

Finally, in their sensitivity analysis for empirical results, AvW (2003) found their trade-flow comparative statics were insensitive to varying  $\sigma$  between 2 and 20. Our Monte Carlo analysis can be used to examine the sensitivity of trade-flow and EV comparative statics to varying  $\sigma$  under laboratory conditions. Appendix Tables A1a, A1b and A2a, A2b provide comparable information to that in Tables 3a, 3b for values of  $\sigma \in \{3, 10\}$ . The notable finding is that the trade-flow and EV comparative static estimates are highly sensitive to the value of the elasticity of substitution. For the 40-country-world for a given trade-cost shock, the trade-flow elasticity is almost 200% for  $\sigma=10$  whereas the trade-flow elasticity is only 5% for  $\sigma=3$ ; the former is 40 times the latter.<sup>33</sup> Both the comparative static trade-flow and the EV estimates vary considerably depending upon the value of  $\sigma$ . The key difference here is that our approach estimates the value of  $\sigma$  and AvW uses an assumed value. So if the true elasticity differs from the assumed one, our approach can capture this whereas the comparative static estimates from AvW will be biased.

#### 4.4. Monte Carlo results assuming asymmetric bilateral trade costs

For asymmetric trade costs (ABTC), we summarize the findings from the Monte Carlo analysis in Tables 4a and 4b. Table 4a reports information about true and estimated parameters from a gravity equation as before, and Table 4b reports the comparative static estimates. For brevity, we assume errors to be uncorrelated with trade costs in all the estimates, as the previous analysis explained the

<sup>33</sup> See Appendix Tables A1b and A2b at the mean of  $\Delta \frac{X_{ij} Y_{ij}}{Y_i Y_j}$ .

impact of correlated errors. Also, in both AvW specifications – one using the approach based upon Eq. (1) labeled ABTC-AvW and one using the approach based upon Eq. (2) labeled SBTC-AvW – we estimate the trade-cost parameter using country fixed effects (FE), since NLS and FE will yield identical estimates with uncorrelated errors. Again, we focus on the bottom third of each table, the 40-country-world.

For brevity, the main results from this Monte Carlo exercise are summarized. First, regarding parameter estimates in Table 4a, the alternative methods perform comparably to their respective counterparts for uncorrelated errors under SBTC in Table 3a. This is not surprising, even for SBTC-AvW, since we have estimated the AvW model using region fixed effects. The MC-IR model permits an unbiased and precise estimate of the elasticity of substitution,  $\sigma$ , for ABTC as well as SBTC. Second, ABTC-AvW provides unbiased and precise estimates of trade-flow comparative statics – under the *assumption* that we know the true value of  $\sigma$  (in Table 4b, the true  $\sigma=5$ ). However, we know from the previous section that the trade-flow comparative static estimates will be much different if the assumed value of  $\sigma$  is incorrect, and such trade-flow comparative statics can increase by as much as 40 times as the value of  $\sigma$  increases from 3 to 10. Third, Table 4b indicates that – when trade costs are bilaterally asymmetric – only our method and ABTC-AvW (when  $\sigma$  is *known*) provide unbiased and precise estimates of EV changes. However, as Appendix Tables 1b and 2b report, such comparative statics are biased using AvW with an assumed value of  $\sigma$  different from the true one.

## 5. Conclusions

Theoretical foundations for estimating gravity equations were enhanced recently in Eaton and Kortum (2002), Anderson and van Wincoop (2003), and Helpman et al. (2008). Though elegant and isomorphic, not all of the approaches provide unbiased estimates of the trade-cost elasticity, a key parameter in computing general equilibrium comparative statics. We use the workhorse Krugman monopolistic competition and increasing returns model of trade to motivate estimating gravity equation coefficients, the elasticity of substitution in consumption, and general equilibrium trade-flow and economic welfare comparative statics. In applying our framework empirically to the well-known McCallum “border-puzzle” as in Anderson and van Wincoop (2003), we generate an unbiased estimate of the elasticity of substitution and economically plausible trade-flow and economic welfare comparative statics that differ significantly from those provided using AvW’s technique (which *assumes* an elasticity of substitution) and are sensitive to using bilaterally symmetric or asymmetric trade costs.

However, the paper has not addressed several issues, which should be examined in future research. Notably, the model should be extended to incorporate heterogeneous productivities and fixed market entry costs as in Melitz (2003) and zeros in trade flows as in Helpman et al. (2008). Moreover, future work should extend the analysis to multiple sectors and multiple factors, subjects also beyond the scope of the present paper.

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## Appendix A. Supplementary data

Supplementary data to this article can be found online at <http://dx.doi.org/10.1016/j.jinteco.2012.05.005>.

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