THEORETICAL SUPPLEMENT

to

THE SCOPE, GROWTH, AND CAUSES OF INTRA-INDUSTRY INTERNATIONAL TRADE
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The model assumes the existence of many products and firms in a single industry. The two countries have differing absolute endowments of the single factor labor. Consumers are assumed to have identical tastes within and across countries to ensure that intra-industry trade (IIT) is not caused by taste differences. All firms within and across countries have identical, neoclassical cost curves to ensure that IIT is not caused by technology differences. The assumption of a single factor of production ensures that trade is not caused by relative factor endowment differences. The market structure is Chamberlinian monopolistic competition. Assume zero transport costs and no artificial trade barriers; foreign country variables are denoted by "*".

The demand side parallels recent work by Paul Krugman.[1] Each consumer, regardless of country, shares the common constant elasticity of substitution (CES) utility function:

$$\text{(A1)} \qquad U = \left[\sum_{n=1}^{N+N^*} a_n c_n^{\Theta} \right]^{1/\Theta}$$

where N (N*) is the number of products in the industry produced domestically (abroad), a_n is the relative importance in utility of the n^{th} product, c_n is the amount of product n consumed by the representative household, and $\Theta = (\rho-1)/\rho$ where ρ is the (constant) elasticity of substitution.[2] All products are close, but imperfect, substitutes; hence, $\Theta < 1$. To ensure a taste for diversity, all products are assumed to enter utility symmetrically; hence $a_n = a$ for all n.

Assume that the home (foreign) country is comprised of M (M*) laborers who consume all output of the industry and that $M \neq M^*$. Assume each individual is constrained by an identical budget constraint:

$$\text{(A2)} \qquad \sum_{n=1}^{N+N^*} p_n c_n = y$$

where p_n is the price of product n and y is the representative consumer's budget for this industry. Because consumers across countries are identical, total output of each product (X_n) is consumed proportionately across consumers:

(A3) $X_n = (M + M^*)c_n$ for all n.

Maximizing equation (A1) subject to (A2) yields demand curves for each individual in each product. For the representative consumer for product q:

$$\text{(A4)} \qquad p_q = \lambda^{-1} \left[\sum_{n=1}^{N+N^*} a_n c_n^{\Theta} \right]^{\frac{1}{\Theta} - 1} a_q c_q^{\Theta - 1}$$

where λ is the marginal utility of income (i.e. LaGrangean multiplier). Equations (A3) and (A4) suggest the market demand curve facing producer q:

$$\text{(A5)} \qquad p_q = \lambda^{-1} \left[\sum_{n=1}^{N+N^*} a_n X_n^{\Theta} \right]^{\frac{1}{\Theta} - 1} a_q X_q^{\Theta - 1}$$

which implies a price elasticity of demand facing firm q (one of many firms in the industry):

$$\text{(A6)} \qquad \varepsilon_q = 1/(1-\Theta)$$

in the limit $(N + N^* \rightarrow \infty)$.

On the supply side, all firms -- regardless of country -- are assumed to have identical, neoclassical cost functions, C(X), representable by the general form:

$$\text{(A7)} \qquad C(X_q) = \delta_0 + \delta_1 X_q + \delta_2 X_q^{2} + \delta_3 X_q^{3}$$

With parameter restrictions,

$$\delta_0, \delta_1, \delta_3 > 0 \qquad \delta_2 < 0 \qquad (\delta_2)^2 < 3\delta_1\delta_3$$

imposed, the cost function suggests typical U-shapes for its long-run average (AC) and marginal (MC) cost curves.

The downward sloping portion of the AC curve implies the presence of increasing returns -- up to a point. It will turn out that the representative firm chooses output in the region of increasing returns. It will be useful to note that the degree of increasing returns at a particular level of output (i.e. $X_q = \alpha m_q^{1+\beta}$, where m_q is the number of laborers to produce product q) can be measured by the elasticity of scale (β), where:

(A8) $$1 + \beta = AC(X_q)/MC(X_q)$$

With many firms in the industry, M (M*) laborers in the home (foreign) country are distributed across n = 1,..., N (n = N + 1,... N + N*) differentiated products:

(A9) $$M = \sum_{n=1}^{N} m_n \qquad M^* = \sum_{n=N+1}^{N+N^*} m_n$$

The theory of monopolistic competition is characterized by two long-run equilibrium conditions. Profit maximization by each firm suggests maximizing:

(A10) $$\pi_q = p_q X_q - C(X_q)$$

subject to the market demand equation and cost function. This yields the first equilibrium condition:

(A11) $$p_q = \Theta^{-1} MC(X_q)$$

The second condition is that firms enter the (barrier-free) market until economic profits are driven to zero:

(A12) $$\pi_q = p_q X_q - C(X_q) = 0$$

This second equilibrium condition can be rewritten as:

(A13) $$p_q = AC(X_q)$$

Long-run equilibrium output and price choices by the q^{th} firm are found in solving (A11) and (A13) for $\bar{X}$ and $\bar{p}$. The equilibrium output for the representative firm is:

(A14) $$AC(\bar{X})/MC(\bar{X}) = \Theta^{-1}$$

As noted, imperfect product substitution ensures $\Theta < 1$. Hence, the representative firm is supplying output in the downward-sloping portion of the AC curve. Since cost functions are identical across firms, output and price are identical across firms.

The stability of the equilibrium is ensured by an appropriate restriction on parameter values for δ_0, δ_2, and δ_3 (i.e., $\delta_2 + 3\delta_3 X > 0 > \delta_2 + 2\delta_3 X - \delta_0 X^{-2}$). Equilibrium output is just short of the minimum-average-cost output, where MC is rising ($C''(X) = \delta_2 + \delta_3 X > 0$) and AC is still falling

($AC'(X) = \delta_2 + 2\delta_3 X - \delta_0 X^{-2} < 0$).

Three final points are noteworthy. First, the number of products produced by each country can be solved for since all products are produced in the same volume using the same number of laborers ($M = N \cdot m$):

(A15) $N = M/C(\bar{X})$ $\qquad$ $N^* = M^*/C(\bar{X})$

if the wage rate is assumed equal to unity.

Second, note that the number of types of products produced in each country is proportional to the absolute factor endowment. Since all goods have the same price and are viewed symmetrically in utility, home country exports (foreign imports) are proportional to the share of all goods produced by the home country:

(A16) $EX = [N/(N + N^*)]Y^*$

where EX is the value of home country exports and Y* is the foreign country's expenditures for this industry. Home country imports are:

(A17) $IM = [N^*/(N + N^*)]Y$

where IM is the value of home country imports and Y is the home country's expenditures for this industry. IIT exists.

Third, equilibrium output ($\bar{X}$) must satisfy equation (A14) and, consequently, equation (A8). Hence, in equilibrium, the degree of increasing returns is a positive (negative) function of the degree of product differentiation (elasticity of substitution):

(A18) $1 + \beta = \theta^{-1}$

Thus, in the regression analysis, a measure of the degree of product differentiation can be ignored since -- in equilibrium -- the degree of increasing returns (or elasticity of scale) and product differentiation are positively related.

FOOTNOTES FOR THEORETICAL SUPPLEMENT

[1]See the following by Krugman: "Increasing Returns, Monopolistic Competition, and International Trade," Journal of International Economics, 9 (1979), pp. 469-479; "Scale Economies, Product Differentiation, and the Pattern of Trade," American Economic Review, 70 (1980), pp. 950-959; "Intraindustry Specialization and the Gains from Trade," Journal of Political Economy, 89 (1981), pp. 959-973.

[2]This model, like earlier ones, assumes a single world industry. This assumption is tantamount to assuming each consumer's utility (and, hence, expenditures) is separable between this industry and others.