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The purpose of this note is to formalize some -- though clearly not all -- of the theoretical relationships described in Part III. For brevity, the note focuses on the relationship between spot exchange rate variation and relative price (interest rate) elasticities of demand for various financial assets in a small open economy. Although the text emphasizes that price elasticities of demand for goods and relative speeds of adjustment across markets are also important to exchange rate determination, formalizations of these aspects are beyond the scope of this note and can be found elsewhere. To emphasize the importance of asset price elasticities of demand but ensure analytical tractability, the model is a log-linear extension of a portfolio-balance model originated by William Branson.[1] Initially, expectations are static; however, the model will be extended for nonstatic expectations later. As in "Exchange Rates in the Short Run," the analysis is limited here to the short-run determination of the exchange rate. In the long run, the exchange rate is assumed to satisfy PPP.

Assume the existence of three financial assets in the domestic economy: noninterest-bearing domestic money (M); interest-bearing domestic bonds (B, "outside" liabilities issued by the Federal government minus central bank and foreign holdings of the debt); and interest-bearing foreign bonds (F, accumulation of capital outflows). Domestic supplies of each asset are initially fixed. Unlike in the text, the foreign exchange market is not explicit (to keep the number of assets limited to three). However, implicitly the foreign exchange market can be considered to have fixed supplies of each currency (implying demand curves for each currency having unit elasticities). The exchange rate, E, is the home currency value of a unit of foreign exchange.

The domestic money market, domestic bond market, and foreign bond market equilibrium conditions are described by:

$$(1) \quad M = g_1(r, r^* + x, Y)W$$

$$(2) \quad B = g_2(r, r^* + x)W$$

$$(3) \quad EF = g_3(r, r^* + x)W$$

where g_1, g_2, and g_3 are the domestic money, domestic bond, and foreign bond demand functions. Variables r, r^*, and x are the domestic interest rate, foreign interest rate, and the expected rate of depreciation of the home currency, respectively. Let Y be domestic real income and W be domestic wealth. In log-linear terms, equations (1) - (3) are:

$$(4) \quad m = -\alpha_r r - \alpha_{r^*}(r^* + x) + \delta_y y + w$$

$$(5) \quad b = \beta_r r - \beta_{r^*}(r^* + x) + w$$

$$(6) \quad e + f = -\phi_r r + \phi_{r^*}(r^* + x) + w$$

where lower case letters denote logarithms of variables <u>except</u> for r, r^*, and x. Consequently, α, β, and ϕ are interest rate (semi-)elasticities of demand for the various assets ($\alpha, \beta, \phi > 0$); δ_y is the income elasticity of money demand ($\delta_y > 0$).

With domestic wealth held in domestic money, domestic bonds, and foreign bonds, the balance sheet constraint:

(7) $W = M + B + EF$

along with the assumption of gross asset substitutability imply:

(8a) $\beta_r(B/W) = \phi_r(F/W) + \alpha_r(M/W)$

(8b) $\phi_{r*}(F/W) = \beta_{r*}(B/W) + \alpha_{r*}(M/W)$

The comparative statics of the short-run equilibrium are found by differentiating and solving only two of equations (4) - (6) because of the balance sheet constraint. Differentiating equations (4) and (5) and grouping terms in matrix form yields:

$$(9) \quad \begin{bmatrix} F/W & -\alpha_r \\ F/W & \beta_r \end{bmatrix} \begin{bmatrix} de \\ dr \end{bmatrix} = \begin{bmatrix} 1-M/W & -B/W & -F/W & \alpha_{r*} & -\delta_y \\ -M/W & 1-B/W & -F/W & \beta_{r*} & 0 \end{bmatrix} \begin{bmatrix} dm \\ db \\ df \\ dx \\ dy \end{bmatrix}$$

Solving for de and dr yields:

$$(10) \quad \begin{bmatrix} de \\ dr \end{bmatrix} = (1/[(F/W)(\beta_r + \alpha_r)]) \begin{bmatrix} \beta_r & \alpha_r \\ -F/W & F/W \end{bmatrix} \cdot \begin{bmatrix} (1-M/W)dm - (B/W)db - (F/W)df + \alpha_{r*}dx - \delta_y dy \\ -(M/W)dm + (1-B/W)db - (F/W)df + \beta_{r*}dx \end{bmatrix}$$

The degree of exchange rate response to a monetary or fiscal policy shock can be shown to depend generally upon the interest rate elasticities of demand for the three assets. First, consider a one percent increase in the money supply as a result of an open market domestic bond purchase; hence, $dM = -dB$. Since $dm = -(B/M)db$, the exchange rate response to the open market purchase is:

$$(11) \quad de/dm \Big|_{dM=-dB} = \phi_r/[(B/W)(\beta_r + \alpha_r)]$$

However, equation (11) can be rewritten to show the condition necessary for exchange rate overshooting:

$$(12) \quad de/dm \Big|_{dM=-dB} = [(F/W)\phi_r + (1 - F/W)\phi_r]/[(F/W)\phi_r + (1-F/W)\alpha_r]$$

Assuming that the exchange rate adjusts proportionately to the price level increase and money supply increase in the long run according to the traditional monetary approach ($d\bar{e}/dm = 1$, where $\bar{e}$ is the long-run equilibrium value of e), equation (12) suggests that the short-run exchange rate will overshoot its long-run value if and only if the interest rate elasticity of foreign bond demand (ϕ_r) exceeds the interest rate elasticity of domestic money demand (α_r) -- that is, if foreign bonds are better substitutes for domestic bonds than money.

Second, consider a one percent increase in the money supply resulting from the central bank fully monetizing government debt issued to finance a budget deficit (dm = 1). The exchange rate response is:

$$(13)\quad de/dm = (\beta_r + \phi_r)/(\beta_r + \alpha_r)$$

Again, the necessary condition for overshooting is that the interest rate elasticity of foreign bond demand exceeds the interest rate elasticity of domestic money demand.[2]

Third, consider a one percent increase in the domestic bond supply <u>not</u> monetized at all by the central bank (db = 1). The exchange rate response is:

$$(14)\quad de/db = (\alpha_r - \phi_r)/(\alpha_r + \beta_r)$$

The exchange rate will appreciate (depreciate) as long as the interest rate elasticity of foreign bond demand is greater (less) than the interest rate elasticity of domestic money demand.

Finally, consider a policy shock when exchange rate expectations are <u>not</u> static. Recall x is defined as the expected rate of depreciation of the home currency:

$$x = (\hat{E} - E)/E$$

where E is the current value of the exchange rate and $\hat{E}$ is the current expectation of the exchange rate in the next period. With x no longer exogenous, the comparative static result of a one percent rise in the exchange rate on the expected rate of depreciation of E is:

$$(15)\quad dx/de = (\hat{E}/E)(\psi - 1)$$

where ψ is the elasticity of the expected future exchange rate with respect to the current exchange rate.

Given this modification, equations (4) - (6), (9), and (10) can be altered accordingly. Consider a one percent increase in the money supply as a result of an open market domestic bond purchase; hence, dM = -dB. Since dm = - (B/M)db, the exchange rate response to the open market purchase is:

$$(16)\quad de/dm\Big|_{dM = -dB} = \phi_r/[(B/W)(\beta_r + \alpha_r) - (\psi - 1)(B/F)(x+1)(\beta_r\alpha_{r*} + \alpha_r\beta_{r*})]$$

which, assuming x > -1, is greater (less) than equation (11) if ψ is greater (less) than one.

Crude estimates of ψ were made using generalized least squares regressions of:

$$(17)\quad \ln \hat{E} = \lambda + \psi \ln E + \varepsilon$$

for seven different exchange rates. The three-month forward exchange rate was used as a proxy for the expected rate ($\hat{E}$), E is the spot exchange rate, λ is a constant, ψ is defined above, and ε is the error term which was assumed to follow a first-order autoregressive process. Iterative Cochrane-Orcutt transformations of the variables were made until Durbin-Watson statistics indicated an absence of serial correlation. Table 1 describes the regression results for the seven exchange rates. In six of the seven cases, estimates of ψ were slightly less than one. However, estimated elasticities were significantly different from one (at the 1% statistical significance level) in only two cases. These estimates are consistent with the view that the static expectations assumption ($\psi = 1$) is valid.

TABLE 1

Regression Estimates of the Elasticity of the Expected Future Exchange Rate With Respect to the Current Exchange Rate

Exchange Rate	Constant	Elasticity Estimate	t-statistic Testing Difference from Unity	Durbin-Watson Statistic	R^2	n
U.S. - Germany	-0.00006 (0.00013)	0.986 (0.0049)	-2.836*	2.019	0.988	494
U.S. - Japan	-0.0008 (0.0030)	0.996 (0.0088)	-0.414	1.798	0.965	467
U.S. - U.K.	-0.0002 (0.0001)	1.0008 (0.0076)	1.081	1.974	0.973	494
U.S. - France	-0.0007 (0.0006)	0.994 (0.0103)	-0.618	1.953	0.950	494
U.S. - Canada	-0.00004 (0.00004)	0.999 (0.0085)	-0.160	1.983	0.966	494
U.S. - Switzerland	0.0003 (0.00008)	0.999 (0.0035)	-0.391	2.071	0.994	494
U.S. - Netherlands	-0.0003 (0.0001)	0.975 (0.0062)	-4.064*	2.025	0.981	494

Source: Board of Governors of the Federal Reserve System, Division of International Finance, Macro Data Base.

Notes: Standard errors are in parentheses. Regressions used iterative Cochrane-Orcutt transformations of variables to eliminate serial correlation. Data are weekly observations beginning in July 1973 and ending in December 1982 (except for Japan where observations begin in January 1974). Asterisks denote significantly less than one at the 1% significance level (in two-tail tests); n denotes number of observations. Durbin-Watson statistics all exceeded the upper bound at the 5% significance level (1.69).

Footnotes

[1]Branson's model is discussed in William Branson, Hannu Halttunen, and Paul Masson, "Exchange Rates in the Short-Run," European Economic Review, vol. 10, 1977, pp. 304-324 and William Branson, "Exchange Rate Dynamics and Monetary Policy," in Assar Lindbeck (ed.), Inflation and Employment in Open Economies, Amsterdam: North-Holland Publishing Co., 1979. Because of the similarity of this model and Branson's, stability conditions for this model are treated adequately in those two papers.

[2]These two results are consistent with recent results in the extension by Jacob Frenkel and Carlos Rodriguez ("Exchange Rate Dynamics and the Overshooting Hypothesis," IMF Staff Papers, vol. 29, March 1982, pp. 1-30) of Dornbusch's dynamic monetary model. Frenkel-Rodriguez extended Dornbusch's model to account for finite speeds of adjustment in goods and capital markets. Defining a term (they called) β as the speed of capital market adjustment, β measured the speed at which interest rate parity attained and they considered this a proxy for the degree of capital mobility. They found that when β was large (small) relative to the interest rate semielasticity of money demand, the short-run exchange rate would overshoot (undershoot) its long-run value in response to one percent money supply increase. In our model, overshooting (undershooting) simply results as long as foreign bond demand is more (less) elastic than domestic money demand to changes in the domestic interest rate in response to a one percent money supply increase.