The conditional factor demands for final goods production are given by:

\[ K^*_X = F_X \left( \frac{w_{U_i}}{r_i} \right)^{1-\alpha} \alpha \frac{\alpha(\chi-1)-\chi}{T_{1i}} \]

\[ S^*_X = F_X \left( \frac{w_{U_i}}{w_{Si}} \right)^{1-\alpha} \alpha \frac{\alpha(\chi-1)-\chi}{T_{2i}} \]

\[ U^*_X = F_X \left( \frac{r_i}{w_{U_i}} \right)^{1-\alpha} \alpha \frac{\alpha(\chi-1)-\chi}{(T_{1i}^{\chi} + T_{2i}^{\chi})} \]

where \( B \) is a constant and we introduce definitions:

\[ T_{1i} = 1 + \left( \frac{r_i}{w_{Si}} \right)^{1-\chi} \]

\[ T_{2i} = 1 + \left( \frac{w_{Si}}{r_i} \right)^{1-\chi} \]

We assume that, in equilibrium, all factors are fully employed for each country \( i \) (\( i = 1,2,3 \)), so that:

\[ K_i \geq a_{KXi} \left[ n_i \sum_{j=1}^{3} x_{ij} + x_{ii} \left( \sum_{j=1}^{3} h_{3,j} \right) + \sum_{j \neq i} v \left( \sum_{j=1}^{3} x_{ij} \right) \right] + a_{Kn_i} n_i + a_{Kmi} \left[ 3 + \sum_{j \neq i} v \left( \sum_{j=1}^{3} h_{3,j} \right) + [1 + v_{ij}] v_{ij} \right] \]

\[ S_i \geq a_{SX_i} \left[ n_i \sum_{j=1}^{3} x_{ij} + x_{ii} \left( \sum_{j=1}^{3} h_{3,j} \right) + \sum_{j \neq i} v \left( \sum_{j=1}^{3} x_{ij} \right) \right] + a_{Sn_i} n_i + a_{Smi} (h_{3,i} + \sum_{j \neq i} v_{ij}) \]

\[ U_i \geq a_{UX_i} \left[ n_i \sum_{j=1}^{3} x_{ij} + x_{ii} \left( \sum_{j=1}^{3} h_{3,j} \right) + \sum_{j \neq i} v \left( \sum_{j=1}^{3} x_{ij} \right) \right] + a_{UY_i} \sum_{j=1}^{3} t_{ij} Y_{ij} \]
Multilateral current account balance for each country \(i\) \((i=1,2,3; i \neq j \neq k)\) requires the following to hold:

\[
(n_i + v_{ji} + v_{ki}) p_{X_i} (x_{ij} + x_{ik}) + p_{Y_j} Y_j + p_{Y_k} Y_k
\]

\[
+ \frac{1}{1 - \epsilon} (h_{3,j} p_{X_j} x_{jj} + h_{3,k} p_{X_k} x_{kk})
\]

\[
+ \frac{1}{1 - \epsilon} (v_{ij} p_{X_j} [x_{jj} + x_{ji} + x_{jk}] + v_{ik} p_{X_k} [x_{kk} + x_{ki} + x_{kj}])
\]

\[
= (n_j + v_{ij} + v_{kj}) p_{X_j} x_{ji} + (n_k + v_{ik} + v_{jk}) p_{X_k} x_{ki} + p_{Y_i} (Y_{ji} + Y_{ki})
\]

\[
+ \frac{1}{1 - \epsilon} (h_{3,j} + h_{3,k}) p_{X_i} x_{ii}
\]

\[
+ \frac{1}{1 - \epsilon} (v_{ji} + v_{ki}) p_{X_i} (x_{ji} + x_{ij} + x_{ik})
\]

\[(A4)\]

The first line in equation (A4) represents the exports of goods of country i. The second and third lines represent income earned on capital invested by country i in horizontal and vertical affiliates, respectively, in countries j and k (k denoting the ROW for country i). The fourth line represents country i’s imports of goods from j and k. The fifth and sixth lines represent i’s repatriation of income on capital of countries j and k invested in country i in horizontal and vertical affiliates, respectively.
Appendix B

This appendix is intended to report the results of some sensitivity analysis of our findings. Of course, the potential number of alternative calibrations is virtually unlimited; resource constraints prevent us from exploring an exhaustive analysis of permutations from our base calibration. Nevertheless, we believe it is important to show that our results are relatively robust to empirically relevant parameter selections.

1. Robustness of Simulation Results to Assuming Headquarters (Plant) Setups use Physical (Human) Capital

Two critical assumptions used to generate coexistence in Section 5 are the existence of physical capital and the assumption that headquarters (plant) setups require human (physical) capital. We proved in section 5 analytically the importance of the third factor for coexistence of FAS, FDI and bilateral trade across a wide range of parameter values for firm and plant setup costs, trade costs, and investment costs. However, the results in Sections 5 and 6 may be sensitive to assuming headquarters (plant) setups require human (physical) capital. To illustrate the results are robust to an alternative assumption, we ran the model assuming plant (headquarters) setups require human (physical) capital. At the initial world endowments, FDI and FAS went to zero because of the scarcity of human capital. After increasing the world endowment of human capital (with again three symmetric economies), qualitatively identical results obtained for Figures 4b-4d. See, for example, Figures B1a-B1c.

2. Robustness of Simulation Results to Variation in Capital-Skill Elasticity-of-Substitution Parameter

The theoretical model has only four “parameters” (in the utility and production functions). On the technology side, there are two parameters in the differentiated goods production function: the elasticity of substitution between knowledge and physical capital and the share of capital (versus labor) in gross output. While empirical data guided the selection of the last parameter (0.2), the selection of the first parameter – the elasticity of substitution between knowledge and physical capital – was based upon econometric studies. As discussed in the text, there is wide-ranging evidence – not just from the production literature, but also the MNE literature (cf., Slaughter, 2000) – that physical capital and skills are complements (in a relative sense). However, physical capital and skills may be substitutes.

For an alternative calibration, we made capital and skills substitutes, choosing a value for $\chi$ of 0.167 which implied an elasticity of substitution of 1.2 (instead of a value for $\chi$ of -0.25, implying an elasticity of 0.8, in the baseline case). The results corresponding to Figures 4b-4d are presented in Figures B2a-B2c. We found that the overall GDP size and similarity relationships were qualitatively the same.

3. Robustness of Simulation Results to Variation in Elasticity of Substitution of Differentiated Goods

On the preferences side, there are only two parameters in the utility function: the elasticity of substitution among differentiated goods and the share of expenditure on differentiated goods (versus the homogenous good). The expenditure share was determined by empirical data. Evidence from econometric studies usually suggests an elasticity of substitution among differentiated goods ranging from 5 to 10 (cf., Anderson and van Wincoop, 2004, p. 716). Our base case of an elasticity of substitution of 6 ($\varepsilon = -5$) was premised upon this.

Consequently, we considered alternative calibrations with two different elasticities of substitution for differentiated goods, 5 and 10. The results are provided in Figures B3a-B3c and B4a-B4c, respectively. The relationships between economic size, economic similarity, and the bilateral interactions were qualitatively identical to the base case when the elasticity of substitution was lowered to 5 (which assumes a value for $\varepsilon = -4$). When the elasticity of substitution among differentiated goods was raised to 10 ($\varepsilon = -9$), the FAS and FDI relationships were qualitatively identical as well. However, while trade flows were still positively related to economic size of i and j, trade flows were negatively related to their similarity when GDPs of i and j were small, which would be inconsistent with typical trade gravity equations. At such a high elasticity of substitution, the net effect of “complementarity” becomes diminished. Of course, a small decrease in trade costs can be shown to reverse this readily and ensure that complementarity still obtains over a wide range of parameter values.
4. Robustness of Simulation Results to Varying Investment Costs and MNE/NE Ratio Firm Setup Costs

The relationships depicted in figures are, of course, sensitive to values chosen for “exogenous” variables as discussed in section 4. In almost all cases, we tried to use empirical data to guide choices. For instance, levels of transport costs and tariff rates were selected based upon averages of actual data. In some cases, it is difficult to choose parameter values due to the absence of empirical evidence; two difficult choices were the selection of exogenous investment costs ($\gamma$) and the ratio of headquarters setup costs for an MNE relative to an NE. In the base simulations, we chose a “tax rate” (i.e., the “tax” on home capital to invest in a plant in another country) of 40 percent for capital investments by a developed country (DC) MNE in another DC; we assumed these investment barriers do not generate revenue. In order to bias our results against finding “complementarity” of trade and FAS (in particular, zero trade for identical economies), we assumed the minimum feasible ratio of setting up an MNE headquarters relative to an NE headquarters suggested in Markusen (2002) – 1.01 – indicating that an MNE consumed only 1 percent more skilled labor for firm setup relative to an NE.

In a sensitivity analysis, we experimented with lower investment costs. We found that trade was still complementary to FAS when investment costs were half of the base values; when lowered by 60 percent of their base value, final goods trade went to zero when countries i and j had identical GDPs. However, we also found that – even with investment costs at 40 percent of base values – small, and plausible, increases in MNE/NE headquarters setup costs offset investment costs, yielding complementarity of trade, FDI and FAS to economic size and similarity. As one example, Figures B5a-B5c illustrate that the complementarity relationships are maintained when investment costs are 50 percent of the initial base levels.
Robustness: plant set-up uses skilled labor $S$ whereas firm set-up uses capital $K$
Robustness: parameter of elasticity of substitution among K and S is $\chi = 0.167$ (instead of $\chi = -0.25$)
Robustness: parameter of elasticity of substitution among manufactures is $\varepsilon = -4$ (instead of $\varepsilon = -5$)
Robustness: parameter of elasticity of substitution among manufactures is $\varepsilon = -9$ (instead of $\varepsilon = -5$)
Robustness: parameter of additional foreign plant set-up costs ($\gamma$) is half its original size