Appendix A

The technique described in the paper, BV-OLS, yields virtually identical gravity equation coefficient estimates to those estimated using region-specific fixed effects (which are unbiased estimates). However, fixed effects cannot be used to generate general equilibrium comparative statics. Because BV-OLS yields linear approximations, it does not provide precise estimates of the region-specific multilateral resistance (MR) terms (with or without borders). However, one need not have to estimate the entire system of equations using custom nonlinear least squares to generate the exact same estimates of the MR terms as with A-vW's NLLS estimation. Given initial estimates of the MR terms using BV-OLS, a version of fixed-point iteration can be used to generate *identical* MR terms as under the NLLS technique, and fixed-point iteration is computationally much less intensive than the A-vW NLLS technique. In particular, even though the system of equations that determines the MR terms is non-linear, our fixed-point iteration method does not require computation of the Jacobian of the system of equations, nor does it require that the inverse of the Jacobian exists. We show that our approach requires nothing more than simple matrix manipulation in *STATA*, *GAUSS*, or any similar matrix programming language.

The approach can be calculated for MR terms with or without borders; for demonstration here, we assume borders are present. First, BV-OLS yields estimates of multilateral resistance terms $P_i^{1-\sigma}$ for i=1,..., N regions (with borders) based upon the log-linear approximation. Denote V_0 as the Nx1 vector of these $P_i^{1-\sigma}$ terms and V_0^- as the Nx1 vector of their inverses ($P_i^{\sigma-1}$). The functional equation we solve is $f(V) = V - BV^-$, where B is an NxN matrix of GDP-share-weighted trade costs where each element, b_{ij}, equals $\theta_j t_{ij}^{1-\sigma}$, where t_{ij} are defined in section 2. Evaluated at the equilibrium values of the MR terms, V^E and V^{-E}, then $f(V^E) = V - BV^{-E} = 0$.

The fixed-point iteration method we use has essentially only two steps. First, use point estimates from BV-OLS to construct B, V_0, and V_0^-. Second, compute V_{k+1} according to:

$$V_{k+1} = zBV_k^{-} + (1-z)V_k \tag{A1}$$

starting a k = 0 until successive approximations are less than some predetermined value of ε (say, 1x10^{-9}), where $\varepsilon = |V_{k+1} - V_k|$ and z is a damping factor with $z \in (0,1)$. The estimated V_{k+1} satisfying this second step is *identical* to the V estimated using A-vW's custom NLLS estimation.

The remainder of this appendix proves in mathematical detail why our version of the fixed-point iteration converges to a solution. First, the standard approach for fixed-point iteration is to start with an initial guess V_0 and iterate on:

$$V_{k+1} = BV_k^{-} \tag{A2}$$

starting at k=0. The above equation converges as long as BV^- is a contraction map; that is, a necessary condition for a fixed-point iteration to converge is that – for each row of the Jacobian of BV^- – the sum of the absolute values of each element is less than unity, cf., Gerald and Wheatley (1990). This condition is unlikely to hold in general and it certainly does not hold for the McCallum-A-vW-Feenstra data. Even if it is a contraction map, it may not be the case that iterating induces convergence to the fixed point.

To see why this iteration process will not work in this context, consider a simple univariate mapping of:

$$v = (1/2)v^{-1} \tag{A3}$$

Trivially, the fixed point of this mapping is $v^E = 1/\sqrt{2}$. Clearly, the Jacobian satisfies the necessary condition for the fixed-point iteration to converge. However, with any initial guess of $v_0 \neq 1/\sqrt{2}$, the iteration produces a periodic cycle. For example, choose $v_0 = 2$ and the "solution" iterates between

$$v_i = \begin{cases} 1/4 \; i \; odd \\ 2 \; i \; even \end{cases}$$

and convergence does not obtain. To induce convergence in this system, we simply add a damping factor z (z = 0.5) and iterate on:

$$v_{k+1} = z(1/2)v_k^{-1} + (1-z)v_k \tag{A4}$$

With an initial estimate of v_0 = 2 for k=0, iterating on (A4) causes convergence of v to the true value (within ten decimal places) in three iterations.

Consequently, to induce convergence in our context, we introduce the damping factor z, where $z \in (0,1)$, and (A2) becomes:

$$V_{k+1} = zBV_k^{-} + (1-z)V_k \tag{A5}$$

Note this implies that $V_{k+1} = V_k - z\,f(V_k)$. For an initial guess in the range of V^E, the fixed-point iteration will converge to $f(V^E) = 0$ if z is contracting (since z is less than unity), cf., Nirenberg (1975). Thus, for the class of models discussed in A-vW the solution to the price terms can be obtained by fixed-point iteration with a damping factor of $z \in (0,1)$. Note how similar this is to the Gauss-Newton iteration scheme discussed in Judd (1998). Unlike the Gauss-Newton iteration, this procedure does not require computing the Jacobian or its inverse, if the latter exists.

We applied this procedure to the McCallum-A-vW-Feenstra Canadian-U.S. data set, using a stopping rule of $\epsilon < 1\text{x}10^{-9}$ for all elements of V. If we use the parameter values in A-vW of $\rho(1-\sigma) = -0.79$ and $\alpha(1-\sigma) = -1.65$, convergence is achieved after 25 iterations (for both cases, with *and* without the border) and the correlation of the multilateral resistance terms with the multilateral resistance terms constructed by A-vW is 1.0 (reported to seven decimal places). If we use the parameter values in BV-OLS of $\rho(1-\sigma) = -1.25$ and $\alpha(1-\sigma) = -1.54$, convergence is achieved after 21 iterations (for both cases, with and without the border) and the correlation of the multilateral resistance terms with the multilateral resistance terms constructed by the A-vW NLLS methodology is 1.0 (reported to seven decimal places). Given that this methodology replicates perfectly the MR terms calculated by A-vW, the comparative statics are identical to those reported by A-vW (2003, 187).