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PRODUCTIVITY, FACTOR ENDOWMENTS, MILITARY EXPENDITURES, AND NATIONAL PRICE LEVELS

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1. Introduction

Economists have found systematic evidence that the general level of prices across countries at a point in time varies dramatically. Irving B. Kravis, Alan W. Heston, and Robert Summers (1982), for example, report that some countries' national price levels are no more than one-third the U.S. price level. Pioneering work by Kravis and Robert E. Lipsey (1983, 1987, 1988) has demonstrated that a positive correlation between the price level and (real) per capita gross domestic product is robust across numerous cross-sectional specifications.

Recently, several studies using the United Nations International Comparisons Program (ICP) data have attempted to disentangle theoretically and empirically the relative effects of productivity differentials, relative factor endowment differences, and the nonhomotheticity of tastes upon national price levels to better understand the "structural" channels through which per capita income differences between countries influence their relative price structures, especially the variation across countries in the relative prices of nontradables (services) to tradables (commodities).

Empirical work in Christopher Clague (1986) and Kravis and Lipsey (1987) assumed that per capita income differences between countries reflected relative productivity differences or relative factor endowment differences. These studies concentrated upon other presumably exogenous variables that might help per capita income explain cross-country variation in general price levels. These variables

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included, for instance, the trade balance, tourism receipts' share of GDP, minerals' share of GDP, and the share of nontradables in GDP.

Studies by Clague (1985), Jeffrey H. Bergstrand (1991) and Rodney E. Falvey and Norman Gemmell (1991, 1992) have focused upon understanding the channels through which per capita income differences themselves influence national price level differences. All these studies have in common the role of relative productivity differences, relative factor endowment differences, and the share of expenditures on services as important determinants of relative national price levels. Clague (1985) focused theoretically only on how factor endowment differentials, productivity differentials, and the share of spending on services affected national price levels. Bergstrand (1991) showed theoretically and empirically that per capita income differences influenced relative national price levels as much through relative demand differentials as through relative supply (i.e., relative factor endowment and relative productivity) differences. Falvey and Gemmell (1991, 1992) support theoretically and empirically the conclusions of Bergstrand (1991) in the context of exogenous populations and multiple factors, but endogenous per capita incomes.

None of these studies, however, has contemplated the role of exogenous fiscal spending, and its influence on endogenous per capita disposable income and national price level differences. This study primarily investigates theoretically and empirically the role of fiscal policy in particular, defence spending – on relative national price levels, in the spirit of Bergstrand (1991, 1992). A secondary goal is of topical interest: How will the imminent reductions of defence spending in numerous industrialized nations affect relative national price levels?

The organization of the remainder of the paper is as follows. Section 2 discusses how the supply of civilian nontradables relative to civilian tradables might be influenced by factor endowments, sectoral productivity levels, military absorption of factors, and relative prices, to derive an estimable relative supply function. Section 3 discusses how the demand for civilian nontradables relative to civilian tradables might be influenced by per capita income, per capita military expenditures, and relative prices, to derive an estimable relative demand function, and discusses the reduced-form relative price level function. Section 4 provides empirical estimates of coefficients of the reduced-form relative price function and of the structural relative demand and supply functions. Section 5 provides conclusions.

2. Supply

I assume a standard simple general equilibrium framework similar to that in Ronald W. Jones (1965) for the production of two goods, tradables (T) and nontradables (N). Tradables and nontradables are consumed by both civilian (X_C^T, X_C^N) and military (X_M^T, X_M^N) sectors. Military absorption is exogenous similar to government absorption in the Frenkel and Razin (1987) framework. Tradables and nontradables are produced using two factors: capital (K) and consumer-workers (L), the endowment of which is fixed intratemporally. Factors are mobile between industries, but not internationally. Perfectly competitive firms are assumed to minimize costs given the constant-returns-to-scale technology, yielding the optimum input requirements per unit of output \( \beta_{ij} \) (i = K, L; j = N, T). Each \( \beta_{ij} \) is a function of the relative factor price (i.e., the wage rate, \( W \), relative to the rental rate on capital, \( R \)) and the state of productivity of factor i in industry j(\( \tau_{ij} \)). An assumption of full employment of both factors yields:

\[
\beta_{LT}X_C^T + \beta_{LT}X_M^T + \beta_{LN}X_C^N + \beta_{LN}X_M^N = L \tag{1}
\]
\[
\beta_{KT}X_C^T + \beta_{KT}X_M^T + \beta_{KN}X_C^N + \beta_{KN}X_M^N = K \tag{2}
\]

In a competitive equilibrium with tradables and nontradables produced, unit costs must reflect market prices of the goods:

\[
\beta_{LT}W + \beta_{KT}R = P_T \tag{3}
\]
\[
\beta_{LN}W + \beta_{ KN}R = P_N \tag{4}
\]

where all factor prices (\( W, R \)) and goods prices (\( P_N, P_T \)) are expressed in terms of a monetary unit.

The four equations (1) - (4) can be differentiated and mathematically manipulated to derive the percentage differences between two countries in key variables; let \( \dot{x} \) denote dx/x. Consider first the relationships between goods prices and factor prices. The first-order conditions from profit-maximization are given by equations (3) and (4). Differentiating (4) and dividing both sides by \( P_N \) yields:

\[
\theta_{LN}\dot{W} + \theta_{KN}\dot{R} + \theta_{LN}\dot{W} + \theta_{KN}\dot{R} = \dot{P}_N \tag{5}
\]

where \( \theta_{LN} = \beta_{LN}W/P_N \) and \( \theta_{KN} = \beta_{KN}R/P_N \) are the (average)
shares of labor and capital, respectively, in nontradables. Differentiating (3) and dividing both sides by $P_T$ yields:

$$\theta_{LT} \dot{W} + \theta_{KT} \dot{R} + \theta_{LT} \dot{\theta}_{LT} + \theta_{KT} \dot{\theta}_{KT} = \dot{P}_T$$

(6)

where $\theta_{LT} = \frac{\beta_{LT} W}{P_T}$ and $\theta_{KT} = \frac{\beta_{KT} R}{P_T}$. The $\dot{\theta}_{ij}$'s each represent the percentage difference between two countries in the factor $i$ requirement to produce a unit of output $j$. The factor requirement will differ if relative factor prices ($W/R$) differ or if the factor's productivity differs between the two countries; expressed as in Jones (1965), section 9:

$$\beta_{ij} = \beta_{ij}(W/R, t) \quad i = L, K; j = N, T$$

where $t$ represents the state of productivity or technology. In terms of percentage differences, $\beta_{ij}$ can be expressed as:

$$\beta_{ij} = \gamma_{ij} - \hat{\gamma}_{ij}$$

(7)

where $\gamma_{ij}$ represents the percentage difference between two countries of the input requirement per unit of output owing specifically to differences in relative factor prices for a given level of productivity and $\hat{\gamma}_{ij}$ represents the percentage difference between two countries of productivity of factor $i$ in industry $j$. Higher levels of $\gamma_{ij}$ are associated with lower levels of $\beta_{ij}$ for any given level of relative factor prices; formally, $\hat{\gamma}_{ij} = \beta_{ij}^{-1}(d\beta_{ij}/dt)dt > 0$. Substituting (7) into (5) and (6) and some mathematical manipulation yields:

$$\theta_{LN} \dot{W} + \theta_{KN} \dot{R} + (\theta_{LN} \gamma_{LN} + \theta_{KN} \gamma_{KN}) = \dot{P}_N + (\theta_{LN} \gamma_{LN} + \theta_{KN} \gamma_{KN})$$

(8)

$$\theta_{LT} \dot{W} + \theta_{KT} \dot{R} + (\theta_{LT} \gamma_{LT} + \theta_{KT} \gamma_{KT}) = \dot{P}_T + (\theta_{LT} \gamma_{LT} + \theta_{KT} \gamma_{KT})$$

(9)

A key assumption in these types of models is that the competitive firm is assumed to maximize profits for given factor prices, goods prices and technology. Hence, as in Jones (1965), any differences in the $\gamma_{ij}$'s must satisfy:

$$\gamma_{LN} = \theta_{LN} + \theta_{LT} \gamma_{LT} + \theta_{KT} \gamma_{KT} = 0 \quad \text{or} \quad (W/R) = d\gamma_{LN}/d\gamma_{LT}$$

$$\gamma_{KT} = \theta_{KT} + \theta_{LT} \gamma_{LT} + \theta_{KT} \gamma_{KT} = 0 \quad \text{or} \quad (W/R) = d\gamma_{KT}/d\gamma_{LT}$$

(10)

(11)

Defining $\Theta = [\theta_{LN} \theta_{LT} \theta_{KN} \theta_{KT}]$ and recalling $\theta_{ij} = 1$ for $j = N, T$, the determinant of $\Theta$ is:

$$|\Theta| = \theta_{LN} - \theta_{LT} = \theta_{KT} - \theta_{KN}$$

Substituting $|\Theta|$ into (16) and dividing by $|\Theta|$ yields:

$$(\dot{W} - \dot{R}) = (1/|\Theta|) ([\dot{P}_N - \dot{P}_T] + (\dot{I}_N - \dot{I}_T))$$

(17)

Since in a competitive equilibrium (with internal tangencies) the slope of the isoquant in each industry equals (in absolute terms) the ratio of factor prices, the elasticities of substitution can be defined as:

$$\sigma_N = (\gamma_{KN} - \gamma_{LN})/(\dot{W} - \dot{R}) > 0$$

(18)
Each \( \hat{y}_{ij} \) can now be solved for in terms of an \( \sigma_j \), \( \theta_{ij} \) and the percentage difference in relative factor prices. Solving (10) and (18) simultaneously yields:

\[
\hat{y}_{LT} = -\Theta_{KT}\sigma_T(W - \hat{\nu}) \tag{22}
\]

\[
\hat{y}_{KT} = \Theta_{LT}\sigma_T(W - \hat{\nu}) \tag{23}
\]

Solving (11) and (19) simultaneously yields:

\[
\hat{y}_{LN} = \Theta_{KN}\sigma_N(W - \hat{\nu}) \tag{20}
\]

\[
\hat{y}_{KN} = -\Theta_{LN}\sigma_N(W - \hat{\nu}) \tag{21}
\]

Consider now the factor endowment constraints (1) and (2). Differentiating (1) and dividing both sides of the resulting equation by \( L \) yields:

\[
\lambda_{LT}^L\hat{\beta}_{LT} + \lambda_{LT}^C\hat{\beta}_{LT} + \lambda_{LN}^L\hat{\theta}_{LN} + \lambda_{LN}^C\hat{\theta}_{LN} + \lambda_{LN}^N\hat{\lambda}_{LN} + \lambda_{LT}^N\hat{\lambda}_{LT} = \hat{L} - [(\beta_{LT}/L)dX_T^M + (\beta_{LN}/L)dX_N^M] \tag{24}
\]

where \( \lambda_{ij}^k = \beta_{ij}X_i^k/L \) \((j = N, T; k = C, M)\) and, by equation (1),

\[
\lambda_{LT}^C + \lambda_{LT}^N + \lambda_{LN}^C + \lambda_{LN}^N = \lambda_{LT} + \lambda_{LN} = 1. \]

Similarly, differentiate (2) and divide both sides by \( K \):

\[
\lambda_{KT}^L\hat{\beta}_{KT} + \lambda_{KT}^C\hat{\beta}_{KT} + \lambda_{KN}^L\hat{\theta}_{KN} + \lambda_{KN}^C\hat{\theta}_{KN} + \lambda_{KN}^N\hat{\lambda}_{KN} + \lambda_{KT}^N\hat{\lambda}_{KT} = \hat{K} - [(\beta_{KT}/K)dX_T^M + (\beta_{KN}/K)dX_N^M] \tag{25}
\]

where \( \lambda_{ij}^k = \beta_{ij}X_i^k/K \) \((j = N, T; k = C, M)\) and, by equation (2),

\[
\lambda_{KT}^C + \lambda_{KT}^N + \lambda_{KN}^C + \lambda_{KN}^N = \lambda_{KT} + \lambda_{KN} = 1. \]

The two bracketed terms in equations (24) and (25) can be simplified by considering first \( dX_T^M > 0 \) when \( \beta_{LT} = dX_T^C = \beta_{LN} = dX_N^C = dX_N^M = dL = d\beta_{KT} = d\beta_{KN} = dK = 0 \):

\[
-[(\beta_{LT}/L)dX_T^M = -[(\beta_{LT}/K)dX_T^M \text{ or } \beta_{LT} = (L/K)\beta_{KT} \tag{28}
\]

The above two expressions can be substituted into (24) and (25) to yield:

\[
\lambda_{LT}^L\hat{\beta}_{LT} + \lambda_{LT}^C\hat{\beta}_{LT} + \lambda_{LN}^L\hat{\theta}_{LN} + \lambda_{LN}^C\hat{\theta}_{LN} + \lambda_{LN}^N\hat{\lambda}_{LN} + \lambda_{LT}^N\hat{\lambda}_{LT} = \hat{L} - (\hat{X}_T^M + \hat{X}_N^M) \tag{26}
\]

\[
\lambda_{KT}^L\hat{\beta}_{KT} + \lambda_{KT}^C\hat{\beta}_{KT} + \lambda_{KN}^L\hat{\theta}_{KN} + \lambda_{KN}^C\hat{\theta}_{KN} + \lambda_{KN}^N\hat{\lambda}_{KN} + \lambda_{KT}^N\hat{\lambda}_{KT} = \hat{K} - (\hat{X}_T^M + \hat{X}_N^M) \tag{27}
\]

Substitution of equation (7) into (26) and (27) and some mathematical manipulation yields:

\[
\lambda_{LT}^L\hat{\beta}_{LT} + \lambda_{LT}^C\hat{\beta}_{LT} + \lambda_{LN}^L\hat{\theta}_{LN} + \lambda_{LN}^C\hat{\theta}_{LN} + \lambda_{LN}^N\hat{\lambda}_{LN} = \hat{L} + [(\lambda_{LT} + \lambda_{LT}^M)\hat{\beta}_{LT} + (\lambda_{LT} + \lambda_{LT}^N)\hat{\lambda}_{LT}] - \hat{X}_T^M \tag{28}
\]

\[
\lambda_{KT}^L\hat{\beta}_{KT} + \lambda_{KT}^C\hat{\beta}_{KT} + \lambda_{KN}^L\hat{\theta}_{KN} + \lambda_{KN}^C\hat{\theta}_{KN} + \lambda_{KN}^N\hat{\lambda}_{KN} = \hat{K} + [(\lambda_{KT} + \lambda_{KT}^M)\hat{\beta}_{KT} + (\lambda_{KT} + \lambda_{KT}^N)\hat{\lambda}_{KT}] - \hat{X}_N^M \tag{29}
\]

Let

\[
\hat{\beta}_L = (\lambda_{LT}^L + \lambda_{LT}^M)\hat{\beta}_{LT} + (\lambda_{LT}^L + \lambda_{LT}^N)\hat{\lambda}_{LT} \tag{28}
\]

Second, consider \( dX_T^M > 0 \) when \( \beta_{LT} = dX_T^C = \beta_{LN} = dX_N^C = dX_N^M = dL = d\beta_{KT} = d\beta_{KN} = dK = 0 \):

\[
-(\beta_{LN}/L)dX_N^M = -(\beta_{KN}/K)dX_N^M \text{ or } \beta_{LN} = (L/K)\beta_{KN} \tag{26}
\]

Using the above, we can define:

\[
\tilde{X}_T^M = \beta_{LT}dX_T^M = \beta_{LT}d(X_T^M/L) = (\beta_{LT}/L)dX_T^M = (\beta_{KT}/K)dX_T^M \tag{28}
\]

\[
\tilde{X}_N^M = \beta_{LN}dX_N^M = \beta_{LN}d(X_N^M/L) = (\beta_{LN}/L)dX_N^M = (\beta_{KT}/K)dX_N^M \tag{28}
\]
be the “labor saving” in factor-productivity differences between countries and

\[ \bar{\Pi}_K = (\lambda_{KT}^C + \lambda_{KT}^M)\bar{\gamma}_{KT} + (\lambda_{KN}^C + \lambda_{KN}^M)\bar{\gamma}_{KN} \]

be the “capital saving” in factor-productivity differences between countries. Substituting these two terms, respectively, into equations (28) and (29) yields:

\[ \lambda_{LT}^C \bar{X}_T^C + \lambda_{LN}^C \bar{X}_N^C = \bar{L} + \bar{\Pi}_L - (\bar{X}_T^M + \bar{X}_N^M) - (\lambda_{LT}^C + \lambda_{LT}^M)\bar{\gamma}_{LT} + (\lambda_{LN}^C + \lambda_{LN}^M)\bar{\gamma}_{LN} \]

\[ \lambda_{KT}^C \bar{X}_T^C + \lambda_{KN}^C \bar{X}_N^C = \bar{K} + \bar{\Pi}_K - (\bar{X}_T^M + \bar{X}_N^M) - (\lambda_{KT}^C + \lambda_{KT}^M)\bar{\gamma}_{KT} + (\lambda_{KN}^C + \lambda_{KN}^M)\bar{\gamma}_{KN} \]

(30) and (31)

In matrix form, (30) and (31) are:

\[
\begin{bmatrix}
\lambda_{LT}^C & \lambda_{LN}^C \\
\lambda_{KT}^C & \lambda_{KN}^C \\
\end{bmatrix}
\begin{bmatrix}
\bar{X}_T^C \\
\bar{X}_N^C \\
\end{bmatrix}
= \begin{bmatrix}
\bar{L} + \bar{\Pi}_L - (\bar{X}_T^M + \bar{X}_N^M) - (\lambda_{LT}^C + \lambda_{LT}^M)\bar{\gamma}_{LT} + (\lambda_{LN}^C + \lambda_{LN}^M)\bar{\gamma}_{LN} \\
\bar{K} + \bar{\Pi}_K - (\bar{X}_T^M + \bar{X}_N^M) - (\lambda_{KT}^C + \lambda_{KT}^M)\bar{\gamma}_{KT} + (\lambda_{KN}^C + \lambda_{KN}^M)\bar{\gamma}_{KN} \\
\end{bmatrix}
\]

(32)

Premultiplying both sides of (32) by the inverse of

\[
\begin{bmatrix}
\lambda_{LT}^C & \lambda_{LN}^C \\
\lambda_{KT}^C & \lambda_{KN}^C \\
\end{bmatrix}
\]

which is:

\[
\begin{bmatrix}
\lambda_{LT}^C & \lambda_{LN}^C \\
\lambda_{KT}^C & \lambda_{KN}^C \\
\end{bmatrix}^{-1} = (\lambda_{LT}^C \lambda_{KN}^C - \lambda_{LN}^C \lambda_{KT}^C)^{-1} \begin{bmatrix}
\lambda_{Kn}^C & -\lambda_{Lm}^C \\
-\lambda_{Km}^C & \lambda_{Lt}^C \\
\end{bmatrix}
\]

yields:

\[
\begin{bmatrix}
\bar{X}_T^C \\
\bar{X}_N^C \\
\end{bmatrix} = (\lambda_{LT}^C \lambda_{KN}^C - \lambda_{LN}^C \lambda_{KT}^C)^{-1} \begin{bmatrix}
\lambda_{Kn}^C & -\lambda_{Lm}^C \\
-\lambda_{Km}^C & \lambda_{Lt}^C \\
\end{bmatrix} \begin{bmatrix}
\bar{L} + \bar{\Pi}_L - (\bar{X}_T^M + \bar{X}_N^M) - (\lambda_{LT}^C + \lambda_{LT}^M)\bar{\gamma}_{LT} + (\lambda_{LN}^C + \lambda_{LN}^M)\bar{\gamma}_{LN} \\
\bar{K} + \bar{\Pi}_K - (\bar{X}_T^M + \bar{X}_N^M) - (\lambda_{KT}^C + \lambda_{KT}^M)\bar{\gamma}_{KT} + (\lambda_{KN}^C + \lambda_{KN}^M)\bar{\gamma}_{KN} \\
\end{bmatrix}
\]

(33)

Simplifying the above yields:

\[
\bar{X}_T^C = (\lambda_{LT}^C \lambda_{KN}^C - \lambda_{LN}^C \lambda_{KT}^C)^{-1} \lambda_{LT}^C [\bar{L} + \bar{\Pi}_L - (\bar{X}_T^M + \bar{X}_N^M)]
\]

\[
-[(\lambda_{LT}^C + \lambda_{LT}^M)\bar{\gamma}_{LT} + (\lambda_{LN}^C + \lambda_{LN}^M)\bar{\gamma}_{LN}]
\]

\[
+ (\lambda_{LT}^C \lambda_{Km}^C - \lambda_{LN}^C \lambda_{Km}^C)^{-1} \lambda_{LT}^C [\bar{K} + \bar{\Pi}_K - (\bar{X}_T^M + \bar{X}_N^M)]
\]

\[-[(\lambda_{KT}^C + \lambda_{KT}^M)\bar{\gamma}_{KT} + (\lambda_{KN}^C + \lambda_{KN}^M)\bar{\gamma}_{KN}]
\]

(34)

In matrix form, (33) are:

\[
\begin{bmatrix}
\lambda_{LT}^C & \lambda_{LN}^C \\
\lambda_{KT}^C & \lambda_{KN}^C \\
\end{bmatrix}
\begin{bmatrix}
\bar{X}_T^C \\
\bar{X}_N^C \\
\end{bmatrix}
= \begin{bmatrix}
\bar{L} + \bar{\Pi}_L - (\bar{X}_T^M + \bar{X}_N^M) - (\lambda_{LT}^C + \lambda_{LT}^M)\bar{\gamma}_{LT} + (\lambda_{LN}^C + \lambda_{LN}^M)\bar{\gamma}_{LN} \\
\bar{K} + \bar{\Pi}_K - (\bar{X}_T^M + \bar{X}_N^M) - (\lambda_{KT}^C + \lambda_{KT}^M)\bar{\gamma}_{KT} + (\lambda_{KN}^C + \lambda_{KN}^M)\bar{\gamma}_{KN} \\
\end{bmatrix}
\]

(35)

Substitution of equations (20)-(23) for \( \bar{\gamma}_{LT}, \bar{\gamma}_{KT}, \bar{\gamma}_{LT}, \) and \( \bar{\gamma}_{KT} \), respectively, and \( -(\lambda_{KT}^C \lambda_{LN}^C - \lambda_{LT}^C \lambda_{KN}^C)^{-1} \) for \( (\lambda_{LT}^C \lambda_{KN}^C - \lambda_{LN}^C \lambda_{KT}^C)^{-1} \), into above two equations yields:

\[
\bar{X}_T^C = (\lambda_{LT}^C \lambda_{KN}^C - \lambda_{LN}^C \lambda_{KT}^C)^{-1} \lambda_{LT}^C [\bar{L} + \bar{\Pi}_L - (\bar{X}_T^M + \bar{X}_N^M)]
\]

\[
+ (\lambda_{LT}^C + \lambda_{LT}^M)\bar{\gamma}_{LT} + (\lambda_{LN}^C + \lambda_{LN}^M)\bar{\gamma}_{LN}
\]

\[-(\lambda_{LT}^C \lambda_{Km}^C - \lambda_{LN}^C \lambda_{Km}^C)^{-1} \lambda_{LT}^C [\bar{K} + \bar{\Pi}_K - (\bar{X}_T^M + \bar{X}_N^M)]
\]

\[-(\lambda_{KT}^C + \lambda_{KT}^M)\bar{\gamma}_{KT} + (\lambda_{KN}^C + \lambda_{KN}^M)\bar{\gamma}_{KN}
\]

(36)

\[
\bar{X}_T^C = -\lambda_{KT}^C \lambda_{LN}^C - \lambda_{LT}^C \lambda_{KN}^C)^{-1} \lambda_{KN}^C [\bar{L} + \bar{\Pi}_L - (\bar{X}_T^M + \bar{X}_N^M)]
\]

(37)
Letting $\lambda_c^\circ = \lambda_{LT}^c \lambda_{LT}^c - \lambda_{LT}^c \lambda_{LT}^c$, $\lambda_c^\circ = \lambda_{LT}^c + \lambda_{LT}^c$ (for all $i = K, L$ and $j = N, T$), and subtracting equation (36) from (35) yields:

$$\bar{X}_c = \bar{X}_c^T - \bar{X}_c^N = (\lambda_c^\circ)^{-1} \{ (\lambda_{LT}^c + \lambda_{LT}^c) \bar{L} + (\lambda_{LT}^c + \lambda_{LT}^c) \bar{L} \}$$

$$+ (\lambda_{LT}^c + \lambda_{LT}^c) (\lambda_{LT}^c \theta_{LT} \sigma_T) (\bar{W} - \bar{R}) - (\lambda_{LT}^c + \lambda_{LT}^c) \theta_{LT} \sigma_T (\bar{W} - \bar{R})$$

$$- (\lambda_{LT}^c + \lambda_{LT}^c) \theta_{LT} \sigma_T (\bar{W} - \bar{R}) - (\lambda_{LT}^c + \lambda_{LT}^c) \theta_{LT} \sigma_T (\bar{W} - \bar{R})$$

(36)

Letting $\lambda_c^\circ = \lambda_{LT}^c \lambda_{LT}^c - \lambda_{LT}^c \lambda_{LT}^c$, $\lambda_c^\circ = \lambda_{LT}^c + \lambda_{LT}^c$ (for all $i = K, L$), and subtracting equation (36) from (35) yields:

$$\bar{X}_c = \bar{X}_c^T - \bar{X}_c^N = (\lambda_c^\circ)^{-1} \{ (\lambda_{LT}^c + \lambda_{LT}^c) \bar{L} + (\lambda_{LT}^c + \lambda_{LT}^c) \bar{L} \}$$

$$+ (\lambda_{LT}^c + \lambda_{LT}^c) (\lambda_{LT}^c \theta_{LT} \sigma_T) (\bar{W} - \bar{R}) - (\lambda_{LT}^c + \lambda_{LT}^c) \theta_{LT} \sigma_T (\bar{W} - \bar{R})$$

$$- (\lambda_{LT}^c + \lambda_{LT}^c) \theta_{LT} \sigma_T (\bar{W} - \bar{R}) - (\lambda_{LT}^c + \lambda_{LT}^c) \theta_{LT} \sigma_T (\bar{W} - \bar{R})$$

(37)

Consolidating and reordering terms, and letting $\lambda_c^\circ = \lambda_{LT}^c + \lambda_{LT}^c$ (for $i = K, L$, yields:

$$\bar{X}_c = (\lambda_c^\circ)^{-1} \{ (\lambda_{LT}^c + \lambda_{LT}^c) \bar{L} + (\lambda_{LT}^c + \lambda_{LT}^c) \bar{L} \}$$

$$+ (\lambda_{LT}^c + \lambda_{LT}^c) (\lambda_{LT}^c \theta_{LT} \sigma_T) (\bar{W} - \bar{R}) - (\lambda_{LT}^c + \lambda_{LT}^c) \theta_{LT} \sigma_T (\bar{W} - \bar{R})$$

$$- (\lambda_{LT}^c + \lambda_{LT}^c) \theta_{LT} \sigma_T (\bar{W} - \bar{R}) - (\lambda_{LT}^c + \lambda_{LT}^c) \theta_{LT} \sigma_T (\bar{W} - \bar{R})$$

(38)

Substituting equation (17) into (38) yields:

$$\bar{X}_c = (\lambda_c^\circ)^{-1} \{ (\lambda_{LT}^c + \lambda_{LT}^c) \bar{L} + (\lambda_{LT}^c + \lambda_{LT}^c) \bar{L} \}$$

$$+ (\lambda_{LT}^c + \lambda_{LT}^c) (\lambda_{LT}^c \theta_{LT} \sigma_T) (\bar{W} - \bar{R}) - (\lambda_{LT}^c + \lambda_{LT}^c) \theta_{LT} \sigma_T (\bar{W} - \bar{R})$$

$$- (\lambda_{LT}^c + \lambda_{LT}^c) \theta_{LT} \sigma_T (\bar{W} - \bar{R}) - (\lambda_{LT}^c + \lambda_{LT}^c) \theta_{LT} \sigma_T (\bar{W} - \bar{R})$$

(39)

Substituting equation (17) into (38) yields:

$$\bar{X}_c = (\lambda_c^\circ)^{-1} \{ (\lambda_{LT}^c + \lambda_{LT}^c) \bar{L} + (\lambda_{LT}^c + \lambda_{LT}^c) \bar{L} \}$$

$$+ (\lambda_{LT}^c + \lambda_{LT}^c) (\lambda_{LT}^c \theta_{LT} \sigma_T) (\bar{W} - \bar{R}) - (\lambda_{LT}^c + \lambda_{LT}^c) \theta_{LT} \sigma_T (\bar{W} - \bar{R})$$

$$- (\lambda_{LT}^c + \lambda_{LT}^c) \theta_{LT} \sigma_T (\bar{W} - \bar{R}) - (\lambda_{LT}^c + \lambda_{LT}^c) \theta_{LT} \sigma_T (\bar{W} - \bar{R})$$

(40)
In matrix form,

\[
\begin{bmatrix}
\lambda_L^G & \lambda_L^E
\end{bmatrix}
= (\theta_L \lambda_{KN} + \theta_K \lambda_{LN})(\lambda_L^G \tilde{G} - \lambda_L^E \tilde{E})
\]

\[
+ (\theta_L \lambda_{KT} + \theta_K \lambda_{LT})(\lambda_L^G \tilde{T} - \lambda_L^E \tilde{E})
\]

\[
+ (\lambda_L - \lambda_{KN})(\lambda_L^G \theta_{KN} \tilde{G} + \lambda_L^E \theta_{KN} \tilde{E} - \lambda_L^G \theta_{LT} \tilde{T} - \lambda_L^E \theta_{KT} \tilde{E})
\]

Letting \([\lambda_L^G, \lambda_L^E]^{-1}\) denote the generalized inverse of \([\lambda_L^G, \lambda_L^E]\), cf., Henri Theil, Principles of Econometrics, 1971, pp. 269-273, premultiplying both sides of (41) by \([1 1]\) yields:

\[
\tilde{T} = (\theta_L \lambda_{KN} + \theta_K \lambda_{LN})(\lambda_L^G \tilde{G} - \lambda_L^E \tilde{E})
\]

\[
+ (\theta_L \lambda_{KT} + \theta_K \lambda_{LT})(\lambda_L^G \tilde{T} - \lambda_L^E \tilde{E})
\]

\[
+ (\lambda_L - \lambda_{KN})(\lambda_L^G \theta_{KN} \tilde{G} + \lambda_L^E \theta_{KN} \tilde{E} - \lambda_L^G \theta_{LT} \tilde{T} - \lambda_L^E \theta_{KT} \tilde{E})
\]

Assuming productivity differentials are Hicks neutral, \(\tilde{T} = 0\), and recalling the definitions of \(\Pi_N\) and \(\Pi_T\) in equations (12) and (13):

\[
\Pi_L - \Pi_K = (\lambda_L - \lambda_{KN})(\Pi_N - \Pi_T)
\]

or

\[
[1 1]\begin{bmatrix}
\Pi_L & -\Pi_K
\end{bmatrix}
= (\lambda_L - \lambda_{KN})[1 1]\begin{bmatrix}
\Pi_N & -\Pi_T
\end{bmatrix}
\]

Premultiplying both sides by \([\lambda_L^G, \lambda_L^E][1 1]^{-1}\) yields:

\[
\lambda_L^G \Pi_L - \lambda_L^E \Pi_K = (\lambda_L - \lambda_{KN})\lambda_L^G \Pi_N - (\lambda_L - \lambda_{KN})\lambda_L^E \Pi_T
\] (42)

Substituting (42) into (39) yields:

\[
\tilde{X}^C = \alpha_1 \tilde{P} - \alpha_2 \tilde{K} + \alpha_3 \tilde{L} + \alpha_4 \tilde{N} - \alpha_5 \Pi_T - \alpha_6 \left[ \tilde{X}_M^M + \tilde{X}_N^M \right]
\] (43)

Finally, indefinite integration of equation (43), treating \(\alpha_1\) through \(\alpha_6\) as constant parameters, yields:

\[
(1nX^C)^S = \alpha_0 + \alpha_1 1nP - \alpha_2 1nK + \alpha_3 1nL + \alpha_4 1nN - \alpha_5 1n\Pi_T - \alpha_6 \left[ 1nX_M^M + 1nX_N^M \right].
\]

from which estimable equation (44) is readily derived:

\[
1nP = \alpha_0 + (1/\alpha_1)(1nX^C)^S + (\alpha_2/\alpha_1)1nK - (\alpha_3/\alpha_1)1nL
\]

\[- (\alpha_4/\alpha_1)1nN + (\alpha_5/\alpha_1)1n\Pi_T + (\alpha_6/\alpha_1)(1nX_M^M + 1nX_N^M)
\] (44)

3. Demand

I assume a representative consumer maximizes the nonhomothetic Stone-Geary utility function for civilian goods:

\[
u = \left[ \frac{\tilde{x}_C}{\tilde{x}_T^\delta} \right]^\delta \left[ \frac{\tilde{x}_N}{\tilde{x}_N^\delta} \right]^{1-\delta} \quad 0 < \delta < 1
\] (45)
where \( x_C^C \) (\( x_N^C \)) is the per capita amount consumed of tradables (non-tradables) in the civilian sector and \( x_C^C \) (\( x_N^C \)) is an exogenous minimum-consumption requirement for the civilian sector for the tradable (non-tradable), common to the Stone-Geary utility function. As in Frenkel and Razin (1987), the consumer derives no utility from the government purchases. Assume the budget constraint

\[
y = x_C^C + px_N^M + \text{taxes}
\]

(46)

where \( y \) is per capita income and taxes = \( x_M^M + px_N^M \), both expressed in terms of the tradable (the numeraire). Per capita military expenditures are \( x_M^M + px_N^M \).

Maximization of utility function (44) subject to income constraint (46) yields:

\[
\frac{\partial u}{\partial x_C^C} = (x_N^C - \bar{x}_C^C)^{1-\delta} \delta (x_C^C - \bar{x}_C^C)^{\delta-1} - \lambda = 0 \quad (47)
\]

\[
\frac{\partial u}{\partial x_N^C} = (x_C^C - \bar{x}_C^C \delta (1-\delta)(x_M^M - \bar{x}_N^C)^{-\delta} - \lambda = 0 \quad (48)
\]

\[
\frac{\partial u}{\partial \lambda} = y - x_C^C - px_N^C - x_M^M - px_N^M = 0 \quad (49)
\]

Equating (47) and (48) yields:

\[
\delta (x_N^C - \bar{x}_N^C)^{1-\delta}(x_C^C - \bar{x}_C^C)^{\delta-1} = p^{-1} (x_C^C - \bar{x}_C^C)^{\delta}(1-\delta)(x_M^M - \bar{x}_N^C)^{-\delta} \quad (50)
\]

Solving (50) for \( x_C^C \) yields:

\[
x_C^C = [(1-\delta)/\delta] p^{-1} (x_C^C - \bar{x}_C^C) + \bar{x}_C^C \quad (51)
\]

Substituting (51) into the budget constraint and solving for \( x_M^M \) yields:

\[
x_C^C = \delta y + (1-\delta) \bar{x}_C^C - \delta \bar{x}_N^C - \delta (x_M^M + px_N^M) \quad (52)
\]

Substituting (52) into the budget constraint and solving for \( x_N^C \) yields:

\[
x_N^C = (1-\delta)p^{-1}y - (1-\delta)p^{-1} \bar{x}_C^C + \delta \bar{x}_N^C - (1-\delta) (p^{-1}x_M^M + px_N^M) \quad (53)
\]

Differentiating equation (52), dividing the resulting equation by \( x_M^M \), and some mathematical manipulation yields:

\[
\dot{z}^C = \frac{\delta (\bar{x}_N^C + x_M^M)}{x_M^M} \dot{p} + \left[ \frac{1 - (1-\delta) \bar{x}_N^C - \delta \bar{x}_N^C / x_M^M - \delta (x_M^M + px_N^M)}{x_M^M} \right] \dot{y} - \frac{\delta (x_M^M + px_N^M)}{x_M^M} (x_M^M + px_N^M) \quad (54)
\]

where

\[
(x_M^M + px_N^M) = (dx_M^M + pdx_N^M)/(x_M^M + px_N^M).
\]

Differentiating equation (53), dividing the resulting equation by \( x_N^C \), and some mathematical manipulation yields:

\[
\dot{z}_N^C = - \frac{1 + (1-\delta) p^{-1} (x_M^M + px_N^M) - \delta z_N^C}{x_N^C} \quad (55)
\]

Subtracting equation (54) from (55) yields:

\[
\dot{z}^c = \dot{z}_N^C - \dot{z}_C^C = \dot{z}_N^C - \dot{z}_C^C = - \phi_1 \dot{p} + \phi_2 \dot{y} - \phi_3 (x_M^M + px_N^M) \quad (56)
\]

where

\[
\phi_1 = 1 + \frac{[(1-\delta) p^{-1} (x_M^M + px_N^M) - \delta z_N^C] x_M^M}{x_M^M} - \delta (z_N^C + x_M^M)/x_M^M
\]

\[
\phi_2 = (a_M^M + px_N^M) [(1-\delta) \dot{z}_N^C - \delta \dot{z}_N^C]/px_N^M \dot{z}_N^C
\]

\[
\phi_3 = \phi_3 + (a_M^M + px_N^M) [(1-\delta) \dot{z}_N^C - \delta \dot{z}_N^C]/px_N^M \dot{z}_N^C
\]

Finally, indefinite integration of equation (56), treating \( \phi_1 \), \( \phi_2 \) and \( \phi_3 \) as constant parameters, yields:

\[
(1nX^C)^D = \phi_0 - \phi_1 n \dot{p} + \phi_2 n \dot{y} - \phi_3 n (x_M^M + px_N^M)
\]
from which estimable equation (57) is readily derived:

\[ 1np = \phi_0 - (1/\phi_1)(\ln X_C^D) + (\phi_2/\phi_1)\ln y - (\phi_3/\phi_1)\ln (x_T^M + px_N^M) \]  

(57)

Finally, the relative price of nontradables in terms of tradables, or real exchange rate, can be solved for as a reduced-form function of the intratemporal supply and demand factors:

\[ 1np = (\alpha_1 + \phi_1)^{-1}[(\phi_0\phi_1 - \alpha_0\alpha_1) + \alpha_5\ln\Pi_T - \alpha_4\ln\Pi_N + \alpha_2\ln K - \alpha_3\ln L + \phi_2\ln y + (\alpha_6 - \phi_3)\ln (x_T^M + px_N^M)] \]  

(58)

The reduced form is obtained by equating (44) and (57) and solving for \( 1np \), allowing \( \ln (x_T^M + px_N^M) \) to represent the unmeasurable variable \( \ln (x_T^M + px_N^M) \). First, higher productivity in tradables (nontradables) will be associated with a higher (lower) real exchange rate, consistent with the Balassa (1964) productivity differential model. Second, a higher capital (labor) stock will be associated with a higher (lower) real exchange rate, if nontradables are labor intensive, consistent with the Bhagwati (1984) relative-factor-endowments theory. Third, a higher per capita income will be associated with a higher real exchange rate, if tastes are nonhomothetic and nontradables (tradables) are luxuries (necessities). Fourth, higher per capita military expenditures will alter the real exchange rate, depending upon the relative factor intensities of civilian and military goods and of civilian tradables and nontradables, and whether civilian nontradables (tradables) are luxuries or necessities in consumption.

4. Empirical Evidence

Reduced-form equation (58) and structural equations (44) and (57) are in forms estimable by ordinary least squares and two-stage least squares, respectively. Data for only 21 countries could be obtained for all of the variables. Data for each country for the price of nontradables relative to that of tradables (relative to that of the United States, US=100), the output of nontradables relative to that of tradables, and per capita GDP in 1975 are from Table 6-12 in Kravis et al. (1982). Capital and labor (LABOR1) for 1975 are from Leamer (1984 appendix table B.1). The level of productivity in nontradables (services) is approximated by the ratio of national output in services in 1975 from Kravis et al. (1982) to the level of employment in services industries in 1975 from the International Labour Organization’s Year Book of Labour Statistics (1979) (relative to that of the United States). Per capita military expenditures were obtained by taking the share of military expenditures in gross national product of each country times its per capita gross national product; the military shares were obtained from the SIPRI Yearbook.

Estimation of reduced-form equation (58) yielded:

\[ \ln p = -1.10 + 0.12 \ln\Pi_T - 0.28\ln\Pi_N + 0.09 \ln K - 0.11 \ln L + 0.28 \ln y + 0.04 \ln (x_T^M + px_N^M); \]  

(0.74)

\( \text{see} = 0.10; \ R^2 = 0.95; \ Adjusted \ R^2 = 0.93 \)

where numbers in parentheses are absolute values of t-statistics and SEE is the standard error of the regression. A one percent reduction in per capita military spending tends to reduce the price of nontradables relative to that of tradables by only 0.04 percent. Note that all other coefficient estimates’ signs conform to the productivity-differential, relative-factor-endowments, and nonhomothetic-tastes theories for departures from absolute PPP, and estimates for \( \Pi_T, \ Pi_N \)

\[ ^3 \text{The 21 countries are India, Sri Lanka, Thailand, the Philippines, Korea, Colombia, Jamaica, Brasil, Yugoslavia, Ireland, Italy, Spain, the United Kingdom, Japan, Austria, the Netherlands, Belgium, France, Denmark, Germany, and the United States.} \]
and y are statistically significant at 10 percent (two-tail t-tests).

Although the reduced-form estimates suggest that the impact of military spending reductions on the relative national price level would be economically insignificant, this result does not imply that the intratemporal supply and demand channels are unimportant. In fact, the structural equations' estimates suggest that endogenous supply decisions and nonhomothetic tastes are economically significant channels, but their effects on relative prices are offsetting. Two-stage least squares estimation of equations (43) and (56) yields:

\[
\ln p = -0.97 + 0.46 (\ln X^C)^S + 0.22 \ln K - 0.23 \ln L - 0.31 \ln \Pi_N \\
(1.71) \quad (1.31) \quad (3.96) \quad (4.26) \quad (2.16) \\
+ 0.14 \ln \Pi_T + 0.04 \ln (x^M_T + px^N_T) \\
(1.59) \quad (2.60)
\]

\[\text{see} = 0.15; \ R^2 = 0.89; \ Adjusted \ R^2 = 0.84 \]

\[
\ln p = -2.22 - 0.75 (\ln X^C)^D + 0.58 \ln y - 0.11 \ln (x^M + px^N) \\
(15.79) \quad (2.90) \quad (7.46) \quad (1.84) \\
\]

\[\text{see} = 0.12; \ R^2 = 0.92; \ Adjusted \ R^2 = 0.90 \]

5. Conclusions

Recently, several studies have focused upon better understanding the robust cross-country relationship between per capita incomes and national price levels, or per capita incomes and the price of nontradables relative to tradables. Efforts have been made to disentangle the relative supply versus the relative demand influences of per capita income on relative price levels, in some cases treating per capita incomes as endogenous.

This study has attempted to extend this literature, examining theoretically and empirically the potential channels through which fiscal spending – in particular, exogenous military expenditures – affect per capita disposable income, the absorption of labor and capital endowments, and ultimately the national price level. Theoretically, the effect of military spending reductions on these variables is ambiguous; the effects depend upon the relative factor intensity in production of civilian versus military goods and of civilian tradable versus nontradable goods, and upon the relative importance in utility of civilian tradable versus nontradable goods. Empirically, the model suggests that the effects of reduced military spending on the relative demand for and relative supply of nontradables to tradables are economically and statistically significant. However, because military spending reductions will tend to increase the relative supply only slightly more than the relative demand, the price of nontradables relative to tradables is
predicted to decline by only a small amount. Consequently, lower military expenditures are predicted to result in a small real depreciation of a country's currency, and thus only a minor fall in the national price level relative to the world average.

References


