Online Appendices

to

"Increasing Marginal Costs, Firm Heterogeneity, and the Gains from "Deep" International Trade Agreements"

April 1, 2023

A Appendix A

A.1 Pricing Rule and Firm Revenue

As in Feenstra (2010), we let $p_{ij}(\varphi)$ and $q_{ij}(\varphi)$ denote the (free-on-board or fob) price received and the quantity shipped by the firm at the factory gate, respectively. A firm with productivity φ in country i serving country j maximizes profits by choosing the factory-gate price p_{ij}:

$$\max_{p_{ij}} \pi_{ij}(\varphi) = p_{ij}(\varphi)q_{ij}(\varphi) - \tilde{w}_i \left[f_{ij} + \frac{q_{ij}(\varphi)^{\frac{1+\gamma}{\gamma}}}{\varphi} \right]. \tag{A.1}$$

where $\tilde{w}_i \equiv w_i/A_i$. By the definition of iceberg trade costs, we have that the quantity produced after the "iceberg melt" is equal to the quantity consumed: $q_{ij}(\varphi)/\tau_{ij} = c_{ij}(\varphi)$. Furthermore, because firms charge $p_{ij}(\varphi)$ per unit *produced*, consumers pay $p_{ij}^c(\varphi) \equiv \tau_{ij}p_{ij}(\varphi)$ per unit *consumed*. Combining these results and making use of the demand function in equation (2) in the paper, we can express output as:

$$q_{ij}(\varphi) = \tau_{ij}c_{ij}(\varphi) = \tau_{ij}E_jP_j^{\sigma-1}b_i^{1-\sigma}p_{ij}^c(\varphi)^{-\sigma} = E_jP_j^{\sigma-1}\tau_{ij}^{1-\sigma}b_i^{1-\sigma}p_{ij}(\varphi)^{-\sigma}. \tag{A.2}$$

Substituting this last result into equation (A.1) yields

$$\max_{p_{ij}} \pi_{ij}(\varphi) = E_jP_j^{\sigma-1}\tau_{ij}^{1-\sigma}b_i^{1-\sigma}p_{ij}(\varphi)^{1-\sigma} - \tilde{w}_i f_{ij} - \frac{\tilde{w}_i}{\varphi}\left[E_jP_j^{\sigma-1}\tau_{ij}^{1-\sigma}b_i^{1-\sigma}p_{ij}(\varphi)^{-\sigma}\right]^{\frac{1+\gamma}{\gamma}}.$$

Because each firm produces only one of a continuum of varieties, a change in p_{ij} has a negligible effect on the price index P_j. As a result, the first order condition for the profit-maximization problem is:

$$\frac{\partial \pi_{ij}}{\partial p_{ij}} = (1-\sigma)E_jP_j^{\sigma-1}b_i^{1-\sigma}\tau_{ij}^{1-\sigma}p_{ij}(\varphi)^{-\sigma} + \sigma\left(\frac{1+\gamma}{\gamma}\right)\frac{\tilde{w}_i}{\varphi}\left(E_jP_j^{\sigma-1}b_i^{1-\sigma}\tau_{ij}^{1-\sigma}\right)^{\frac{1+\gamma}{\gamma}} p_{ij}(\varphi)^{-\sigma\left(\frac{1+\gamma}{\gamma}\right)-1} = 0,$$

Simplifying the equation above yields:

$$p_{ij}(\varphi) = \left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)\frac{\tilde{w}_i}{\varphi}\left[E_j P_j^{\sigma-1} b_i^{1-\sigma} \tau_{ij}^{1-\sigma} p_{ij}(\varphi)^{-\sigma}\right]^{\frac{1}{\gamma}}.$$

From equation (A.2) we can replace with $q_{ij}(\varphi)$ the last term in the squared brackets in the equation above to obtain the optimal factory-gate price:

$$p_{ij}(\varphi) = \left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)\frac{\tilde{w}_i}{\varphi} q_{ij}(\varphi)^{\frac{1}{\gamma}}. \tag{A.3}$$

We can use this result to derive optimal firm-destination revenue as follows:

$$r_{ij}(\varphi) = p_{ij}(\varphi) q_{ij}(\varphi) = \left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)\frac{\tilde{w}_i q_{ij}(\varphi)^{\frac{1+\gamma}{\gamma}}}{\varphi}. \tag{A.4}$$

As explained earlier, firms charge $p_{ij}(\varphi)$ per unit *produced* such that consumers pay $p_{ij}^c(\varphi) \equiv \tau_{ij} p_{ij}(\varphi)$ per unit *consumed*. From equation (A.3), consumers pay a price per unit consumed of:

$$p_{ij}^c(\varphi) \equiv \tau_{ij} p_{ij}(\varphi) = \left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)\frac{\tilde{w}_i \tau_{ij}}{\varphi} q_{ij}(\varphi)^{\frac{1}{\gamma}}. \tag{A.5}$$

Finally, we note that our solution for optimal consumer price converges to the benchmark result as $\gamma \to \infty$:

$$\lim_{\gamma \to \infty} p_{ij}^c(\varphi) = \left(\frac{\sigma}{\sigma-1}\right)\frac{\tilde{w}_i \tau_{ij}}{\varphi}.$$

A.2 Firm Profits

From equation (A.1), we have:

$$\begin{aligned}
\pi_{ij}(\varphi) &= p_{ij}(\varphi) q_{ij}(\varphi) - \tilde{w}_i \left[f_{ij} + \frac{q_{ij}(\varphi)^{\frac{1+\gamma}{\gamma}}}{\varphi}\right] \\
&= r_{ij}(\varphi) - \tilde{w}_i f_{ij} - \left(\frac{\gamma}{1+\gamma}\right)\left(\frac{\sigma-1}{\sigma}\right)\left[\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)\frac{\tilde{w}_i q_{ij}(\varphi)^{\frac{1+\gamma}{\gamma}}}{\varphi}\right] \\
&= r_{ij}(\varphi) - \tilde{w}_i f_{ij} - \left(\frac{\gamma}{1+\gamma}\right)\left(\frac{\sigma-1}{\sigma}\right) r_{ij}(\varphi) \\
&= \left[1 - \left(\frac{\gamma}{1+\gamma}\right)\left(\frac{\sigma-1}{\sigma}\right)\right] r_{ij}(\varphi) - \tilde{w}_i f_{ij} \\
&= \left(\frac{\sigma+\gamma}{1+\gamma}\right)\frac{r_{ij}(\varphi)}{\sigma} - \tilde{w}_i f_{ij} \qquad \text{(A.6)}
\end{aligned}$$

where the third line uses the definition of optimal revenue in equation (A.4). We note that our solution for profits converges to the benchmark result as $\gamma \to \infty$:

$$\lim_{\gamma \to \infty} \pi_{ij}(\varphi) = \frac{r_{ij}(\varphi)}{\sigma} - \tilde{w}_i f_{ij}.$$

A.3 Cutoff Productivity

Together, the profit function defined in equation (A.1) and the zero-profit condition $\pi_{ij}(\varphi_{ij}^*) = 0$ imply that:

$$\left(\frac{\sigma+\gamma}{1+\gamma}\right) \frac{r_{ij}(\varphi_{ij}^*)}{\sigma} = \tilde{w}_i f_{ij}. \tag{A.7}$$

Substituting into this last equation optimal revenue, as defined in equation (A.4), yields:

$$\left(\frac{\sigma+\gamma}{1+\gamma}\right)\left(\frac{1}{\sigma}\right)\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right) \frac{\tilde{w}_i q_{ij}(\varphi_{ij}^*)^{\frac{1+\gamma}{\gamma}}}{\varphi_{ij}^*} = \tilde{w}_i f_{ij}, \tag{A.8}$$

which, after rearranging, yields an expression for the optimal output of the cutoff firm:

$$q_{ij}(\varphi_{ij}^*) = \left[\left(\frac{\gamma}{\sigma+\gamma}\right)(\sigma-1) f_{ij} \varphi_{ij}^*\right]^{\frac{\gamma}{1+\gamma}}. \tag{A.9}$$

We can substitute this last result into equation (A.3) to obtain an expression for the optimal factory-gate price for the cutoff firm:

$$\begin{aligned} p_{ij}(\varphi_{ij}^*) &= \left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right) \frac{\tilde{w}_i}{\varphi_{ij}^*} \left[\left(\frac{\gamma}{\sigma+\gamma}\right)(\sigma-1) f_{ij} \varphi_{ij}^*\right]^{\frac{1}{1+\gamma}} \\ &= \left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)\left[\left(\frac{\gamma}{\sigma+\gamma}\right)(\sigma-1) f_{ij}\right]^{\frac{1}{1+\gamma}} \tilde{w}_i (\varphi_{ij}^*)^{\frac{-\gamma}{1+\gamma}}. \end{aligned} \tag{A.10}$$

From equation (A.2), we can express firm revenue as:

$$r_{ij}(\varphi) = p_{ij}(\varphi) q_{ij}(\varphi) = E_j P_j^{\sigma-1} b_i^{1-\sigma} \tau_{ij}^{1-\sigma} p_{ij}(\varphi)^{1-\sigma}.$$

Using this last result, we can express the zero-profit condition in equation (A.7) as:

$$\left(\frac{\sigma+\gamma}{1+\gamma}\right) \frac{E_j P_j^{\sigma-1} b_i^{1-\sigma} \tau_{ij}^{1-\sigma} p_{ij}(\varphi_{ij}^*)^{1-\sigma}}{\sigma} = \tilde{w}_i f_{ij}. \tag{A.11}$$

Substituting for the factory-gate price in equation (A.11) using equation (A.10), we can solve for the zero-cutoff-profit productivity:

$$\tilde{w}_i f_{ij} = \left(\frac{\sigma+\gamma}{1+\gamma}\right) \frac{E_j P_j^{\sigma-1} b_i^{1-\sigma} \tau_{ij}^{1-\sigma} \left\{ \left(\frac{1+\gamma}{\gamma}\right) \left(\frac{\sigma}{\sigma-1}\right) \left[\left(\frac{\gamma}{\sigma+\gamma}\right)(\sigma-1) f_{ij}\right]^{\frac{1}{1+\gamma}} \tilde{w}_i (\varphi_{ij}^*)^{\frac{-\gamma}{1+\gamma}} \right\}^{1-\sigma}}{\sigma}$$

$$\Rightarrow (\varphi_{ij}^*)^{(\sigma-1)\left(\frac{\gamma}{1+\gamma}\right)} = \left(\frac{1+\gamma}{\sigma+\gamma}\right) \left(\frac{\sigma \tilde{w}_i f_{ij}}{E_j P_j^{\sigma-1} b_i^{1-\sigma} \tau_{ij}^{1-\sigma}}\right) \left[\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right) \tilde{w}_i\right]^{\sigma-1} \left[\left(\frac{\gamma}{\sigma+\gamma}\right)(\sigma-1) f_{ij}\right]^{\frac{\sigma-1}{1+\gamma}}$$

$$\Rightarrow \varphi_{ij}^* = \left[\frac{\left(\frac{1+\gamma}{\gamma}\frac{\sigma}{\sigma-1}\tilde{w}_i\right)^{\sigma}}{E_j P_j^{\sigma-1} b_i^{1-\sigma}}\right]^{\frac{1}{\frac{\gamma}{1+\gamma}(\sigma-1)}} \left[\frac{\gamma}{\sigma+\gamma}(\sigma-1) f_{ij}\right]^{\frac{\frac{1+\gamma}{\gamma}}{\frac{1+\gamma}{\sigma+\gamma}(\sigma-1)}} \tau_{ij}^{\frac{1+\gamma}{\gamma}}. \qquad \text{(A.12)}$$

Again, when $\gamma \to \infty$ we obtain the benchmark result:

$$\lim_{\gamma\to\infty} \varphi_{ij}^* = \left[\left(\frac{\sigma}{\sigma-1}\right)^{\sigma} (\sigma-1) \frac{f_{ij}\tilde{w}_i^{\sigma}}{E_j P_j^{\sigma-1} b_i^{1-\sigma}}\right]^{\frac{1}{\sigma-1}} \tau_{ij} = \frac{\sigma^{1+\frac{1}{\sigma-1}} \tilde{w}_i^{1+\frac{1}{\sigma-1}} f_{ij}^{\frac{1}{\sigma-1}} b_i \tau_{ij}}{(\sigma-1) E_i^{\frac{1}{\sigma-1}} P_j}$$

$$= \left(\frac{\sigma}{\sigma-1}\right) \frac{\tilde{w}_i b_i \tau_{ij}}{P_j} \left(\frac{\sigma w_i f_{ij}}{E_j}\right)^{\frac{1}{\sigma-1}}.$$

A.4 Average Profits

In our model, the relationship between the relative revenues of two firms in country i serving the domestic market and their relative productivities is similar to – but nontrivially different from – the constant marginal cost case. From equation (A.2) and the pricing rule (A.5), we can express the ratio of output between any firm and the cutoff firm as follows

$$\frac{q_{ij}(\varphi)}{q_{ij}(\varphi_{ij}^*)} = \left(\frac{\varphi}{\varphi_{ij}^*}\right)^{\sigma\left(\frac{\gamma}{\sigma+\gamma}\right)}, \qquad \text{(A.13)}$$

which differs from the constant marginal cost case because of the extra term in the exponent (i.e., $\gamma/(\sigma+\gamma)$). However, when $\gamma \to \infty$ the result is the same as in Melitz (2003). Using equation (A.3) to define the ratio of prices and multiplying by the ratio of quantities to obtain relative revenues yields:

$$\frac{r_{ij}(\varphi)}{r_{ij}(\varphi_{ij}^*)} = \frac{p_{ij}(\varphi)}{p_{ij}(\varphi_{ij}^*)} \times \frac{q_{ij}(\varphi)}{q_{ij}(\varphi_{ij}^*)} = \left[\frac{q_{ij}(\varphi)^{\frac{1}{\gamma}}/\varphi}{q_{ij}(\varphi_{ij}^*)^{\frac{1}{\gamma}}/\varphi_{ij}^*}\right] \left[\frac{q_{ij}(\varphi)}{q_{ij}(\varphi_{ij}^*)}\right] = \left(\frac{\varphi}{\varphi_{ij}^*}\right)^{(\sigma-1)\left(\frac{\gamma}{\sigma+\gamma}\right)} \qquad \text{(A.14)}$$

where the last equality follows from equation (A.13). Note that when $\gamma \to \infty$, the relationship is identical to the constant marginal cost case. The sufficient condition here for a positive relationship between productivity and revenue is $\sigma\left(\frac{1+\gamma}{\sigma+\gamma}\right) > 1$, instead of the typical assumption $\sigma > 1$.

From the zero-profit condition $\pi_{ij}(\varphi_{ij}^*) = 0$ and the definition of profits in equation (A.6), we have:

$$\pi_{ij}(\varphi_{ij}^*) = 0 \quad \Leftrightarrow \quad r_{ij}(\varphi_{ij}^*) = \left(\frac{1+\gamma}{\sigma+\gamma}\right)\sigma\tilde{w}_i f_{ij}. \tag{A.15}$$

Using this result and equation (A.14), we obtain:

$$r_{ij}(\varphi) = \left(\frac{\varphi}{\varphi_{ij}^*}\right)^{(\sigma-1)\left(\frac{\gamma}{\sigma+\gamma}\right)} r_{ij}(\varphi_{ij}^*) = \left(\frac{1+\gamma}{\sigma+\gamma}\right)\left(\frac{\varphi}{\varphi_{ij}^*}\right)^{(\sigma-1)\left(\frac{\gamma}{\sigma+\gamma}\right)} \sigma\tilde{w}_i f_{ij}, \tag{A.16}$$

which shows clearly that firm revenue is increasing in firm productivity. Using this last result, we can express average revenue for a country i firm selling to country j as:

$$\begin{aligned}
\overline{r}_{ij}(\varphi_{ij}^*) &= \int_{\varphi_{ij}^*}^{\infty} r_{ij}(\varphi)\mu_{ij}(\varphi)d\varphi \\
&= \left(\frac{1+\gamma}{\sigma+\gamma}\right)\left(\frac{1}{\varphi_{ij}^*}\right)^{(\sigma-1)\left(\frac{\gamma}{\sigma+\gamma}\right)} \sigma\tilde{w}_i f_{ij} \int_{\varphi_{ij}^*}^{\infty} \varphi^{(\sigma-1)\left(\frac{\gamma}{\sigma+\gamma}\right)}\mu_{ij}(\varphi)d\varphi \\
&= \left(\frac{1+\gamma}{\sigma+\gamma}\right)\left[\frac{\tilde{\varphi}_{ij}(\varphi_{ij}^*)}{\varphi_{ij}^*}\right]^{(\sigma-1)\left(\frac{\gamma}{\sigma+\gamma}\right)} \sigma\tilde{w}_i f_{ij}
\end{aligned} \tag{A.17}$$

where

$$\mu_{ij}\left(\varphi\right) = \begin{cases} \frac{g(\varphi)}{1-G\left(\varphi_{ij}^*\right)} = \theta(\varphi_{ij}^*)^{\theta}\varphi^{-\theta-1}, & \text{if } \varphi \geq \varphi_{ij}^*, \\ 0 & \text{otherwise} \end{cases} \tag{A.18}$$

is the Pareto distribution of firm productivity, and

$$\tilde{\varphi}_{ij}(\varphi_{ij}^*) = \left[\int_{\varphi_{ij}^*}^{\infty} \varphi^{(\sigma-1)\left(\frac{\gamma}{\sigma+\gamma}\right)}\mu_{ij}(\varphi)d\varphi\right]^{\left(\frac{1}{\sigma-1}\right)\frac{\sigma+\gamma}{\gamma}}. \tag{A.19}$$

defines an aggregate productivity level as a function of the cutoff level φ_{ij}^*.

Using equation (A.19), we can define average profit for each destination market as follows:

$$\begin{aligned}\overline{\pi}_{ij}(\varphi^*_{ij}) &= \int_{\varphi^*_{ij}}^{\infty} \pi_{ij}(\varphi)\mu_{ij}(\varphi)d\varphi = \int_{\varphi^*_{ij}}^{\infty} \left[\left(\frac{\sigma+\gamma}{1+\gamma}\right)\frac{r_{ij}(\varphi)}{\sigma} - \tilde{w}_i f_{ij}\right]\mu_{ij}(\varphi)d\varphi \\ &= \left(\frac{\sigma+\gamma}{1+\gamma}\right)\int_{\varphi^*_{ij}}^{\infty}\frac{r_{ij}(\varphi)}{\sigma}\mu_{ij}(\varphi)d\varphi - \tilde{w}_i f_{ij} = \left(\frac{\sigma+\gamma}{1+\gamma}\right)\frac{\overline{r}_{ij}(\varphi^*_{ij})}{\sigma} - \tilde{w}_i f_{ij} \\ &= \left\{\left[\frac{\tilde{\varphi}_{ij}(\varphi^*_{ij})}{\varphi^*_{ij}}\right]^{(\sigma-1)\left(\frac{\gamma}{\sigma+\gamma}\right)} - 1\right\}\tilde{w}_i f_{ij}. \end{aligned} \tag{A.20}$$

This result is analogous to the zero-cutoff-profit condition in Melitz (2003), with $\overline{\pi}_{ij}$ a negative function of φ^*_{ij}. The nontrivial difference is the necessary condition that $\sigma\left(\frac{1+\gamma}{\sigma+\gamma}\right) > 1$.

By definition, the average profit of an incumbent firm is the sum of the average profits from sales to all markets:

$$\overline{\pi}_i = \sum_{j=1}^{N}\left[\frac{1-G(\varphi^*_{ij})}{1-G(\varphi^*_{ii})}\right]\overline{\pi}_{ij}(\varphi^*_{ij}) = \sum_{j=1}^{N}\left(\frac{\varphi^*_{ij}}{\varphi^*_{ii}}\right)^{-\theta}\overline{\pi}_{ij}(\varphi^*_{ij}), \tag{A.21}$$

where the last equality follows from the Pareto distribution assumption. This expression includes domestic profits (i.e., when $i=j$). Using equation (A.20) in (A.21), we can express average total firm profit (under the Pareto distribution assumption) as:

$$\overline{\pi}_i = \sum_{j=1}^{N}\left(\frac{\varphi^*_{ij}}{\varphi^*_{ii}}\right)^{-\theta}\left\{\left[\frac{\tilde{\varphi}_{ij}(\varphi^*_{ij})}{\varphi^*_{ij}}\right]^{(\sigma-1)\left(\frac{\gamma}{\sigma+\gamma}\right)} - 1\right\}\tilde{w}_i f_{ij}. \tag{A.22}$$

We can further simplify this expression using the definition of average productivity in equation (A.19), which implies that:

$$\begin{aligned}\left[\tilde{\varphi}_{ij}(\varphi^*_{ij})\right]^{(\sigma-1)\left(\frac{\gamma}{\sigma+\gamma}\right)} &= \int_{\varphi^*_{ij}}^{\infty}\varphi^{(\sigma-1)\left(\frac{\gamma}{\sigma+\gamma}\right)}\mu_{ij}(\varphi)d\varphi = \int_{\varphi^*_{ij}}^{\infty}\varphi^{(\sigma-1)\left(\frac{\gamma}{\sigma+\gamma}\right)}\frac{\theta\varphi^{-\theta-1}}{(\varphi^*_{ij})^{-\theta}}d\varphi \\ &= \theta(\varphi^*_{ij})^{\theta}\int_{\varphi^*_{ij}}^{\infty}\varphi^{\sigma\left(\frac{\gamma+1}{\sigma+\gamma}\right)-\theta-2}d\varphi = \left[\frac{\theta}{\theta-(\sigma-1)\left(\frac{\gamma}{\sigma+\gamma}\right)}\right](\varphi^*_{ij})^{(\sigma-1)\left(\frac{\gamma}{\sigma+\gamma}\right)}. \end{aligned} \tag{A.23}$$

Using this last result in equation (A.22) yields:

$$\overline{\pi}_i = \sum_{j=1}^{N} \left(\frac{\varphi_{ij}^*}{\varphi_{ii}^*}\right)^{-\theta} \left\{ \left[\frac{\theta}{\theta - (\sigma-1)\left(\frac{\gamma}{\sigma+\gamma}\right)} \right] - 1 \right\} \tilde{w}_i f_{ij} = \frac{(\sigma-1)\left(\frac{\gamma}{\sigma+\gamma}\right)}{\theta - (\sigma-1)\left(\frac{\gamma}{\sigma+\gamma}\right)} \sum_{j=1}^{N} \left(\frac{\varphi_{ii}^*}{\varphi_{ij}^*}\right)^{\theta} \tilde{w}_i f_{ij}. \tag{A.24}$$

A.5 Masses of Firms

Consumers have no taste for leisure, so the supply of labor is fixed at L_i. There are three sources of demand for labor: labor for entry costs (f^e), labor for fixed trade costs (f_{ij}), and labor for production. Therefore, the labor-market-clearing condition is given by:

$$L_i = \frac{M_i^e f^e}{A_i} + \sum_{j=1}^{N} M_{ij} \int_{\varphi_{ij}^*}^{\infty} \frac{1}{A_i} \left[f_{ij} + \frac{q_{ij}(\varphi)^{\frac{1+\gamma}{\gamma}}}{\varphi} \right] \mu_{ij}(\varphi) d\varphi, \tag{A.25}$$

where M_i^e is the mass of firms attempting to enter the industry in country i, M_{ij} is the mass of firms based in i that serve market j, and

$$\mu_{ij}(\varphi) = \begin{cases} \frac{g(\varphi)}{1-G(\varphi_{ij}^*)} = \theta(\varphi_{ij}^*)^{\theta}\varphi^{-\theta-1}, & \text{if } \varphi \geq \varphi_{ij}^*, \\ 0 & \text{otherwise} \end{cases} \tag{A.26}$$

is the Pareto distribution of firms' productivities.

Multiplying both sides of equation (A.25) by w_i, yields:

$$w_i L_i = \tilde{w}_i M_i^e f^e + \tilde{w}_i \sum_{j=1}^{N} M_{ij} f_{ij} + \tilde{w}_i \sum_{j=1}^{N} M_{ij} \int_{\varphi_{ij}^*}^{\infty} \frac{q_{ij}(\varphi)^{\frac{1+\gamma}{\gamma}}}{\varphi} \mu_{ij}(\varphi) d\varphi. \tag{A.27}$$

From the optimal revenue equation (A.4), we can show that:

$$\frac{\tilde{w}_i q_{ij}(\varphi)^{\frac{1+\gamma}{\gamma}}}{\varphi} = \left(\frac{\gamma}{1+\gamma}\right)\left(\frac{\sigma-1}{\sigma}\right) r_{ij}(\varphi).$$

Using this result in equation (A.27) yields:

$$w_i L_i = \tilde{w}_i M_i^e f^e + \tilde{w}_i \sum_{j=1}^{N} M_{ij} f_{ij} + \left(\frac{\gamma}{1+\gamma}\right)\left(\frac{\sigma-1}{\sigma}\right) \sum_{j=1}^{N} M_{ij} \int_{\varphi_{ij}^*}^{\infty} r_{ij}(\varphi)\mu_{ij}(\varphi) d\varphi. \tag{A.28}$$

As in Feenstra (2010) and Redding (2011), zero expected profits imply that aggregate revenue is equal to expenditure such that:

$$w_i L_i = \sum_{j=1}^{N} M_{ij} \int_{\varphi_{ij}^*}^{\infty} r_{ij}(\varphi)\mu_{ij}(\varphi)d\varphi. \tag{A.29}$$

Substituting with this result for the last term on the right-hand-side of equation (A.28) yields:

$$\begin{aligned} w_i L_i &= \tilde{w}_i M_i^e f^e + \tilde{w}_i \sum_{j=1}^{N} M_{ij} f_{ij} + \left(\frac{\gamma}{1+\gamma}\right)\left(\frac{\sigma-1}{\sigma}\right) w_i L_i \\ \Leftrightarrow \quad \left[1 - \left(\frac{\gamma}{1+\gamma}\right)\left(\frac{\sigma-1}{\sigma}\right)\right] w_i L_i &= \tilde{w}_i M_i^e f^e + \tilde{w}_i \sum_{j=1}^{N} M_{ij} f_{ij}. \end{aligned} \tag{A.30}$$

Substituting the left-hand-side of equation (A.30) for the first two terms on the right-hand-side of equation (A.27) yields:

$$\begin{aligned} w_i L_i &= \left[1 - \left(\frac{\gamma}{1+\gamma}\right)\left(\frac{\sigma-1}{\sigma}\right)\right] w_i L_i + \tilde{w}_i \sum_{j=1}^{N} M_{ij} \int_{\varphi_{ij}^*}^{\infty} \frac{q_{ij}(\varphi)^{\frac{1+\gamma}{\gamma}}}{\varphi} \mu_{ij}(\varphi)d\varphi \\ \Leftrightarrow \quad \left(\frac{\gamma}{1+\gamma}\right)\left(\frac{\sigma-1}{\sigma}\right) w_i L_i &= \tilde{w}_i \sum_{j=1}^{N} M_{ij} \int_{\varphi_{ij}^*}^{\infty} \frac{q_{ij}(\varphi)^{\frac{1+\gamma}{\gamma}}}{\varphi} \mu_{ij}(\varphi)d\varphi \end{aligned} \tag{A.31}$$

We can now solve for $\tilde{w}_i \sum_{j=1}^{N} M_{ij} f_{ij}$. From equation (A.13), we can express the output for any firm as a function of the output of the cutoff firm as follows:

$$q_{ij}(\varphi) = \left(\frac{\varphi}{\varphi_{ij}^*}\right)^{\frac{\sigma\gamma}{\sigma+\gamma}} q_{ij}(\varphi_{ij}^*). \tag{A.32}$$

Using this result and the Pareto distribution, we can solve the integral on the right-hand-side of equation (A.31):

$$\int_{\varphi_{ij}^*}^{\infty} \frac{q(\varphi)^{\frac{1+\gamma}{\gamma}}}{\varphi} \mu_{ij}(\varphi) d\varphi = \int_{\varphi_{ij}^*}^{\infty} \frac{\left[q(\varphi_{ij}^*)\left(\frac{\varphi}{\varphi_{ij}^*}\right)^{\frac{\sigma\gamma}{\sigma+\gamma}}\right]^{\frac{1+\gamma}{\gamma}}}{\varphi} \mu_{ij}(\varphi) d\varphi$$

$$= \int_{\varphi_{ij}^*}^{\infty} \frac{q(\varphi_{ij}^*)^{\frac{1+\gamma}{\gamma}} \left(\frac{\varphi}{\varphi_{ij}^*}\right)^{\sigma\left(\frac{1+\gamma}{\sigma+\gamma}\right)}}{\varphi} \left[\frac{\theta\varphi^{-\theta-1}}{(\varphi_{ij}^*)^{-\theta}}\right] d\varphi$$

$$= q(\varphi_{ij}^*)^{\frac{1+\gamma}{\gamma}} \left(\frac{1}{\varphi_{ij}^*}\right)^{\sigma\frac{1+\gamma}{\sigma+\gamma}-\theta} \theta \int_{\varphi_{ij}^*}^{\infty} \varphi^{\frac{\gamma}{\sigma+\gamma}(\sigma-1)-(\theta+1)} d\varphi$$

$$= q(\varphi_{ij}^*)^{\frac{1+\gamma}{\gamma}} \left(\frac{1}{\varphi_{ij}^*}\right)^{\sigma\frac{1+\gamma}{\sigma+\gamma}-\theta} \left[\frac{\theta}{\theta - \frac{\gamma}{\sigma+\gamma}(\sigma-1)}\right] \left[\left(\frac{1}{\infty}\right)^{\theta-\frac{\gamma}{\sigma+\gamma}(\sigma-1)} - (\varphi_{ij}^*)^{\frac{\gamma}{\sigma+\gamma}(\sigma-1)}\right]$$

$$= \left[\frac{\theta}{\theta - \frac{\gamma}{\sigma+\gamma}(\sigma-1)}\right] q(\varphi_{ij}^*)^{\frac{1+\gamma}{\gamma}} (\varphi_{ij}^*)^{\frac{\gamma}{\sigma+\gamma}(\sigma-1)-\sigma\frac{1+\gamma}{\sigma+\gamma}}$$

$$= \left[\frac{\theta}{\theta - (\sigma-1)\left(\frac{\gamma}{\sigma+\gamma}\right)}\right] \frac{q(\varphi_{ij}^*)^{\frac{1+\gamma}{\gamma}}}{\varphi_{ij}^*}. \tag{A.33}$$

Importantly, note that, for a positive integral, we require only that $\theta > \frac{\gamma}{\sigma+\gamma}(\sigma-1)$ and not $\theta > \sigma - 1$, as in the standard constant marginal cost Melitz models.

Rearranging equation (A.9), we can show that:

$$\frac{q_{ij}(\varphi_{ij}^*)^{\frac{1+\gamma}{\gamma}}}{\varphi_{ij}^*} = \left(\frac{\gamma}{\sigma+\gamma}\right)(\sigma-1) f_{ij}.$$

Using this result in the equation just above it yields:

$$\int_{\varphi_{ij}^*}^{\infty} \frac{q_{ij}(\varphi)^{\frac{1+\gamma}{\gamma}}}{\varphi} \mu_{ij}(\varphi) d\varphi = \left[\frac{\theta\left(\frac{\gamma}{\sigma+\gamma}\right)(\sigma-1)}{\theta - \frac{\gamma}{\sigma+\gamma}(\sigma-1)}\right] f_{ij}, \tag{A.34}$$

which implies that:

$$\tilde{w}_i \sum_{j=1}^{N} M_{ij} \int_{\varphi_{ij}^*}^{\infty} \frac{q_{ij}(\varphi)^{\frac{1+\gamma}{\gamma}}}{\varphi} \mu_{ij}(\varphi) d\varphi = \left[\frac{\theta\left(\frac{\gamma}{\sigma+\gamma}\right)(\sigma-1)}{\theta - \frac{\gamma}{\sigma+\gamma}(\sigma-1)}\right] \tilde{w}_i \sum_{j=1}^{N} M_{ij} f_{ij}. \tag{A.35}$$

Substituting with this last result into equation (A.31) yields:

$$\left(\frac{\gamma}{1+\gamma}\right)\left(\frac{\sigma-1}{\sigma}\right)w_iL_i = \left[\frac{\theta\left(\frac{\gamma}{\sigma+\gamma}\right)(\sigma-1)}{\theta-\frac{\gamma}{\sigma+\gamma}(\sigma-1)}\right]\tilde{w}_i\sum_{j=1}^{N}M_{ij}f_{ij}$$

$$\Rightarrow \quad \tilde{w}_i\sum_{j=1}^{N}M_{ij}f_{ij} = \left(\frac{\gamma}{1+\gamma}\right)\left(\frac{\sigma-1}{\sigma}\right)\left[\frac{\theta-\frac{\gamma}{\sigma+\gamma}(\sigma-1)}{\theta\left(\frac{\gamma}{\sigma+\gamma}\right)(\sigma-1)}\right]w_iL_i. \tag{A.36}$$

Substituting this result into equation (A.35) yields:

$$\tilde{w}_i\sum_{j=1}^{N}M_{ij}\int_{\varphi_{ij}^*}^{\infty}\frac{q_{ij}(\varphi)}{\varphi}^{\frac{1+\gamma}{\gamma}}\mu_{ij}(\varphi)d\varphi = \left(\frac{\gamma}{1+\gamma}\right)\left(\frac{\sigma-1}{\sigma}\right)w_iL_i. \tag{A.37}$$

We can now solve for M_i^e. Substituting equations (A.36) and (A.37) into equation (A.27), and eliminating out the w_i, yields:

$$\begin{aligned}
L_i &= \frac{1}{A_i}M_i^ef^e + \frac{1}{A_i}\sum_{j=1}^{N}M_{ij}f_{ij} + \frac{1}{A_i}\sum_{j=1}^{N}M_{ij}\int_{\varphi_{ij}^*}^{\infty}\frac{q_{ij}(\varphi)}{\varphi}^{\frac{1+\gamma}{\gamma}}\mu_{ij}(\varphi)d\varphi \\
&= \frac{M_i^ef^e}{A_i} + \left(\frac{\gamma}{1+\gamma}\right)\left(\frac{\sigma-1}{\sigma}\right)\left[\frac{\theta-\frac{\gamma}{\sigma+\gamma}(\sigma-1)}{\theta\left(\frac{\gamma}{\sigma+\gamma}\right)(\sigma-1)}\right]L_i + \left(\frac{\gamma}{1+\gamma}\right)\left(\frac{\sigma-1}{\sigma}\right)L_i \\
&= \frac{M_i^ef^e}{A_i} + \left(\frac{\gamma}{1+\gamma}\right)\left(\frac{\sigma-1}{\sigma}\right)\left[1+\frac{\theta-\frac{\gamma}{\sigma+\gamma}(\sigma-1)}{\theta\left(\frac{\gamma}{\sigma+\gamma}\right)(\sigma-1)}\right]L_i \\
&= \frac{M_i^ef^e}{A_i} + \left(\frac{\gamma}{1+\gamma}\right)\left(\frac{\sigma-1}{\sigma}\right)\left[\frac{\theta\left(\frac{\gamma}{\sigma+\gamma}\right)(\sigma-1)+\theta-\frac{\gamma}{\sigma+\gamma}(\sigma-1)}{\theta\left(\frac{\gamma}{\sigma+\gamma}\right)(\sigma-1)}\right]L_i \\
&= \frac{M_i^ef^e}{A_i} + \left[\frac{(\theta-1)\left(\frac{\gamma}{\sigma+\gamma}\right)(\sigma-1)+\theta}{\theta\sigma\left(\frac{1+\gamma}{\sigma+\gamma}\right)}\right]L_i
\end{aligned}$$

which implies that:

$$
\begin{aligned}
M_i^e &= \left[1 - \frac{(\theta-1)\left(\frac{\gamma}{\sigma+\gamma}\right)(\sigma-1)+\theta}{\theta\sigma\left(\frac{1+\gamma}{\sigma+\gamma}\right)}\right]\frac{A_iL_i}{f^e} \\
&= \left[\frac{\theta\sigma\left(\frac{1+\gamma}{\sigma+\gamma}\right)-(\theta-1)\left(\frac{\gamma}{\sigma+\gamma}\right)(\sigma-1)-\theta}{\theta\sigma\left(\frac{1+\gamma}{\sigma+\gamma}\right)}\right]\frac{A_iL_i}{f^e} \\
&= \left(\frac{1}{\theta\sigma}\right)\left(\frac{\sigma+\gamma}{1+\gamma}\right)\left(\frac{\theta\sigma+\theta\sigma\gamma-\theta\sigma\gamma+\theta\gamma+\gamma\sigma-\gamma-\theta\gamma-\theta\sigma}{\sigma+\gamma}\right)\frac{A_iL_i}{f^e} \\
&= \left(\frac{\gamma}{1+\gamma}\right)\left(\frac{\sigma-1}{\sigma}\right)\frac{A_iL_i}{\theta f^e}. \qquad \text{(A.38)}
\end{aligned}
$$

We now solve for M_{ii}. As standard, we assume a fraction δ of existing firms M_{ii} exit the industry. In a steady state equilibrium, the mass of new entrant (M_i^e) must replace firms hit by the exogenous shock and forced to exit the industry. Hence, in a steady state:

$$[1-G(\varphi_{ii}^*)]M_i^e = \delta M_{ii} \qquad \text{(A.39)}$$

where $[1-G(\varphi_{ii}^*)] = (\varphi_{ii}^*)^{-\theta}$ is the probability of successful entry. It follows that:

$$M_{ii} = \frac{[1-G(\varphi_{ii}^*)]M_i^e}{\delta} = \frac{M_i^e}{\delta(\varphi_{ii}^*)^\theta} = \left(\frac{\gamma}{1+\gamma}\right)\left(\frac{\sigma-1}{\sigma}\right)\frac{A_iL_i}{\theta\delta f^e(\varphi_{ii}^*)^\theta} \qquad \text{(A.40)}$$

Finally, we can solve for the mass of exporting firms M_{ij}. A successful entrant in country-i will export to country j if it is is productive enough to be profitable in the foreign country. This implies that:

$$M_{ij} = \left[\frac{1-G(\varphi_{ij}^*)}{1-G(\varphi_{ii}^*)}\right]M_{ii} = \left(\frac{\gamma}{1+\gamma}\right)\left(\frac{\sigma-1}{\sigma}\right)\frac{A_iL_i}{\theta\delta f^e(\varphi_{ij}^*)^\theta}. \qquad \text{(A.41)}$$

A.6 Price Index

In this section, we solve for the price index. Substituting equation (A.2) into optimal pricing rule (A.5) we obtain:

$$
\begin{aligned}
p_{ij}^{c}(\varphi) &= \left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)\frac{\tilde{w}_i\tau_{ij}}{\varphi}q_{ij}(\varphi)^{\frac{1}{\gamma}} \\
&= \left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)\frac{\tilde{w}_i\tau_{ij}}{\varphi}\left[E_j P_j^{\sigma-1} b_i^{1-\sigma} p_{ij}^{c}(\varphi)^{-\sigma}\right]^{\frac{1}{\gamma}} \\
&= \left[\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)\frac{\tilde{w}_i\tau_{ij}}{\varphi}\right]^{\frac{\gamma}{\sigma+\gamma}} E_j^{\frac{1}{\sigma+\gamma}} P_j^{\frac{\sigma-1}{\sigma+\gamma}} b_i^{\frac{1-\sigma}{\sigma+\gamma}} \qquad \text{(A.42)}
\end{aligned}
$$

Substituting this result into the definition of the price index

$$
P_j = \left[\int_{\nu\in\Omega_j} b_i^{1-\sigma} p_j^{c}(\nu)^{1-\sigma} d\nu\right]^{\frac{1}{1-\sigma}}, \qquad \text{(A.43)}
$$

and rearranging, we obtain:

$$
\begin{aligned}
P_j^{1-\sigma} &= \int_{\nu\in\Omega_j} b_i^{1-\sigma} p_j^{c}(\nu)^{1-\sigma} d\nu = \sum_i M_{ij}\int_{\varphi_{ij}^{*}}^{\infty} b_i^{1-\sigma} p_{ij}^{c}(\varphi)^{1-\sigma}\mu_{ij}(\varphi)d\varphi \\
&= \sum_i M_{ij}\int_{\varphi_{ij}^{*}}^{\infty}\left\{\left[\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)\frac{\tilde{w}_i\tau_{ij}}{\varphi}\right]^{\frac{\gamma}{\sigma+\gamma}} E_j^{\frac{1}{\sigma+\gamma}} P_j^{\frac{\sigma-1}{\sigma+\gamma}} b_i^{\frac{1-\sigma}{\sigma+\gamma}}\right\}^{1-\sigma}\mu_{ij}(\varphi)d\varphi \\
&= \sum_i M_{ij}\left\{\left[\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)\tilde{w}_i\tau_{ij}\right]^{\frac{\gamma}{\sigma+\gamma}} E_j^{\frac{1}{\sigma+\gamma}} P_j^{\frac{\sigma-1}{\sigma+\gamma}} b_i^{\frac{1-\sigma}{\sigma+\gamma}}\right\}^{1-\sigma}\int_{\varphi_{ij}^{*}}^{\infty}\varphi^{(\sigma-1)\left(\frac{\gamma}{\sigma+\gamma}\right)}\mu_{ij}(\varphi)d\varphi \\
&= \sum_i M_{ij}\left\{\left[\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)\frac{\tilde{w}_i\tau_{ij}}{\varphi_{ij}^{*}}\right]^{\frac{\gamma}{\sigma+\gamma}} E_j^{\frac{1}{\sigma+\gamma}} P_j^{\frac{\sigma-1}{\sigma+\gamma}} b_i^{\frac{1-\sigma}{\sigma+\gamma}}\right\}^{1-\sigma}\left[\frac{\theta}{\theta-(\sigma-1)\left(\frac{\gamma}{\sigma+\gamma}\right)}\right] \\
&= \left[\frac{\theta}{\theta-(\sigma-1)\left(\frac{\gamma}{\sigma+\gamma}\right)}\right]\sum_i M_{ij} b_i^{1-\sigma}[p_{ij}^{c}(\varphi_{ij}^{*})]^{1-\sigma} \\
&= \left[\frac{\theta}{\theta-(\sigma-1)\left(\frac{\gamma}{\sigma+\gamma}\right)}\right]\sum_i M_{ij}\tau_{ij}^{1-\sigma} b_i^{1-\sigma}[p_{ij}(\varphi_{ij}^{*})]^{1-\sigma}. \qquad \text{(A.44)}
\end{aligned}
$$

We can use the productivity cutoff in equation (A.12) and the mass of firms in equation (A.41) to obtain an expression also for the price index P_j as a function of the endogenous wages and parameters of the model, given in equation (A.87) below.

A.7 Trade Flows

Using the pricing rule (A.3), the result in equation (A.34), and equation (A.41) for the mass of firms, we can express trade flows as:

$$
\begin{aligned}
X_{ij} &\equiv M_{ij}\int_{\varphi_{ij}^*}^{\infty} r_{ij}(\varphi)\mu_{ij}(\varphi)d\varphi = M_{ij}\int_{\varphi_{ij}^*}^{\infty} p_{ij}(\varphi)q_{ij}(\varphi)\mu_{ij}(\varphi)d\varphi \\
&= M_{ij}\int_{\varphi_{ij}^*}^{\infty}\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)\frac{\tilde{w}_i}{\varphi}q_{ij}(\varphi)^{\frac{1+\gamma}{\gamma}}\mu_{ij}(\varphi)d\varphi \\
&= M_{ij}\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)\tilde{w}_i\int_{\varphi_{ij}^*}^{\infty}\frac{q_{ij}(\varphi)^{\frac{1+\gamma}{\gamma}}}{\varphi}\mu_{ij}(\varphi)d\varphi \\
&= M_{ij}\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)\left[\frac{\theta\left(\frac{\gamma}{\sigma+\gamma}\right)(\sigma-1)}{\theta-\frac{\gamma}{\sigma+\gamma}(\sigma-1)}\right]\tilde{w}_i f_{ij} \\
&= \left(\frac{\gamma}{1+\gamma}\right)\left(\frac{\sigma-1}{\sigma}\right)\frac{A_i L_i}{\theta\delta f^e(\varphi_{ij}^*)^{\theta}}\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)\left[\frac{\theta\left(\frac{\gamma}{\sigma+\gamma}\right)(\sigma-1)}{\theta-\frac{\gamma}{\sigma+\gamma}(\sigma-1)}\right]\frac{w_i f_{ij}}{A_i} \\
&= \left[\frac{(\sigma-1)\left(\frac{\gamma}{\sigma+\gamma}\right)}{\theta-(\sigma-1)\left(\frac{\gamma}{\sigma+\gamma}\right)}\right]\frac{w_i L_i f_{ij}}{\delta f^e(\varphi_{ij}^*)^{\theta}}.
\end{aligned}
\tag{A.45}
$$

By definition, aggregate expenditure in country j is given by:

$$
E_j = \sum_k X_{kj} = \left[\frac{(\sigma-1)\left(\frac{\gamma}{\sigma+\gamma}\right)}{\theta-(\sigma-1)\left(\frac{\gamma}{\sigma+\gamma}\right)}\right]\frac{1}{\delta f^e}\sum_k w_k L_k f_{kj}(\varphi_{kj}^*)^{-\theta}. \tag{A.46}
$$

Therefore, the share of country j's expenditure on goods supplied by country i is given by:

$$
\lambda_{ij} \equiv \frac{X_{ij}}{E_j} = \frac{w_i L_i f_{ij}(\varphi_{ij}^*)^{-\theta}}{\sum_{k=1}^{N} w_k L_k f_{kj}(\varphi_{kj}^*)^{-\theta}}. \tag{A.47}
$$

Adapting equation (A.12), we know:

$$
\varphi_{kj}^* = \left[\frac{\left(\frac{1+\gamma}{\gamma}\frac{\sigma}{\sigma-1}\tilde{w}_k\right)^{\sigma}}{E_j P_j^{\sigma-1} b_i^{1-\sigma}}\right]^{\frac{1}{\frac{\gamma}{1+\gamma}(\sigma-1)}}\left[\frac{\gamma}{\sigma+\gamma}(\sigma-1)f_{kj}\right]^{\frac{\frac{1+\gamma}{\gamma}}{\frac{1+\gamma}{\sigma+\gamma}(\sigma-1)}}\tau_{kj}^{\frac{1+\gamma}{\gamma}}.
$$

Substituting this equation for φ_{kj}^* and equation (A.12) for φ_{ij}^* into equation (A.47), we obtain bilateral trade from i to j as a share of j's expenditures (λ_{ij}):

$$\lambda_{ij} = \frac{w_i L_i f_{ij} \left[\tilde{w}_i^{\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)} b_i^{\frac{1+\gamma}{\gamma}} \tau_{ij}^{\frac{1+\gamma}{\gamma}} f_{ij}^{\left(\frac{1}{\frac{\gamma}{\sigma+\gamma}(\sigma-1)}\right)} \right]^{-\theta}}{\sum_{k=1}^{N} w_k L_k f_{kj} \left[\tilde{w}_k^{\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)} b_k^{\frac{1+\gamma}{\gamma}} \tau_{kj}^{\frac{1+\gamma}{\gamma}} f_{kj}^{\left(\frac{1}{\frac{\gamma}{\sigma+\gamma}(\sigma-1)}\right)} \right]^{-\theta}}$$

$$= \frac{A_i^{\theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)} L_i w_i^{1-\theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)} b_i^{-\theta\left(\frac{1+\gamma}{\gamma}\right)} \tau_{ij}^{-\theta\left(\frac{1+\gamma}{\gamma}\right)} f_{ij}^{1-\frac{\theta\left(\frac{1+\gamma}{\gamma}\right)}{\frac{1+\gamma}{\sigma+\gamma}(\sigma-1)}}}{\sum_{k=1}^{N} A_k^{\theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)} L_k w_k^{1-\theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)} b_k^{-\theta\left(\frac{1+\gamma}{\gamma}\right)} \tau_{kj}^{-\theta\left(\frac{1+\gamma}{\gamma}\right)} f_{kj}^{1-\frac{\theta\left(\frac{1+\gamma}{\gamma}\right)}{\frac{1+\gamma}{\sigma+\gamma}(\sigma-1)}}}. \tag{A.48}$$

A.8 Wage Rates

We now determine aggregate revenue in equilibrium. First, total payments to production workers, which we denote L_i^p, must be equal to the difference between aggregate revenue and aggregate profit such that $w_i L_i^p = R_i - \Pi_i$, where $\Pi_i \equiv M_{ii}\overline{\pi}_i$. Second, in equilibrium, the mass of successful entrants must be equal to the mass of firms forced to exit the industry. This aggregate stability condition requires that $[1 - G(\varphi_{ij}^*)]M_i^e = \delta M_{ii}$. Combining this last result with the free entry condition (A.53) (provided later) implies that total payments to labor used in entry equal total profits: $w_i L_i^e = w_i M_i^e f^e = \Pi_i$. It follows that aggregate revenue, which is the sum of total payments to labor and profits, is equal to payroll $R_i = w_i L_i^p + \Pi_i = w_i L_i$.

The equilibrium wage rate (w_i) in each country can be determined from the requirement that total revenue equals total expenditure on goods produced there:

$$w_i L_i = \sum_{j=1}^{N} \lambda_{ij} w_j L_j.$$

Substituting in equation (A.48) yields the following system of N equations (one for each of N countries):

$$w_i L_i = \sum_{j=1}^{N} \left[\frac{A_i^{\theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)} L_i w_i^{1-\theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)} \tau_{ij}^{-\theta\left(\frac{1+\gamma}{\gamma}\right)} f_{ij}^{1-\frac{\theta\left(\frac{1+\gamma}{\gamma}\right)}{\frac{1+\gamma}{\sigma+\gamma}(\sigma-1)}}}{\sum_{k=1}^{N} A_k^{\theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)} L_k w_k^{1-\theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)} b_k^{-\theta\frac{1+\gamma}{\gamma}} \tau_{kj}^{-\theta\left(\frac{1+\gamma}{\gamma}\right)} f_{kj}^{1-\frac{\theta\left(\frac{1+\gamma}{\gamma}\right)}{\frac{1+\gamma}{\sigma+\gamma}(\sigma-1)}}} \right] w_j L_j \tag{A.49}$$

Equation (A.49) implies a system of N equations in the N unknown wage rates in each country, w_i. Note that this equation takes the same form as equation (3.14) on p. 1734 of

Alvarez and Lucas (2007). Using equation (A.49), we can define the following excess demand system:

$$\Xi(\boldsymbol{w}) = \frac{1}{w_i}\left[\sum_{j=1}^{N} \frac{A_i^{\theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)} L_i w_i^{1-\theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)} b_i^{-\theta\frac{1+\gamma}{\gamma}} \tau_{ij}^{-\theta\left(\frac{1+\gamma}{\gamma}\right)} f_{ij}^{1-\frac{\theta\left(\frac{1+\gamma}{\gamma}\right)}{\frac{1+\gamma}{\sigma+\gamma}(\sigma-1)}}}{\sum_{k=1}^{N} A_k^{\theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)} L_k w_k^{1-\theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)} b_k^{-\theta\frac{1+\gamma}{\gamma}} \tau_{kj}^{-\theta\left(\frac{1+\gamma}{\gamma}\right)} f_{kj}^{1-\frac{\theta\left(\frac{1+\gamma}{\gamma}\right)}{\frac{1+\gamma}{\sigma+\gamma}(\sigma-1)}}} - w_i L_i\right] \tag{A.50}$$

where $\boldsymbol{w}$ denotes the vector of wage rates across countries.

Proposition 1. *There exists a unique wage-rate vector* $\boldsymbol{w} \in \mathbb{R}^N_{++}$ *such that* $\Xi(\boldsymbol{w}) = 0$.

Proof. Note that $\Xi(\boldsymbol{w})$ has the following properties:

1. $\Xi(\boldsymbol{w})$ is continuous (by assumption on the parameters).
2. $\Xi(\boldsymbol{w})$ is homogenous of degree zero.
3. $\boldsymbol{w} \cdot \Xi(\boldsymbol{w}) = 0$ for all $\boldsymbol{w} \in \mathbb{R}^N_{++}$ (Walras Law).
4. There exists a constant $s > 0$ such that $\Xi_i(\boldsymbol{w}) > -s$ for each country i and all $\boldsymbol{w} \in \mathbb{R}^N_{++}$.
5. If $\boldsymbol{w}^m \to \boldsymbol{w}^0$ where $\boldsymbol{w}^0 \neq 0$ and $w_i^0 = 0$ for some country i, then $\max_j\{\Xi_j(\boldsymbol{w})\} \to \infty$.
6. $\Xi(\boldsymbol{w})$ satisfies the gross substitutes property

$$\frac{\partial \Xi_i(\boldsymbol{w})}{\partial w_j} > 0,\ i \neq j, \quad \text{and} \quad \frac{\partial \Xi_i(\boldsymbol{w})}{\partial w_i} < 0, \forall\, \boldsymbol{w} \in \mathbb{R}^N_{++}.$$

Under these conditions, Propositions 17.C.1 and 17.F.3 of Mas-Colell et al. (1995) or Theorems 1-3 of Alvarez and Lucas (2007) hold, such that there exists a unique vector of wage rates $\boldsymbol{w} \in \mathbb{R}^N_{++}$ that satisfies the clearing conditions $\Xi(\boldsymbol{w}) = 0$. □

A.9 Free Entry

There is an unbounded set of potential entrants in the industry. To enter the industry, firms must incur a fixed entry cost of f^e units of labor. That sunk entry cost provides the firm with a blue print for a unique variety and also reveals the firm's productivity, φ, a random draw from a common distribution $G(\varphi)$. Once the fixed entry cost is paid, firms can begin production.

The value of a successful entrant with productivity φ is equal to the discounted sum of lifetime profits. Following Melitz (2003), we assume that each period there is a probability

$\delta \in (0,1)$ that an incumbent firm will be hit by an adverse shock and be forced to exit the industry. In that case, the value of a successful entrant in the industry can be expressed as:

$$V_i(\varphi) = \sum_{t=1}^{\infty}(1-\delta)^t \pi_{it}(\varphi) = \frac{\pi_i(\varphi)}{\delta}, \tag{A.51}$$

where the second equality follows from the fact that profits are constant throughout the lifetime of the firm, i.e., $\pi_{it}(\varphi) = \pi_i(\varphi)$. Therefore, the value of entry as a function of productivity is given by:

$$V_i(\varphi) = \max\left\{0, \frac{\pi_i(\varphi)}{\delta}\right\}. \tag{A.52}$$

Firms with productivity above the domestic cutoff, φ_{ii}^*, will generate enough variable profits to cover the fixed costs. As a result, they stay in the industry and earn a lifetime profit proportional to their per-period profits. Firms with productivity lower than the domestic cutoff would earn negative profits if they remain in the industry. Hence, they prefer to exit the industry and get a null return.

In a free entry equilibrium, the expected value of entry, V_i^e, must be equal to the cost of entry such that:

$$V_i^e = [1 - G(\varphi_{ii}^*)]\frac{\overline{\pi}_i}{\delta} = \tilde{w}_i f^e. \tag{A.53}$$

The expected value of entry is defined as the product of the probability of successful entry, $1 - G(\varphi_{ii}^*)$, and the lifetime profits of the average incumbent firm, $\overline{\pi}_i/\delta$. The cost of entry is defined as the product of $\tilde{w}_i$ and the fixed entry cost, f^e, defined in units of labor.

By definition, the average profit of an incumbent firm is the sum of the average profits from sales to each market (including the domestic market) multiplied by the probability of entering each market conditional on producing for the domestic market:

$$\overline{\pi}_i = \sum_{j=1}^{N}\left[\frac{1 - G(\varphi_{ij}^*)}{1 - G(\varphi_{ii}^*)}\right]\overline{\pi}_{ij}(\varphi_{ij}^*). \tag{A.54}$$

To obtain an analytical solution, we follow the literature and assume that the productivity distribution is Pareto, such that $G(\varphi) = 1 - \varphi^{-\theta}$. We can combine the zero-cutoff-profit condition $\pi_{ij}(\varphi_{ij}^*) = 0$, the optimal pricing function in equation (6), and the definition of profits in equation (5), to express average total firm profit as:

$$\overline{\pi}_i = \frac{(\sigma-1)\frac{\gamma}{\sigma+\gamma}}{\theta - (\sigma-1)\frac{\gamma}{\sigma+\gamma}} \sum_{j=1}^{N}\left(\frac{\varphi_{ii}^*}{\varphi_{ij}^*}\right)^{\theta} \tilde{w}_i f_{ij}. \tag{A.55}$$

Substituting this last result for average profits into equation (A.53), we obtain an expression for the free-entry condition that depends only on the productivity cutoffs and parameters of the model:

$$V_i^e = \frac{(\sigma-1)\frac{\gamma}{\sigma+\gamma}}{\theta-(\sigma-1)\frac{\gamma}{\sigma+\gamma}} \sum_{j=1}^{N} \frac{f_{ij}}{(\varphi_{ij}^*)^\theta} = \delta f^e, \tag{A.56}$$

where the wage rates have canceled in the expression above. This result shows that the value of entry is proportionate to fixed entry costs (f^e).

A.10 General Equilibrium

As in Bernard et al. (2011), we determine general equilibrium using the recursive structure of the model. The system of equations (A.50) determines a unique equilibrium wage in each country (w_i). Furthermore, the mass of entrants M_i^e is determined as a function of parameters in equation (A.38). With these two equilibrium components, we can solve for all the other endogenous variables as follows. The price index P_j follows from the wage rate as explained in section A.6. The productivity cutoffs then follow from equation (9), the wage rates, the price indexes, and that $E_i = R_i = w_i L_i$ in equilibrium. The mass of firms in each country i serving each destination country j, M_{ij}, follows from equation (11) and the productivity cutoffs. Finally, the trade shares λ_{ij} follow directly from equation (A.47), the wage rates, and the productivity cutoffs. This completes the characterization of the general equilibrium.

A.11 Structural Gravity

In this section, we show how to derive the structural gravity equation from our theoretical model. Substituting equation (A.12) for the ZCP productivity threshold in the solution for bilateral trade flows in equation (A.45), we can solve for:

$$X_{ij} = BA_i^{\theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)} L_i \left(E_j P_j^{\sigma-1}\right)^{\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\theta}{\sigma-1}\right)} w_i^{1-\theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)} b_i^{-\theta\left(\frac{1+\gamma}{\gamma}\right)} \tau_{ij}^{-\theta\left(\frac{1+\gamma}{\gamma}\right)} f_{ij}^{1-\frac{\theta\frac{1+\gamma}{\gamma}}{\frac{1+\gamma}{\sigma+\gamma}(\sigma-1)}}. \tag{A.57}$$

where B is a constant and a function of parameters $\sigma, \gamma, \theta, \delta$, and f^e. By the definition of revenue, it follows that:

$$R_i = \sum_{j=1}^{N} X_{ij} = BA_i^{\theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)} b_i^{-\theta\left(\frac{1+\gamma}{\gamma}\right)} L_i w_i^{1-\theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)} \tilde{\Pi}_i^{-\theta\left(\frac{1+\gamma}{\gamma}\right)} \tag{A.58}$$

where

$$\tilde{\Pi}_i^{-\theta\left(\frac{1+\gamma}{\gamma}\right)} \equiv \tilde{\Pi}_i^{-\varepsilon_\tau} = \sum_{j=1}^{N} \left(E_j^{\frac{1}{\sigma-1}} P_j\right)^{\theta\left(\frac{1+\gamma}{\gamma}\right)} \phi_{ij} \equiv \sum_{j=1}^{N} \left(E_j^{\frac{1}{\sigma-1}} P_j\right)^{\varepsilon_\tau} \phi_{ij} \tag{A.59}$$

and

$$\phi_{ij} \equiv \tau_{ij}^{-\theta\left(\frac{1+\gamma}{\gamma}\right)} f_{ij}^{1-\frac{\theta\frac{1+\gamma}{\gamma}}{\frac{1+\gamma}{\sigma+\gamma}(\sigma-1)}}.$$

Rearranging equation (A.58) to solve for $R_i\tilde{\Pi}_i^{\varepsilon_\tau}$ (where $\varepsilon_\tau \equiv \theta\frac{1+\gamma}{\gamma}$) yields:

$$R_i\tilde{\Pi}_i^{\varepsilon_\tau} = BA_i^{\theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)} b_i^{-\theta\left(\frac{1+\gamma}{\gamma}\right)} L_i w_i^{1-\theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)}. \tag{A.60}$$

Substituting the LHS term from above for the RHS term into equation (A.57) yields:

$$X_{ij} = \frac{R_i}{\tilde{\Pi}_i^{-\varepsilon_\tau}} \left(E_j^{\frac{1}{\sigma-1}} P_j\right)^{\varepsilon_\tau} \phi_{ij}. \tag{A.61}$$

We can define $\tilde{\Phi}_j$ such that:

$$E_j\tilde{\Phi}_j^{\varepsilon_\tau} \equiv \left(E_j^{\frac{1}{\sigma-1}} P_j\right)^{\varepsilon_\tau}. \tag{A.62}$$

Substituting the LHS term for the RHS term in equation (A.61) yields:

$$X_{ij} = \frac{R_i}{\tilde{\Pi}_i^{-\varepsilon_\tau}} \frac{E_j}{\tilde{\Phi}_j^{-\varepsilon_\tau}} \phi_{ij}. \tag{A.63}$$

Using the definition of $\tilde{\Phi}_j$ in equation (A.62), we can rewrite the multilateral resistance term $\tilde{\Pi}_i$ using equation (A.59) as follows:

$$\tilde{\Pi}_i^{-\varepsilon_\tau} = \sum_{j=1}^{N} \frac{E_j}{\tilde{\Phi}_j^{-\varepsilon_\tau}} \phi_{ij}. \tag{A.64}$$

Finally, by definition of expenditure and equation (A.63) it follows that:

$$E_j = \sum_{i=1}^{N} X_{ij} = \sum_{i=1}^{N} \frac{R_i}{\tilde{\Pi}_i^{-\varepsilon_\tau}} \frac{E_j}{\tilde{\Phi}_j^{-\varepsilon_\tau}} \phi_{ij}. \tag{A.65}$$

This result implies that:

$$\tilde{\Phi}_j^{-\varepsilon_\tau} = \sum_{i=1}^{N} \frac{R_i}{\tilde{\Pi}_i^{-\varepsilon_\tau}} \phi_{ij}. \tag{A.66}$$

If we define $\Pi_i \equiv \tilde{\Pi}_i^{-\varepsilon_\tau}$ and $\Phi_j \equiv \tilde{\Phi}_j^{-\varepsilon_\tau}$, the system of equations (A.63), (A.64), and (A.66) forms a structural gravity-equation equivalent to equation (2) in Head and Mayer (2014).

A.12 Elasticity of Trade with respect to *Ad Valorem* Variable Trade Costs

First, we determine the elasticity of trade with respect to *ad valorem* variable trade costs. By definition, aggregate bilateral trade flows are given by:

$$X_{ij} \equiv M_{ij} \int_{\varphi_{ij}^*}^{\infty} r_{ij}(\varphi)\mu_{ij}(\varphi)d\varphi = M_{ij}[1 - G(\varphi_{ij}^*)]^{-1} \int_{\varphi_{ij}^*}^{\infty} r_{ij}(\varphi)g(\varphi)d\varphi. \tag{A.67}$$

It follows that:

$$\begin{aligned} \frac{\partial X_{ij}}{\partial \tau_{ij}} = \frac{\partial M_{ij}}{\partial \tau_{ij}} \frac{X_{ij}}{M_{ij}} &+ M_{ij}[1 - G(\varphi_{ij}^*)]^{-2} \frac{\partial G(\varphi_{ij}^*)}{\partial \varphi} \frac{\partial \varphi_{ij}^*}{\partial \tau_{ij}} [1 - G(\varphi_{ij}^*)] \frac{X_{ij}}{M_{ij}} \\ &- M_{ij}[1 - G(\varphi_{ij}^*)]^{-1} r_{ij}(\varphi_{ij}^*) g(\varphi_{ij}^*) \frac{\partial \varphi_{ij}^*}{\partial \tau_{ij}} \\ &+ M_{ij}[1 - G(\varphi_{ij}^*)]^{-1} \int_{\varphi_{ij}^*}^{\infty} \frac{\partial r_{ij}(\varphi)}{\partial \tau_{ij}} g(\varphi)d\varphi. \end{aligned} \tag{A.68}$$

From this last result, it is straightforward to define the elasticity as follows:

$$\begin{aligned} \varepsilon_\tau \equiv -\frac{\partial X_{ij}}{\partial \tau_{ij}} \frac{\tau_{ij}}{X_{ij}} = \frac{\partial M_{ij}}{\partial \tau_{ij}} \frac{X_{ij}}{M_{ij}} \frac{\tau_{ij}}{X_{ij}} &+ M_{ij}[1 - G(\varphi_{ij}^*)]^{-2} \frac{\partial G(\varphi_{ij}^*)}{\partial \varphi} \frac{\partial \varphi_{ij}^*}{\partial \tau_{ij}} [1 - G(\varphi_{ij}^*)] \frac{X_{ij}}{M_{ij}} \frac{\tau_{ij}}{X_{ij}} \\ &- M_{ij} \frac{\tau_{ij}}{X_{ij}} [1 - G(\varphi_{ij}^*)]^{-1} r_{ij}(\varphi_{ij}^*) g(\varphi_{ij}^*) \frac{\partial \varphi_{ij}^*}{\partial \tau_{ij}} \\ &+ M_{ij} \frac{\tau_{ij}}{X_{ij}} [1 - G(\varphi_{ij}^*)]^{-1} \int_{\varphi_{ij}^*}^{\infty} \frac{\partial r_{ij}(\varphi)}{\partial \tau_{ij}} g(\varphi)d\varphi \\ = -\Bigg\{ \underbrace{\frac{\partial M_{ij}}{\partial \tau_{ij}} \frac{\tau_{ij}}{M_{ij}}}_{\text{extensive}} &+ \underbrace{\frac{g(\varphi_{ij}^*)\varphi_{ij}^*}{1 - G(\varphi_{ij}^*)} \left[1 - \frac{r_{ij}(\varphi_{ij}^*)}{X_{ij}/M_{ij}}\right] \frac{\partial \varphi_{ij}^*}{\partial \tau_{ij}} \frac{\tau_{ij}}{\varphi_{ij}^*}}_{\text{compositional}} \\ &+ \underbrace{\int_{\varphi_{ij}^*}^{\infty} \frac{\partial r_{ij}(\varphi)}{\partial \tau_{ij}} \frac{\tau_{ij}}{X_{ij}/M_{ij}} \mu_{ij}(\varphi)d\varphi}_{\text{intensive}} \Bigg\}, \end{aligned} \tag{A.69}$$

where the last equality follows from simplifying and rearranging terms.

We now calculate each component of equation (A.69) separately. From equation (9), we have:

$$\frac{\partial \varphi_{ij}^*}{\partial \tau_{ij}} = \left(\frac{1+\gamma}{\gamma}\right) \frac{\varphi_{ij}^*}{\tau_{ij}}, \tag{A.70}$$

which implies that:

$$\frac{\partial \varphi_{ij}^*}{\partial \tau_{ij}} \frac{\tau_{ij}}{\varphi_{ij}^*} = \frac{1+\gamma}{\gamma}. \tag{A.71}$$

Using equations (11) and (A.70), we have:

$$\frac{\partial M_{ij}}{\partial \tau_{ij}} = -\theta \left(\frac{M_{ij}}{\varphi_{ij}^*}\right) \frac{\partial \varphi_{ij}^*}{\partial \tau_{ij}} = -\theta \left(\frac{M_{ij}}{\varphi_{ij}^*}\right) \left(\frac{1+\gamma}{\gamma}\right) \frac{\varphi_{ij}^*}{\tau_{ij}} = -\theta \left(\frac{1+\gamma}{\gamma}\right) \frac{M_{ij}}{\tau_{ij}}.$$

This last result implies that:

$$\frac{\partial M_{ij}}{\partial \tau_{ij}} \frac{\tau_{ij}}{M_{ij}} = -\theta \left(\frac{1+\gamma}{\gamma}\right). \tag{A.72}$$

Under the Pareto distribution assumption it follows that:

$$\frac{g(\varphi_{ij}^*)\varphi_{ij}^*}{1-G(\varphi_{ij}^*)} = \frac{\theta(\varphi_{ij}^*)^{-\theta-1}\varphi_{ij}^*}{(\varphi_{ij}^*)^{-\theta}} = \theta, \tag{A.73}$$

where the last equality uses equation (A.70).

Next, using the solution for the equilibrium mass of firms in equation (11) and cutoff-firm revenue:

$$r_{ij}(\varphi_{ij}^*) = \left(\frac{1+\gamma}{\sigma+\gamma}\right) \sigma \tilde{w}_i f_{ij}, \tag{A.74}$$

which, as shown in section A.4, is obtained from the zero profit condition, we can show that:

$$1 - \frac{r_{ij}(\varphi_{ij}^*)}{X_{ij}/M_{ij}} = 1 - \frac{1}{\theta}\left[\theta - \left(\frac{\gamma}{\sigma+\gamma}\right)(\sigma-1)\right] = \frac{1}{\theta}\left(\frac{\gamma}{\sigma+\gamma}\right)(\sigma-1). \tag{A.75}$$

Finally, as shown in section A.4, it is possible to express firm revenue as a function of the cutoff productivity as follows:

$$r_{ij}(\varphi) = \left(\frac{\varphi}{\varphi_{ij}^*}\right)^{(\sigma-1)\frac{\gamma}{\sigma+\gamma}} r_{ij}(\varphi_{ij}^*) = \left(\frac{\varphi}{\varphi_{ij}^*}\right)^{(\sigma-1)\frac{\gamma}{\sigma+\gamma}} \sigma \tilde{w}_i f_{ij}.$$

Using this result, we get:

$$\frac{\partial r_{ij}(\varphi_{ij})}{\partial \tau_{ij}} = -\left[\sigma\left(\frac{1+\gamma}{\sigma+\gamma}\right) - 1\right] \frac{r_{ij}(\varphi_{ij})}{\varphi_{ij}^*} \frac{\partial \varphi_{ij}^*}{\partial \tau_{ij}} = -(\sigma-1)\left(\frac{1+\gamma}{\sigma+\gamma}\right) \frac{r_{ij}(\varphi_{ij})}{\tau_{ij}}. \tag{A.76}$$

It then follows that:

$$\begin{aligned}\int_{\varphi_{ij}^*}^{\infty} \frac{\partial r_{ij}(\varphi)}{\partial \tau_{ij}} \frac{\tau_{ij}}{X_{ij}/M_{ij}} \mu_{ij}(\varphi) d\varphi &= -\int_{\varphi_{ij}^*}^{\infty} (\sigma-1)\left(\frac{1+\gamma}{\sigma+\gamma}\right) \frac{r_{ij}(\varphi_{ij})}{\tau_{ij}} \frac{\tau_{ij}}{X_{ij}/M_{ij}} \mu_{ij}(\varphi) d\varphi \\ &= -(\sigma-1)\left(\frac{1+\gamma}{\sigma+\gamma}\right)\left(\frac{1}{X_{ij}}\right) M_{ij} \int_{\varphi_{ij}^*}^{\infty} r_{ij}(\varphi_{ij}) \mu_{ij}(\varphi) d\varphi \\ &= -(\sigma-1)\left(\frac{1+\gamma}{\sigma+\gamma}\right) \frac{X_{ij}}{X_{ij}} = -(\sigma-1)\left(\frac{1+\gamma}{\sigma+\gamma}\right). \end{aligned} \tag{A.77}$$

Substituting results (A.71), (A.72), (A.73), (A.75) and (A.77) into equation (A.69), we get:

$$\begin{aligned}\varepsilon_\tau &= -\left[\underbrace{-\theta\left(\frac{1+\gamma}{\gamma}\right)}_{\text{extensive}} + \underbrace{(1-\sigma)\left(\frac{1+\gamma}{\sigma+\gamma}\right)}_{\text{intensive}} + \underbrace{(\sigma-1)\left(\frac{1+\gamma}{\sigma+\gamma}\right)}_{\text{compositional}}\right] \\ &= \theta\left(\frac{1+\gamma}{\gamma}\right) = \theta\left(1+\frac{1}{\gamma}\right),\end{aligned}$$

which is the result in the paper.

A.13 Elasticity of Trade with respect to Fixed Trade Costs

The computations for the fixed-trade-cost trade elasticity are similar to those for the *ad valorem* variable-trade-cost trade elasticity. From equation (A.67), we get:

$$\begin{aligned}\frac{\partial X_{ij}}{\partial f_{ij}} = \frac{\partial M_{ij}}{\partial f_{ij}} \frac{X_{ij}}{M_{ij}} &+ M_{ij}[1-G(\varphi_{ij}^*)]^{-2} \frac{\partial G(\varphi_{ij}^*)}{\partial \varphi} \frac{\partial \varphi_{ij}^*}{\partial f_{ij}} [1-G(\varphi_{ij}^*)] \frac{X_{ij}}{M_{ij}} \\ &- M_{ij}[1-G(\varphi_{ij}^*)]^{-1} r_{ij}(\varphi_{ij}^*) g(\varphi_{ij}^*) \frac{\partial \varphi_{ij}^*}{\partial f_{ij}} \\ &+ M_{ij}[1-G(\varphi_{ij}^*)]^{-1} \int_{\varphi_{ij}^*}^{\infty} \frac{\partial r_{ij}(\varphi)}{\partial f_{ij}} g(\varphi) d\varphi,\end{aligned} \tag{A.78}$$

such that

$$\begin{aligned}\varepsilon_f \equiv -\frac{\partial X_{ij}}{\partial f_{ij}} \frac{f_{ij}}{X_{ij}} = -\Bigg\{ &\underbrace{\frac{\partial M_{ij}}{\partial f_{ij}} \frac{f_{ij}}{M_{ij}}}_{\text{extensive}} + \underbrace{\frac{g(\varphi_{ij}^*)\varphi_{ij}^*}{1-G(\varphi_{ij}^*)}\left[1-\frac{r_{ij}(\varphi_{ij}^*)}{X_{ij}/M_{ij}}\right] \frac{\partial \varphi_{ij}^*}{\partial f_{ij}} \frac{f_{ij}}{\varphi_{ij}^*}}_{\text{compositional}} \\ &+ \underbrace{\frac{f_{ij}}{X_{ij}/M_{ij}} \int_{\varphi_{ij}^*}^{\infty} \frac{\partial r_{ij}(\varphi)}{\partial f_{ij}} \mu_{ij}(\varphi) d\varphi}_{\text{intensive}} \Bigg\}.\end{aligned} \tag{A.79}$$

Some of the "components" of this last result are the same as those in equation (A.69). So, we calculate only the new components of equation (A.79). First, from equation (9), we have:

$$\frac{\partial \varphi_{ij}^*}{\partial f_{ij}} \frac{f_{ij}}{\varphi_{ij}^*} = \left(\frac{\sigma+\gamma}{\gamma}\right)\left(\frac{1}{\sigma-1}\right). \tag{A.80}$$

Using this result and equations (A.73) and (A.75), it follows that the compositional margin defined in (A.79) simplifies to 1:

$$\begin{aligned} &\frac{g(\varphi_{ij}^*)\varphi_{ij}^*}{1-G(\varphi_{ij}^*)}\left[1-\frac{r_{ij}(\varphi_{ij}^*)}{X_{ij}/M_{ij}}\right]\frac{\partial \varphi_{ij}^*}{\partial f_{ij}}\frac{f_{ij}}{\varphi_{ij}^*} \\ &\qquad = \theta\left[1-1+\frac{1}{\theta}\left(\frac{\gamma}{\sigma+\gamma}\right)(\sigma-1)\right]\left(\frac{\sigma+\gamma}{\gamma}\right)\left(\frac{1}{\sigma-1}\right) = 1. \end{aligned} \tag{A.81}$$

Next, using the definition of firm-level revenue in equation (A.76), we can show that:

$$\frac{\partial r_{ij}(\varphi_{ij})}{\partial f_{ij}} = 0. \tag{A.82}$$

This result implies that the intensive-margin component of the elasticity in (A.79) is equal to 0. Finally, from the equilibrium mass of firms in equation (11), we have:

$$\begin{aligned} \frac{\partial M_{ij}}{\partial f_{ij}} &= -\theta\left(\frac{M_{ij}}{\varphi_{ij}^*}\right)\frac{\partial \varphi_{ij}^*}{\partial f_{ij}} = -\theta\left(\frac{\sigma+\gamma}{\gamma}\right)\left(\frac{1}{\sigma-1}\right)\left(\frac{M_{ij}}{\varphi_{ij}^*}\right)\frac{\varphi_{ij}^*}{f_{ij}} \\ &= -\theta\left(\frac{\sigma+\gamma}{\gamma}\right)\left(\frac{1}{\sigma-1}\right)\frac{M_{ij}}{f_{ij}}. \end{aligned}$$

This last result implies that:

$$\frac{\partial M_{ij}}{\partial f_{ij}}\frac{f_{ij}}{M_{ij}} = -\theta\left(\frac{\sigma+\gamma}{\gamma}\right)\left(\frac{1}{\sigma-1}\right). \tag{A.83}$$

Substituting equations (A.81), (A.82), and (A.83) into equation (A.69), we get:

$$\varepsilon_f = -\left[\underbrace{-\frac{\theta}{\frac{\gamma}{\sigma+\gamma}(\sigma-1)}}_{\text{extensive}} + \underbrace{0}_{\text{intensive}} + \underbrace{1}_{\text{compositional}}\right] = \frac{\theta}{\frac{\gamma}{\sigma+\gamma}(\sigma-1)} - 1 = \frac{\theta\frac{1+\gamma}{\gamma}}{\frac{1+\gamma}{\sigma+\gamma}(\sigma-1)} - 1,$$

which is the result in the paper.

A.14 Welfare

In the model, welfare (W_j) is equal to purchasing power. Letting the consumption aggregate $C_j \equiv U_j$, then by definition of the ideal price index it follows that:

$$P_j C_j = w_j \quad \Leftrightarrow \quad W_j = \frac{w_j}{P_j}. \tag{A.84}$$

To compute welfare, we need to define each term of W_j. We begin with the price index.

From the zero-profit condition $\pi_{ij}(\varphi_{ij}^*) = 0$ and the definition of profits in equation (A.6), we have:

$$\left(\frac{\sigma+\gamma}{1+\gamma}\right)\frac{r_{ij}(\varphi_{ij}^*)}{\sigma} = \tilde{w}_i f_{ij}.$$

Substituting demand function (A.2) into the equation above for $r_{ij}(\varphi_{ij}^*)$ yields:

$$\left(\frac{\sigma+\gamma}{1+\gamma}\right)\frac{E_j P_j^{\sigma-1} b_i^{1-\sigma} p_{ij}(\varphi_{ij}^*)^{1-\sigma}}{\sigma} = \tilde{w}_i f_{ij} \quad \Rightarrow \quad b_i^{1-\sigma} p_{ij}(\varphi_{ij}^*)^{1-\sigma} = \left(\frac{1+\gamma}{\sigma+\gamma}\right)\frac{\sigma \tilde{w}_i f_{ij}}{E_j P_j^{\sigma-1}}. \tag{A.85}$$

Substituting this result into equation (A.44), we obtain:

$$\begin{aligned} P_j^{1-\sigma} &= \left[\frac{\theta}{\theta-(\sigma-1)\left(\frac{\gamma}{\sigma+\gamma}\right)}\right]\left(\frac{1+\gamma}{\sigma+\gamma}\right)\frac{\sigma}{E_j P_j^{\sigma-1}}\sum_i M_{ij}\tilde{w}_i f_{ij} \\ \Leftrightarrow \quad 1 &= \left[\frac{\theta}{\theta-(\sigma-1)\left(\frac{\gamma}{\sigma+\gamma}\right)}\right]\left(\frac{1+\gamma}{\sigma+\gamma}\right)\frac{\sigma}{E_j}\sum_i M_{ij}\tilde{w}_i f_{ij}. \end{aligned} \tag{A.86}$$

Substituting in the equation above with the mass of firms from equation (A.41) yields:

$$1 = \left[\frac{\theta}{\theta-(\sigma-1)\frac{\gamma}{\sigma+\gamma}}\right]\left(\frac{1+\gamma}{\sigma+\gamma}\right)\frac{\sigma}{E_j}\sum_i\left(\frac{\gamma}{1+\gamma}\right)\left(\frac{\sigma-1}{\sigma}\right)\frac{A_i L_i \frac{w_i}{A_i} f_{ij}}{\theta\delta f^e(\varphi_{ij}^*)^\theta}$$

which simplifies to:

$$1 = \left[\frac{\frac{\gamma}{\sigma+\gamma}(\sigma-1)}{\theta-\frac{\gamma}{\sigma+\gamma}(\sigma-1)}\right] E_j^{-1}\sum_i \frac{w_i L_i f_{ij}}{\delta f^e}(\varphi_{ij}^*)^{-\theta}.$$

Substituting in equation (A.12) for φ_{ij}^* in the equation above yields:

$$1=\left[\frac{\frac{\gamma}{\sigma+\gamma}(\sigma-1)}{\theta-\frac{\gamma}{\sigma+\gamma}(\sigma-1)}\right]\frac{E_j^{-1}}{\delta f^e}\sum_i w_iL_if_{ij}\left\{\left[\frac{\left(\frac{1+\gamma}{\gamma}\frac{\sigma}{\sigma-1}\tilde{w}_i\right)^{\sigma}}{E_jP_j^{\sigma-1}b_i^{1-\sigma}}\right]^{\frac{\frac{1+\gamma}{\gamma}}{\sigma-1}}\left[\frac{\gamma}{\sigma+\gamma}(\sigma-1)f_{ij}\right]^{\frac{\frac{1+\gamma}{\gamma}}{\frac{1+\gamma}{\sigma+\gamma}(\sigma-1)}}\tau_{ij}^{\frac{1+\gamma}{\gamma}}\right\}^{-\theta}.$$

Solving the equation above for $P_j^{-\theta\frac{1+\gamma}{\gamma}}$ on the LHS yields:

$$P_j^{-\theta\left(\frac{1+\gamma}{\gamma}\right)}=DE_j^{\theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{1}{\sigma-1}\right)-1}\sum_{i=1}^N A_i^{\theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)}w_iL_iw_i^{-\theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)}b_i^{-\theta\left(\frac{1+\gamma}{\gamma}\right)}\tau_{ij}^{-\theta\left(\frac{1+\gamma}{\gamma}\right)}f_{ij}^{1-\frac{\theta\left(\frac{1+\gamma}{\gamma}\right)}{\frac{1+\gamma}{\sigma+\gamma}(\sigma-1)}}$$

where

$$D=\left[\frac{\frac{\gamma}{\sigma+\gamma}(\sigma-1)}{\theta-\frac{\gamma}{\sigma+\gamma}(\sigma-1)}\right](\delta f^e)^{-1}\left[\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)\left(\frac{\gamma}{\sigma+\gamma}\right)(\sigma-1)\right]^{\frac{-\theta}{\frac{\gamma}{\sigma+\gamma}(\sigma-1)}}$$

is a constant that depends on parameters σ, γ, θ, δ and f^e. It will be convenient to rewrite the equation above as:

$$P_j^{-\theta\left(\frac{1+\gamma}{\gamma}\right)}=DE_j^{\theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{1}{\sigma-1}\right)-1}\sum_{k=1}^N A_k^{\theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)}w_kL_kw_k^{-\theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)}b_k^{-\theta\left(\frac{1+\gamma}{\gamma}\right)}\tau_{kj}^{-\theta\left(\frac{1+\gamma}{\gamma}\right)}f_{kj}^{1-\frac{\theta\left(\frac{1+\gamma}{\gamma}\right)}{\frac{1+\gamma}{\sigma+\gamma}(\sigma-1)}}. \tag{A.87}$$

Having defined the first component of welfare (P_j), we turn to the second component: wage rates. From equation (A.48), we have:

$$\lambda_{jj}=\frac{w_jL_jw_j^{-\theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)}A_j^{\theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)}b_j^{-\theta\left(\frac{1+\gamma}{\gamma}\right)}f_{jj}^{1-\frac{\theta\left(\frac{1+\gamma}{\gamma}\right)}{\frac{1+\gamma}{\sigma+\gamma}(\sigma-1)}}}{\sum_{k=1}^N w_kL_kw_k^{-\theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)}A_k^{\theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)}b_k^{-\theta\left(\frac{1+\gamma}{\gamma}\right)}\tau_{kj}^{-\theta\left(\frac{1+\gamma}{\gamma}\right)}f_{kj}^{1-\frac{\theta\left(\frac{1+\gamma}{\gamma}\right)}{\frac{1+\gamma}{\sigma+\gamma}(\sigma-1)}}}$$

where $\tau_{jj}=1$, as standard in the literature. Dividing both sides by λ_{jj} and multiplying both sides by $w_j^{\theta\frac{1+\gamma}{\gamma}}$ yields:

$$w_j^{\theta\left(\frac{1+\gamma}{\gamma}\right)} = \left(\frac{1}{\lambda_{jj}}\right) \frac{w_j L_j w_j^{-\theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{1}{\sigma-1}\right)} A_j^{\theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)} b_j^{-\theta\left(\frac{1+\gamma}{\gamma}\right)} f_{jj}^{1-\frac{\theta\left(\frac{1+\gamma}{\gamma}\right)}{\frac{1+\gamma}{\sigma+\gamma}(\sigma-1)}}}{\sum_{k=1}^{N} w_k L_k w_k^{-\theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)} A_k^{\theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)} b_k^{-\theta\left(\frac{1+\gamma}{\gamma}\right)} \tau_{kj}^{-\theta\left(\frac{1+\gamma}{\gamma}\right)} f_{kj}^{1-\frac{\theta\left(\frac{1+\gamma}{\gamma}\right)}{\frac{\gamma}{\sigma+\gamma}(\sigma-1)}}}, \tag{A.88}$$

Multiplying equations (A.87) and (A.88) yields:

$$W_j^{\theta\left(\frac{1+\gamma}{\gamma}\right)} = D E_j^{\theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{1}{\sigma-1}\right)-1} \left(\frac{1}{\lambda_{jj}}\right) w_j L_j w_j^{-\theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{1}{\sigma-1}\right)} A_j^{\theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)} b_j^{-\theta\left(\frac{1+\gamma}{\gamma}\right)} f_{jj}^{1-\frac{\theta\left(\frac{1+\gamma}{\gamma}\right)}{\frac{1+\gamma}{\sigma+\gamma}(\sigma-1)}}.$$

Since $E_j = w_j L_j$, then:

$$W_j^{\theta\left(\frac{1+\gamma}{\gamma}\right)} = D\left(\frac{1}{\lambda_{jj}}\right) L_j^{\theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{1}{\sigma-1}\right)} A_j^{\theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)} b_j^{-\theta\left(\frac{1+\gamma}{\gamma}\right)} f_{jj}^{1-\frac{\theta\left(\frac{1+\gamma}{\gamma}\right)}{\frac{1+\gamma}{\sigma+\gamma}(\sigma-1)}}$$

or

$$W_j^{\theta\left(\frac{1+\gamma}{\gamma}\right)} = D^{\frac{1}{\theta\left(\frac{1+\gamma}{\gamma}\right)}} \lambda_{jj}^{-\frac{1}{\theta\left(\frac{1+\gamma}{\gamma}\right)}} L_j^{\frac{1}{\sigma-1}} A_j^{\frac{\sigma}{\sigma-1}} b_j^{-1} f_{jj}^{\left(\frac{1}{1+\gamma}\right)\left(\frac{\gamma}{\theta}-\frac{\sigma+\gamma}{\sigma-1}\right)}.$$

Hence, for any *foreign* shock (i.e., holding constant L_j, A_j, b_j and f_{jj}), then:

$$\hat{W}_j = \hat{\lambda}_{jj}^{-\frac{1}{\theta\left(\frac{1+\gamma}{\gamma}\right)}} \tag{A.89}$$

where the hat denotes the gross change, i.e., W_j'/W_j and $\lambda_{jj}'/\lambda_{jj}$, where W_j' and λ_{jj}' denote the post-shock values of W_j and λ_{jj}, respectively.

Feenstra (2010) insightfully shows that one can interpret the gains from trade in a Melitz model as a gain due to increase in "export variety" or "average productivity." Importantly, the gain reflects the increase in real wage rates due to the productivity improvement as new exporting firms drive out less productive domestic firms, *raising average productivity.*[47]

To make this point, Feenstra (2010) derives a transformation curve between masses of varieties for sale to different markets, M_{ij}, and shows that trade increases real income by allowing the economy to reach more productive output combinations. As shown below in section A.15, we can solve for the concave transformation frontier between the (output-

[47]As Feenstra (2010) notes, because the gains from new imported varieties exactly offset the losses from fewer domestic varieties (under the Pareto distribution assumption), there are no further gains from trade on the consumption side.

adjusted) masses of varieties, $\tilde{M}_{ij}$, as follows:

$$L_i = k_1 (f^e)^{\frac{1}{1+\eta}} \left(\sum_j f_{ij}^{\frac{\eta-\theta}{\eta}} \tilde{M}_{ij}^{\frac{1+\eta}{\eta}} \right)^{\frac{\eta}{1+\eta}}, \tag{A.90}$$

where $k_1 > 0$ is a constant that depends only on parameters of the model (with the exact definition of k_1 provided later in section A.15). The economically important difference between our result under IMC and that in Feenstra (2010) under CMC is that the constant-elasticity-of-transformation (CET) in our model is $\eta = \theta \left(\frac{\sigma}{\sigma-1}\right)\left(\frac{1+\gamma}{\gamma}\right) - 1 > 0$, whereas Feenstra's CET is $\omega = \theta \left(\frac{\sigma}{\sigma-1}\right) - 1 > 0$. All else equal, $\eta \geq \omega$ because $(1+\gamma)/\gamma \geq 1$, with strict inequality when $\gamma < \infty$. Thus, with IMC, the CET curve will be flatter than under CMC as long as $\gamma < \infty$. In fact, we can show:

$$\eta = \omega + (\omega + 1)/\gamma,$$

which reveals the degree to which the CET under IMC is larger. As γ declines from ∞, η increases relative to ω. As γ approaches ∞, $\eta = \omega$, as in Feenstra (2010).

In section A.15 below, we show that aggregate income in our model is a linear function of the (output-adjusted) masses of varieties:

$$R_i = \sum_{j=1}^{N} A_{ij} \tilde{M}_{ij}, \tag{A.91}$$

where, to be consistent (and tractable) with Feenstra (2010), the A_{ij}s now denote demand-shift parameters that depend only on parameters of the model; in the remainder of this section and in the next, we omit any TFP shocks (labeled previously A) and preference shocks (labeled previously b). As explained in Feenstra (2010), the welfare maximizing combination of (output-adjusted) masses of varieties can be obtained by maximizing income in equation (A.91) subject to the transformation curve in equation (A.90).

We can now evaluate the impact of trade liberalization on welfare. For simplicity, consider the two-country case illustrated in Figure A.1 (an extended version of Figure 5 in Feenstra (2010)). As shown in Figure A.1, our transformation curve (the dashed bowed-out line from point A to point B) is flatter compared to that of Feenstra (2010) under CMC (the solid bowed-out line from point A to point B). Point A represents the equilibrium under autarky for both cases. At that point, the mass of (output-adjusted) varieties for sale in the domestic market is positive, $\tilde{M} > 0$, and the mass of (output-adjusted) varieties for sale in the foreign market is null, $\tilde{M}_x = 0$. Autarky income is represented by the straight line closest to the origin, starting at point A. By opening up to trade, the economy can increase

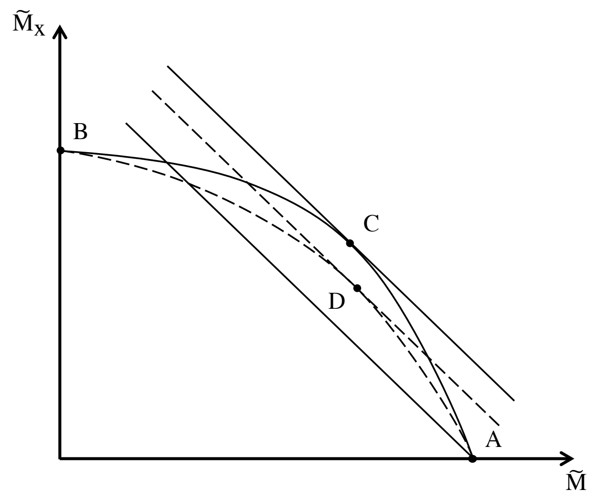

Figure A.1: CET Frontier with Increasing Marginal Costs and Constant Marginal Costs

its mass of (output-adjusted) varieties for sale in the foreign market and reduce its mass of (output-adjusted) varieties for sale in the domestic market. Under CMC, the gain in income is shown by the shift outward of the straight line through point A to the straight line tangent to the (solid-line) transformation curve at point C. Under IMC, the transformation curve is flatter which leads to smaller gains in income, as shown by the shift outward of the straight line through point A to the straight line tangent to the (dashed-line) transformation curve at point D. The difference between the income line tangent to point C and the income line tangent to point D represents the *welfare diminution effect* associated with IMC.

The diminished welfare gains due to IMC can also be interpreted mathematically in the context of Feenstra (2010). In a Melitz model with constant marginal costs, the change in welfare ($\hat{W}_j$) from a reduction in variable trade costs is proportionate to the change in average productivity ($\hat{\tilde{\varphi}}_{ij}$) and the change in the number of varieties ($\hat{M}_{ij}$), cf., Melitz (2003), equation (17). Feenstra (2010) shows also that the change in welfare ($\hat{W}_j$) can be simplified further to be proportionate to the change in output of the zero-cutoff-profit firm ($q_{ij}(\hat{\varphi}^*_{ij})$), cf. Feenstra (2010). As seen in equation (8) in the paper, under IMC the output of the cutoff productivity firm is proportional to the cutoff productivity according to:

$$q_{ij}(\varphi^*_{ij}) = \left[\left(\frac{\gamma}{\sigma+\gamma}\right)(\sigma-1)f_{ij}\varphi^*_{ij}\right]^{\frac{\gamma}{1+\gamma}}.$$

Because a property of the Pareto distribution is that the average productivity, $\tilde{\varphi}_{ij}$, is proportionate to cutoff productivity, φ^*_{ij}, changes in welfare will be proportional to $(\hat{\varphi}^*_{ij})^{\frac{\gamma}{1+\gamma}}$. Under CMC, there is a linear relationship between the productivity cutoff and the output, i.e., as γ approaches ∞, $\frac{\gamma}{1+\gamma}$ approaches 1. However, when we introduce IMC, this relationship becomes concave. As a result, a given change in φ^*_{ij} has a *smaller effect* on output, $q_{ij}(\varphi^*_{ij})$, under IMC than under CMC. This is the intuition underlying the "welfare diminution effect" from increasing marginal costs.

A.15 Constant Elasticity of Transformation

In this section, we derive the constant-elasticity-of-transformation (CET) function for our model. As a first step, we define aggregate revenue in our model. Using equations (A.45), (A.46), and (A.47):

$$R_i = \sum_{j=1}^{N} X_{ij} = \sum_{j=1}^{N} M_{ij} \int_{\varphi^*_{ij}}^{\infty} r_{ij}(\varphi)\mu_{ij}(\varphi)d\varphi. \tag{A.92}$$

In our model, we can solve for $p_{ij}(\varphi) = q_{ij}(\varphi)^{-\frac{1}{\sigma}} \tau_{ij}^{\frac{1-\sigma}{\sigma}} P_j^{\frac{\sigma-1}{\sigma}} (w_j L_j)^{\frac{1}{\sigma}}$. Since $r_{ij}(\varphi) = p_{ij}(\varphi)q_{ij}(\varphi)$ and assuming aggregate revenue (R_i) equals aggregate income ($w_i L_i$), we can write:

$$R_i = w_i L_i = \sum_{j=1}^{N} A_{ij} M_{ij} \int_{\varphi^*_{ij}}^{\infty} q_{ij}(\varphi)^{\frac{\sigma-1}{\sigma}} \mu_{ij}(\varphi)d\varphi = \sum_{j=1}^{N} A_{ij}\tilde{M}_{ij} \tag{A.93}$$

where, analogous to Feenstra (2010):

$$A_{ij} = \tau_{ij}^{\frac{1-\sigma}{\sigma}} P_j \left(\frac{w_j L_j}{P_j}\right)^{\frac{1}{\sigma}} \tag{A.94}$$

and we denote $\tilde{M}_{ij}$ as the "output-adjusted" mass of varieties produced in country i and sold in market j:

$$\tilde{M}_{ij} = M_{ij} \int_{\varphi^*_{ij}}^{\infty} q_{ij}(\varphi)^{\frac{\sigma-1}{\sigma}} \mu_{ij}(\varphi)d\varphi. \tag{A.95}$$

In the context of our model, we know from section A.4 that:

$$\tilde{\varphi}_{ij} = \left[\int_{\varphi^*_{ij}}^{\infty} \varphi^{\frac{\gamma}{\gamma+\sigma}(\sigma-1)} \mu_{ij}(\varphi)d\varphi\right]^{\frac{1}{\frac{\gamma}{\gamma+\sigma}(\sigma-1)}} \tag{A.96}$$

is a measure of average productivity ($\tilde{\varphi}_{ij}$). Using equation (A.13) from section A.4, we can write:

$$q_{ij}(\varphi) = \left(\frac{\varphi}{\tilde{\varphi}_{ij}}\right)^{\sigma\frac{\gamma}{\gamma+\sigma}} q_{ij}(\tilde{\varphi}_{ij}). \tag{A.97}$$

Using equation (A.97) in the middle equality in equation (A.93) yields:

$$\begin{aligned} w_i L_i &= \sum_{j=1}^{N} A_{ij} M_{ij} \int_{\varphi_{ij}^*}^{\infty} \left[\left(\frac{\varphi}{\tilde{\varphi}_{ij}} \right)^{\sigma \frac{\gamma}{\gamma+\sigma}} q_{ij}(\tilde{\varphi}_{ij}) \right]^{\frac{\sigma-1}{\sigma}} \mu_{ij}(\varphi) d\varphi \\ &= \sum_{j=1}^{N} A_{ij} M_{ij} [q_{ij}(\tilde{\varphi}_{ij})]^{\frac{\sigma-1}{\sigma}} \tilde{\varphi}_{ij}^{(1-\sigma)\frac{\gamma}{\gamma+\sigma}} \int_{\varphi_{ij}^*}^{\infty} \varphi^{(\sigma-1)\frac{\gamma}{\gamma+\sigma}} \mu_{ij}(\varphi) d\varphi. \end{aligned} \tag{A.98}$$

Since the integral term in the equation above simplifies to $\tilde{\varphi}_{ij}^{(\sigma-1)\frac{\gamma}{\gamma+\sigma}}$, then:

$$w_i L_i = \sum_{j=1}^{N} A_{ij} M_{ij} [q_{ij}(\tilde{\varphi}_{ij})]^{\frac{\sigma-1}{\sigma}} = \sum_{j=1}^{N} A_{ij} \tilde{M}_{ij} \tag{A.99}$$

where

$$\tilde{M}_{ij} = M_{ij} [q_{ij}(\tilde{\varphi}_{ij})]^{\frac{\sigma-1}{\sigma}}.$$

Using the equations for output and average productivity (A.9) and (A.23), respectively, and inverting equation (A.41) to solve for φ_{ij}^* as a function of M_{ij}, we find:

$$\tilde{M}_{ij} = k_0 f_{ij}^{\frac{\gamma}{\gamma+1} \frac{\sigma-1}{\sigma}} \left(\frac{f^e}{L_i} \right)^{-\frac{\gamma}{1+\gamma} \frac{\sigma-1}{\theta\sigma}} M_{ij}^{1-\frac{\gamma}{\gamma+1} \frac{\sigma-1}{\theta\sigma}}, \tag{A.100}$$

where k_0 is a constant that depends only on parameters σ, γ, θ, and δ:

$$k_0 = \left[\frac{\theta}{\theta - (\sigma-1)\frac{\gamma}{\gamma+\sigma}} \right] \left[\left(\frac{\gamma}{\gamma+\sigma} \right) (\sigma - 1) \right]^{\left(\frac{\gamma}{1+\gamma}\right)\left(\frac{\sigma-1}{\sigma}\right)} \left[\left(\frac{\gamma}{1+\gamma} \right) \left(\frac{\sigma-1}{\sigma} \right) \frac{1}{\theta\delta} \right]^{\frac{1}{\theta}\left(\frac{\gamma}{1+\gamma}\right)\left(\frac{\sigma-1}{\sigma}\right)}.$$

We invert equation (A.100) to solve for the mass of firms as a function of the adjusted mass:

$$M_{ij} = \left(\frac{1}{k_0} \right)^{\frac{1+\eta}{\eta}} f_{ij}^{-\frac{\theta}{\eta}} \left(\frac{f_e}{L_i} \right)^{\frac{1}{\eta}} \tilde{M}_{ij}^{\frac{1+\eta}{\eta}} \tag{A.101}$$

where $\eta = \theta \left(\frac{1+\gamma}{\gamma}\right) \left(\frac{\sigma}{\sigma-1}\right) - 1$.

We can use equation (A.36), from section A.5, to express country i's labor stock as a linear transformation function of masses M_{ij}:

$$L_i = \left(\frac{1+\gamma}{\gamma} \right) \left(\frac{\sigma}{\sigma-1} \right) \left[\frac{\theta(\sigma-1)\frac{\gamma}{\gamma+\sigma}}{\theta - (\sigma-1)\frac{\gamma}{\gamma+\sigma}} \right] \sum_{j=1}^{N} M_{ij} f_{ij}. \tag{A.102}$$

Substituting equation (A.101) into equation (A.102) yields country i's labor stock as a *concave* CET function of the "output-adjusted" masses:

$$L_i = k_1 (f^e)^{\frac{1}{1+\eta}} \left(\sum_j f_{ij}^{\frac{\eta-\theta}{\eta}} \tilde{M}_{ij}^{\frac{1+\eta}{\eta}} \right)^{\frac{\eta}{1+\eta}} \tag{A.103}$$

which is similar – but not identical – to (corrected) equation (3.24) in Feenstra (2010).[48] Note that k_1 is a constant that depends only parameters σ, γ, θ, and k_0:

$$\begin{aligned} k_1 &= \frac{1}{k_0} \left[\left(\frac{1+\gamma}{\gamma} \right) \left(\frac{\sigma}{\sigma - 1} \right) \frac{\theta(\sigma-1)\frac{\gamma}{\gamma+\sigma}}{\theta - (\sigma-1)\frac{\gamma}{\gamma+\sigma}} \right]^{1-\frac{1}{\theta}\left(\frac{\gamma}{1+\gamma}\right)\left(\frac{\sigma-1}{\sigma}\right)} \\ &= \frac{1}{k_0} \left[\frac{\theta\sigma\left(\frac{1+\gamma}{\gamma+\sigma}\right)}{\theta - (\sigma-1)\frac{\gamma}{\gamma+\sigma}} \right]^{1-\frac{1}{\theta}\left(\frac{\gamma}{1+\gamma}\right)\left(\frac{\sigma-1}{\sigma}\right)} . \end{aligned}$$

[48] The exponent for f_{ij}, $1 - \frac{\theta}{\eta}$, differs from, and is a corrected version of, that in Feenstra (2010). Under CMC, the exponent in Feenstra (2010) should be $1 - \frac{\theta}{\omega}$, not $1 + \frac{\theta}{\omega} \left(= 1 + \frac{(\omega+1)(\sigma-1)}{\omega\sigma} \right)$, and was confirmed with Robert Feenstra in email correspondence.

B Appendix B

B.1 The Bergstrand (1985) Model with Increasing Marginal Costs

As noted in numerous studies and in prominent surveys of the gravity equation in international trade, the first formal theoretical foundation for the gravity equation was Anderson (1979). Assuming a frictionless world, Anderson (1979) established theoretically one of the most enduring empirical relationships in international trade – that bilateral trade from i to j (X_{ij}) was proportional to the *product* of both countries' national outputs (Y_iY_j) – using only four assumptions: every country i is endowed with a nationally differentiated output (Y_i), preferences are identical and homothetic across countries, the assumed absence of trade costs allows all prices to be identical across countries, and trade is balanced multilaterally (i.e., markets clear). The first three assumptions implied the demand for i's output in j was proportionate to j's output, $X_{ij} = b_iY_j$, where b_i is every importer's demand for the good of i as a share of its expenditures. Assuming all output of each country is absorbed (i.e., markets clear), $X_{ij} = Y_iY_j/Y_W$, where Y_W is world output. However, once Anderson (1979) introduced (positive) trade costs, he was unable to generate a transparent "structural" gravity equation, such as in Anderson and van Wincoop (2003). In fact, throughout the later sections including his appendix (using CES preferences), Anderson (1979) assumed inappropriately "the convention that all free trade prices are unity" despite his incorporating trade costs (cf., p. 115).

In contrast to Anderson (1979), the main motivation behind Bergstrand (1985) was to address the role of prices in the gravity equation, both theoretically and empirically. Unlike Anderson (1979), Bergstrand (1985) started with a CES utility function to emphasize that products from various markets were imperfect substitutes, as originally hypothesized by Armington. Moreover, he nested a CES utility function among importables inside a CES utility function between importables and the domestic good. On the supply side, he chose not to use the convention of constant marginal costs. Rather, he introduced a constant-elasticity-of-transformation (CET) function for producing output in the domestic market and foreign market, allowing a cost (in terms of labor) for output to be transformed between home and foreign markets. He also used a CET function to allow a cost for foreign output to be transformed between various export markets. He nested the latter CET function inside the former CET function. This formulation motivated upward-sloping supply curves *for each bilateral market* (including the domestic market). Assuming bilateral import demand values equaled bilateral export supply values in general equilibrium, this generated a system of $4N^2 + 3N$ equations in the same number of unknowns.

Assuming each bilateral market was small relative to the other $N^2 - 1$ markets and identical preferences and technologies across countries, Bergstrand (1985) derived the trade gravity equation:

$$X_{ij} = Y_i^{\frac{\sigma-1}{\gamma+\sigma}} Y_j^{\frac{\gamma+1}{\gamma+\sigma}} (C_{ij}T_{ij})^{-\sigma\frac{\gamma+1}{\gamma+\sigma}} E_{ij}^{\sigma\frac{\gamma+1}{\gamma+\sigma}} \left(\sum_{k=1,k\neq i}^{N} p_{ik}^{1+\gamma}\right)^{-\frac{(\sigma-1)(\gamma-\eta)}{(1+\gamma)(\gamma+\sigma)}} \left(\sum_{k=1,k\neq j}^{N} \overline{p}_{kj}^{1-\sigma}\right)^{\frac{(\gamma+1)(\sigma-\mu)}{(1-\sigma)(\gamma+\sigma)}}$$
$$\left[\left(\sum_{k=1,k\neq i}^{N} p_{ik}^{1+\gamma}\right)^{\frac{1+\eta}{1+\gamma}} + p_{ii}^{1+\eta}\right]^{-\frac{\sigma-1}{\gamma+\sigma}} \left[\left(\sum_{k=1,k\neq j}^{N} \overline{p}_{kj}^{1-\sigma}\right)^{\frac{1-\mu}{1-\sigma}} + p_{jj}^{1-\mu}\right]^{-\frac{\gamma+1}{\gamma+\sigma}}, \tag{B.1}$$

where $C_{ij} \geq 1$ is the gross transport (or c.i.f./f.o.b.) factor, $T_{ij} \geq 1$ is the gross tariff rate, E_{ij} is the spot exchange rate (value of j's currency in terms of i's), p_{ik} is the (free-on-board, or f.o.b.) price in i's currency of i's goods sold in k, $\bar{p}_{kj}$ is the (cost-insurance-freight, or c.i.f.) price of k's good in j (including tariffs), σ (μ) is the elasticity of substitution in consumption between importables (between importables and the domestic good), and γ (η) is the elasticity of transformation of output between export markets (between foreign markets and the domestic market).[49] The limitation in Bergstrand (1985) was that – due to the complexity of equation (B.1) – the market-clearing condition of Anderson (1979) could not be imposed.

In the remainder of this appendix, we provide two theoretical results. First, we show that a special case of gravity equation (14) in Bergstrand (1985) – labeled equation (B.1) above – yields that the intensive-margin (and trade) elasticity with respect to τ_{ij} is *identical* to the intensive-margin elasticity in Section 3.1 of this paper (from our modified Melitz model). Second, we show that – allowing the non-nested (single) constant-elasticity-of-transformation in this case to equal infinity and assuming multilateral trade balance – a "structural gravity equation" results.

B.2 Reconciling the Intensive-Margin Elasticity in Bergstrand (1985) with Section 3.1's Intensive-Margin Elasticity

Before we reconcile equation (B.1) with structural gravity, a special case of Bergstrand (1985) yields an intensive-margin (and, in this homogeneous-firm context, trade) elasticity identical to that in Section 3.2. We need only two assumptions. First, assume the elasticities of substitution in consumption in equation (B.1) to be identical ($\sigma = \mu$). Second, assume the elasticities of transformation in equation (B.1) to be identical ($\gamma = \eta$). Simplifying notation

[49] We have replaced here some notation in the original article. We use X_{ij} for the nominal trade flow rather than PX_{ij} and we use p_{ij} rather than P_{ij} to denote bilateral prices.

in equation (B.1) by denoting $\tau_{ij} = C_{ij}T_{ij}/E_{ij}$, these two assumptions yield:

$$X_{ij} = Y_i^{\frac{\sigma-1}{\gamma+\sigma}} Y_j^{\frac{\gamma+1}{\gamma+\sigma}} (\tau_{ij})^{(1-\sigma)\frac{\gamma+1}{\gamma+\sigma}} \left[\left(\sum_{j=1}^{N} p_{ij}^{1+\gamma} \right)^{\frac{1}{1+\gamma}} \right]^{(1-\sigma)\frac{\gamma+1}{\gamma+\sigma}} \left[\left(\sum_{i=1}^{N} (p_{ij}\tau_{ij})^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \right]^{-(1-\sigma)\frac{\gamma+1}{\gamma+\sigma}} . \tag{B.2}$$

From equation (B.2), the (positively-defined) intensive-margin (and trade) elasticity with respect to τ_{ij} is:

$$\varepsilon_\tau = -\frac{\partial X_{ij}}{\partial \tau_{ij}} \frac{\tau_{ij}}{X_{ij}} = -\frac{1+\gamma}{\sigma+\gamma}(1-\sigma) = \frac{1+\gamma}{\sigma+\gamma}(\sigma-1). \tag{B.3}$$

This elasticity is identical to that in Section 3.1 of the current paper. Moreover, this trade elasticity is *scaled down* by $\frac{1+\gamma}{\sigma+\gamma}$ relative to the constant marginal cost case in Anderson (1979) (and analogously in Krugman (1980)). The intuitive explanation for this was provided in the paper's introduction, Section 1, and illustrated in Figure 1.

B.3 Reconciling the Gravity Equation in Bergstrand (1985) with Structural Gravity

The second theoretical result in this appendix is to show that a special case of gravity equation (14) in Bergstrand (1985) is consistent with the structural gravity equation in Anderson and van Wincoop (2003) and in Baier et al. (2017). Building upon the previous section B.2, add two more assumptions. First, assume production is now *costlessly* transformable between markets ($\gamma = \infty$). With this additional assumption, equation (B.2) above simplifies to:

$$X_{ij} = Y_j \left(\frac{p_i \tau_{ij}}{P_j} \right)^{1-\sigma} \tag{B.4}$$

where p_{ij} is replaced by p_i since output is now costlessly transformed between markets and:

$$P_j \equiv \left[\sum_{i=1}^{N} (p_i \tau_{ij})^{1-\sigma} \right]^{1/(1-\sigma)} . \tag{B.5}$$

Equation (B.4) is identical to equation (6) in Anderson and van Wincoop (2003) (ignoring the arbitrary preference parameter β_i in that paper) and to the bilateral import demand functions in structural gravity equations discussed in Baier et al. (2017). Second, structural gravity follows once one assumes also market clearance (trade balance), $Y_i = \sum_{j=1}^{N} X_{ij}$.

Following derivations in Anderson and van Wincoop (2003) and Baier et al. (2017):

$$X_{ij} = \frac{Y_i Y_j}{Y_W} \left(\frac{\tau_{ij}}{\Pi_i P_j} \right)^{1-\sigma} \tag{B.6}$$

where:

$$\Pi_i = \left[\sum_{j=1}^{N} \frac{Y_j}{Y_W} \left(\frac{\tau_{ij}}{P_j} \right)^{1-\sigma} \right]^{1/(1-\sigma)} \tag{B.7}$$

and:

$$P_j = \left[\sum_{i=1}^{N} \frac{Y_j}{Y_W} \left(\frac{\tau_{ij}}{\Pi_i} \right)^{1-\sigma} \right]^{1/(1-\sigma)} . \tag{B.8}$$

Thus, the simplifications of equation (B.1) above from Bergstrand (1985) – along with adding in the market-clearing condition – yield the same structural gravity equation as in Anderson and van Wincoop (2003) and Baier et al. (2017).

C Appendix C

The key distinguishing assumption of our model is that marginal costs are increasing in output. There are many ways to implement this. In section 2.2 of the paper, we motivated the case for marginal costs increasing with respect to destination-specific output. This is one extreme of a range of models. At the other extreme, marginal costs could depend exclusively on the overall output of the firm. In that case, all the destination-specific customization is captured in the fixed export costs, as more common to Melitz models. In this appendix, we develop a model that fits this type of increasing marginal costs, that is, marginal costs are allowed to increase with *total firm output.*

If the marginal costs depend on overall firm output, which itself depends on the endogenous set of countries to which the firm exports, we cannot solve analytically a model with asymmetric country size and asymmetric bilateral trade barriers. As a consequence, in this appendix we assume all countries are identical and develop an extension of the symmetric-country Melitz (2003) model with increasing marginal costs in total firm-level output and a Pareto distribution of firm productivity. We present only key results because the solution method is similar to the one we used to solve the model in the main text; we refer the reader to Appendix A for additional details.

Consider a world with $1 + J$ identical countries. The representative consumer in each country has CES preferences defined over differentiated varieties. The representative consumer maximizes utility subject to the standard income constraint. Hence, the optimal aggregate demand function for each variety ν is given by:

$$c(\nu) = EP^{\sigma-1}p^c(\nu)^{-\sigma}, \quad \text{with} \quad P = \left[\int_{\nu\in\Omega} p^c(\nu)^{1-\sigma}d\nu\right]^{\frac{1}{1-\sigma}} \tag{C.1}$$

where E denotes aggregate expenditure, $p^c(\nu)$ is the unit price of variety ν, and Ω is the set of varieties available for consumption.[50]

Firms face fixed production costs and increasing marginal costs, such that the total labor demand by a firm depends on its total output (q) and whether or not the firm exports as follows:

$$l(\varphi) = f + I_x J f_x + \frac{q^{1+\frac{1}{\gamma}}}{\varphi}, \tag{C.2}$$

where φ denotes the firm's productivity and q is total output defined as:

$$q = q_d + I_x J q_x,$$

[50]Different from the main text and Appendix A, we omit, for brevity, preference parameter b.

where q_d denotes domestic sales and q_x denotes sales to a foreign market.[51] The variable I_x is an indicator function equal to 1 if the firm exports and 0 otherwise. It is important to note that, because countries and (international bilateral) trade costs (τ and f_x) are symmetric, if a firm can export profitably to one market abroad, it will be able to export profitably to all foreign markets.

Using the labor-demand function in equation (C.2), we can express firm-level profits as:

$$\pi(\varphi) = p_d q_d + I_x J p_x q_x - w \left[f + I_x J f_x + \frac{1}{\varphi} \left(q_d + I_x J q_x \right)^{1+\frac{1}{\gamma}} \right], \tag{C.3}$$

where w is the wage rate. It is important to emphasize that, in contrast to the benchmark model, it is not possible to separate total profits into domestic and export components. This key distinction is a direct consequence of the technology. As shown in equation (C.2), labor demand is a non-linear function of total firm output such that it is not possible to separate the costs associated with output for domestic sales from the costs associated with output for foreign sales. As a result, the firm's production costs must be expressed as a function of total output as seen from the last term in square brackets. This implies that we cannot solve for the optimal behavior of a given firm in each market separately, that is, without also taking into account its behavior in other markets. Instead, we need to characterize the optimal behavior of firm as a function of both its market (domestic vs. foreign) and its type (domestic vs. exporter).

Markets are segmented, such that firms can charge different prices in the domestic and foreign markets. Therefore, the firm-level profit maximization problem takes the following form:

$$\max_{p_d, p_x} \pi(\varphi) = p_d q_d + I_x J p_x q_x - w \left[f + I_x J f_x + \frac{1}{\varphi} \left(q_d + I_x J q_x \right)^{1+\frac{1}{\gamma}} \right] \tag{C.4}$$

subject to the demand constraints defined in equation (C.1). The two first-order conditions imply the following pricing rules:

$$\begin{aligned} p_d^D(\varphi) &= \left(\frac{1+\gamma}{\gamma} \right) \left(\frac{\sigma}{\sigma-1} \right) \frac{w}{\varphi} q_d^D(\varphi)^{\frac{1}{\gamma}}, \\ p_d^X(\varphi) &= \left(\frac{1+\gamma}{\gamma} \right) \left(\frac{\sigma}{\sigma-1} \right) \frac{w}{\varphi} \left[\left(1 + J\tau^{1-\sigma} \right) q_d^X(\varphi) \right]^{\frac{1}{\gamma}}, \end{aligned} \tag{C.5}$$

where $p_d^D(\varphi)$ and $q_d^D(\varphi)$ denote, respectively, the optimal domestic sales price and output of a (pure) domestic firm (denoted with superscript D) with productivity φ producing and selling in the domestic market (denoted with subscript d). Let $p_d^X(\varphi)$ and $q_d^X(\varphi)$ denote, respectively, the optimal sales price and output of an exporting firm (denoted with superscript X) with

[51] Different from the main text and Appendix A, we omit, for brevity, the TFP factor A.

productivity φ selling in the domestic market (denoted with subscript d). The results in equation (C.5) imply that, conditional on productivity and total output, exporting firms (located in country d) can charge higher, equal, or lower prices at home relative to (pure) domestic firms, due to opposing effects from productivity differences versus scale effects. The higher productivity of an exporter tends to lower p_d^X relative to p_d^D. However, an exporter serves more markets, tending to raise p_d^X relative to p_d^D.

We define the profitability threshold φ^* as the productivity level at which a (pure) domestic firm makes zero profits: $\pi(\varphi^*|I_x = 0) = 0$, where the profit function $\pi(\cdot)$ is defined in equation (C.3). Using this condition, we can solve for the output and the price of the threshold pure domestic firm as follows:

$$\begin{aligned} q_d^D(\varphi^*) &= \left[\left(\frac{\gamma}{\sigma+\gamma}\right)(\sigma-1)f\varphi^*\right]^{\frac{\gamma}{1+\gamma}}, \\ p_d^D(\varphi^*) &= \left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)\left[\left(\frac{\gamma}{\sigma+\gamma}\right)(\sigma-1)f\right]^{\frac{1}{1+\gamma}} w(\varphi^*)^{\frac{-\gamma}{1+\gamma}}. \end{aligned} \tag{C.6}$$

Similarly, if we define the export profitability threshold as the level of productivity φ_x^* required for an exporting firm to break even, $\pi(\varphi_x^*|I_x = 1) = 0$, we can solve for the domestic price and output of the threshold exporting firm as follows:

$$\begin{aligned} q_d^X(\varphi_x^*) &= \left(\frac{1}{1+J\tau^{1-\sigma}}\right)\left[\left(\frac{\gamma}{\sigma+\gamma}\right)(\sigma-1)(f+Jf_x)\varphi_x^*\right]^{\frac{\gamma}{1+\gamma}}, \\ p_d^X(\varphi_x^*) &= \left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)\left[\left(\frac{\gamma}{\sigma+\gamma}\right)(\sigma-1)(f+Jf_x)\right]^{\frac{1}{1+\gamma}} w(\varphi_x^*)^{\frac{-\gamma}{1+\gamma}}. \end{aligned} \tag{C.7}$$

Note that, when $J = 0$, these last two solutions become equivalent to the domestic firms' solutions in (C.6), as they should. Substituting the results in (C.6) and (C.7) into the zero-profit conditions that define the productivity thresholds and rearranging, we obtain:

$$\begin{aligned} \pi(\varphi^*|I_x = 0) = 0 \quad &\Leftrightarrow \quad r_d^D(\varphi^*) = \left(\frac{1+\gamma}{\sigma+\gamma}\right)\sigma w f, \\ \pi(\varphi_x^*|I_x = 1) = 0 \quad &\Leftrightarrow \quad r_d^X(\varphi_x^*) = \left(\frac{1+\gamma}{\sigma+\gamma}\right)\left(\frac{\sigma w}{1+J\tau^{1-\sigma}}\right)(f+Jf_x). \end{aligned} \tag{C.8}$$

From the definition of revenues ($r(\varphi) = p(\varphi)q(\varphi)$) and the optimal demand function in (C.1), it follows that

$$\frac{r_d^D(\varphi)}{r_d^D(\varphi^*)} = \left[\frac{p_d^D(\varphi)}{p_d^D(\varphi^*)}\right]^{1-\sigma} \quad \text{and} \quad \frac{r_d^X(\varphi)}{r_d^X(\varphi_x^*)} = \left[\frac{p_d^X(\varphi)}{p_d^X(\varphi_x^*)}\right]^{1-\sigma}. \tag{C.9}$$

We can simplify these results using the definitions of prices in equations (C.6) and (C.7) to obtain analytical expressions for the revenue of any firm as a function of the revenue of the threshold firm. Combining these expressions with equations (C.8), it is possible to express the revenue of any domestic and exporting firms, respectively, as follows:

$$\begin{aligned} r_d^D(\varphi) &= \left(\frac{1+\gamma}{\sigma+\gamma}\right)\sigma w f\left(\frac{\varphi}{\varphi^*}\right)^{(\sigma-1)\left(\frac{\gamma}{\sigma+\gamma}\right)}, \\ r_d^X(\varphi) &= \left(\frac{1+\gamma}{\sigma+\gamma}\right)\left(\frac{\sigma w}{1+J\tau^{1-\sigma}}\right)(f+Jf_x)\left(\frac{\varphi}{\varphi_x^*}\right)^{(\sigma-1)\left(\frac{\gamma}{\sigma+\gamma}\right)}. \end{aligned} \tag{C.10}$$

Using equations (C.8), we can obtain a first expression for the ratio of domestic threshold revenue and export threshold revenue,

$$\frac{r_d^D(\varphi^*)}{r_d^X(\varphi_x^*)} = \left(1+J\tau^{1-\sigma}\right)\left(\frac{f}{f+Jf_x}\right). \tag{C.11}$$

We can obtain a second expression for the ratio of domestic threshold revenue and export threshold revenue using the definition of revenue and the optimal demand function as follows:

$$\frac{r_d^D(\varphi^*)}{r_d^X(\varphi_x^*)} = \left[\frac{p_d^D(\varphi^*)}{p_d^X(\varphi_x^*)}\right]^{1-\sigma}. \tag{C.12}$$

Using the definitions of prices in equations (C.6) and (C.7), we obtain:

$$\frac{r_d^D(\varphi^*)}{r_d^X(\varphi_x^*)} = \left(\frac{f}{f+Jf_x}\right)^{\frac{1-\sigma}{1-\gamma}}\left(\frac{\varphi^*}{\varphi_x^*}\right)^{(\sigma-1)\left(\frac{\gamma}{1+\gamma}\right)}. \tag{C.13}$$

Combining our two expressions for the ratio of revenues, (C.11) and (C.13), we can solve for the ratio of the productivity thresholds as follows:

$$\frac{\varphi_x^*}{\varphi^*} = \left(\frac{1}{1+J\tau^{1-\sigma}}\right)^{\left(\frac{1}{\sigma-1}\right)\left(\frac{1+\gamma}{\gamma}\right)}\left(\frac{f+Jf_x}{f}\right)^{\left(\frac{1}{\sigma-1}\right)\left(\frac{\sigma+\gamma}{\gamma}\right)}. \tag{C.14}$$

When $\gamma \to \infty$, the relationship between the two thresholds is analogous to that in the benchmark Melitz (2003) model. We can use the definitions of revenue in (C.10) and the ratio in (C.14) to express average profits as a function of parameters of the model and the profitability threshold φ^*. Using the free entry condition that the expected value of entry is equal to the cost of entry, we can show that there exists a unique equilibrium threshold φ^*.

We are interested in defining the trade elasticities in our model. For convenience, we introduce the term XD to denote aggregate domestic absorption. As a first step, we can define aggregate domestic absorption as follows:

$$XD = M\int_{\varphi^*}^{\infty} r_d(\varphi)\mu(\varphi)d\varphi = M\left[\int_{\varphi^*}^{\varphi_x^*} r_d^D(\varphi)\mu(\varphi)d\varphi + \int_{\varphi_x^*}^{\infty} r_d^X(\varphi)\mu(\varphi)d\varphi\right], \tag{C.15}$$

where M is the equilibrium mass of firms in each country and $\mu(\varphi)$, defined as:

$$\mu(\varphi) = \begin{cases} 0 & \text{if } \varphi < \varphi^*, \\ \frac{g(\varphi)}{1-G(\varphi^*)} & \text{if } \varphi \geq \varphi^*, \end{cases} \tag{C.16}$$

denotes the equilibrium distribution of firm productivities. We assume that the following theoretical restriction on the parameters holds: $\theta > \frac{\gamma}{\sigma+\gamma}(\sigma-1)$. Equation (C.15) shows that domestic absorption depends on the mass of firms M and the average sales of firms in their domestic market. The average sales per firm can be decomposed into the separate contributions of domestic firms and exporting firms, the first and second terms in square brackets, respectively.

To obtain an analytical solution, we assume that firms draw their productivity from a Pareto distribution with parameter θ, such that $G(\varphi) = 1 - \varphi^{-\theta}$. Using this assumption and the definitions of revenue in equation (C.10), we can solve for aggregate domestic absorption as:

$$\begin{aligned} XD = M &\left[\frac{\theta}{\theta - (\sigma-1)\left(\frac{\gamma}{\sigma+\gamma}\right)}\right]\left(\frac{1+\gamma}{\sigma+\gamma}\right)\sigma\theta w f \\ &\times \left[1 - \left(\frac{\varphi_x^*}{\varphi^*}\right)^{\frac{\gamma(\sigma-1)}{\sigma+\gamma}-\theta} + \left(\frac{1}{1+J\tau^{1-\sigma}}\right)\left(\frac{f+Jf_x}{f}\right)\left(\frac{\varphi_x^*}{\varphi^*}\right)^{-\theta}\right]. \end{aligned} \tag{C.17}$$

In a second step, we introduce, for convenience, the term XX to denote aggregate expenditures on foreign goods, noting that – due to symmetry – aggregate imports (from the rest of the world) equal aggregate exports (to the rest of the world). We define aggregate expenditure on foreign goods as:

$$XX = M_x J\int_{\varphi_x^*}^{\infty} r_x^X(\varphi)\mu_x(\varphi)d\varphi = MJ\int_{\varphi_x^*}^{\infty} r_x^X(\varphi)\mu(\varphi)d\varphi, \tag{C.18}$$

where $M_x = [1 - G(\varphi_x^*)]M$ is the equilibrium mass of exporting firms in each country and $\mu_x(\varphi)$, defined as:

$$\mu_x(\varphi) = \begin{cases} 0 & \text{if } \varphi < \varphi_x^*, \\ \frac{g(\varphi)}{1-G(\varphi_x^*)} & \text{if } \varphi \geq \varphi_x^*, \end{cases} \tag{C.19}$$

denotes the equilibrium distribution of exporting firms' productivities, where $\mu_x(\varphi) = \frac{1-G(\varphi^*)}{1-G(\varphi_x^*)}\mu(\varphi)$. Substituting with the definition of revenue in equation (C.10) and using the fact that $r_x^X(\varphi) = \tau^{1-\sigma} r_d^X(\varphi)$ yields:

$$XX = M\left[\frac{\theta}{\theta-(\sigma-1)\left(\frac{\gamma}{\sigma+\gamma}\right)}\right]\left(\frac{1+\gamma}{\sigma+\gamma}\right)\sigma\theta w(f+Jf_x)\left(\frac{J\tau^{1-\sigma}}{1+J\tau^{1-\sigma}}\right)\left(\frac{\varphi_x^*}{\varphi^*}\right)^{-\theta}. \quad \text{(C.20)}$$

We now have separate analytical expressions for expenditures on domestic and foreign goods.

Using E to denote aggregate expenditures ($E = XD + XX$), we can now compute the share of aggregate expenditures on foreign goods (XX/E). Using equations (C.14), (C.17) and (C.20), we obtain:

$$\frac{XX}{E} = \frac{XX}{XD+XX} = \frac{\frac{J\tau^{1-\sigma}}{1+J\tau^{1-\sigma}}}{1+\left(\frac{1}{1+J\tau^{1-\sigma}}\right)^{\left(\frac{\theta}{\sigma-1}\right)\left(\frac{1+\gamma}{\gamma}\right)}\left(\frac{f+Jf_x}{f}\right)^{\left(\frac{\theta}{\sigma-1}\right)\left(\frac{\sigma+\gamma}{\gamma}\right)-1}-\left(\frac{1}{1+J\tau^{1-\sigma}}\right)^{\frac{1+\gamma}{\sigma+\gamma}}}. \quad \text{(C.21)}$$

We can use this last result to derive the trade elasticities. Note that:

$$\varepsilon_\tau \equiv -\frac{\partial(XX/JE)}{\partial\tau}\frac{\tau}{XX/JE} = -\frac{\partial(XX/E)}{\partial\tau}\frac{\tau}{XX/E}, \quad \text{(C.22)}$$

$$\varepsilon_f \equiv -\frac{\partial(XX/JE)}{\partial f_x}\frac{f_x}{XX/JE} = -\frac{\partial(XX/E)}{\partial f_x}\frac{f_x}{XX/E}. \quad \text{(C.23)}$$

It is useful to introduce additional notation to simplify the presentation. Define the following terms:

$$a = \left(\frac{1}{1+J\tau^{1-\sigma}}\right)^{\left(\frac{\theta}{\sigma-1}\right)\left(\frac{1+\gamma}{\gamma}\right)}\left(\frac{f+Jf_x}{f}\right)^{\left(\frac{\theta}{\sigma-1}\right)\left(\frac{\sigma+\gamma}{\gamma}\right)-1}, \quad \text{(C.24)}$$

$$b = \left(\frac{1}{1+J\tau^{1-\sigma}}\right)^{\frac{1+\gamma}{\sigma+\gamma}}, \quad \text{(C.25)}$$

$$c = \frac{J\tau^{1-\sigma}}{1+J\tau^{1-\sigma}}. \quad \text{(C.26)}$$

Then, it is possible to rewrite the share of expenditures on foreign goods (C.21) as follows:

$$\frac{XX}{E} = \frac{c}{1+a-b}. \quad \text{(C.27)}$$

After some tedious, but straightforward, algebra, we can show that:

$$\varepsilon_{\tau} = \theta \left(\frac{1+\gamma}{\gamma}\right) \left[\frac{a}{1+a-b} - \left(\frac{\sigma-1}{\theta}\right)\left(\frac{\gamma}{\sigma+\gamma}\right)\frac{b}{1+a-b}\right] c, \tag{C.28}$$

$$\varepsilon_{f} = \left[\left(\frac{\sigma+\gamma}{\gamma}\right)\left(\frac{\theta}{\sigma-1}\right) - 1\right]\left(\frac{a}{1+a-b}\right) c. \tag{C.29}$$

To gain some insight into these complex equations, we consider the case of a large number of countries. In the limit, when J tends to infinity it can be shown that:[52]

$$\lim_{J\to\infty} b = 0, \quad \lim_{J\to\infty} c = 1, \quad \text{and if } \theta > \gamma \quad \lim_{J\to\infty} a = \infty. \tag{C.30}$$

Together, it can be shown that these results imply:

$$\lim_{J\to\infty} \frac{a}{1+a-b} = 1, \quad \text{and} \quad \lim_{J\to\infty} \frac{b}{1+a-b} = 0. \tag{C.31}$$

Using these results in the definition of the elasticities in (C.28) and (C.29), it follows that:

$$\varepsilon_{\tau} = \theta \left(\frac{1+\gamma}{\gamma}\right), \tag{C.32}$$

$$\varepsilon_{f} = \frac{\theta\left(\frac{1+\gamma}{\gamma}\right)}{\frac{1+\gamma}{\sigma+\gamma}(\sigma-1)} - 1. \tag{C.33}$$

These results show that – as the number of countries increases – the trade elasticities in our symmetric model with increasing marginal costs *defined over total firm output* converge to the elasticities in our benchmark model with asymmetric countries and destination-specific increasing marginal costs.

[52] We provide evidence in Section 5 of the paper, comparing Tables 3 and 4, that estimates of θ exceed estimates of γ within the 10th-75th percentiles of the 568 four-digit industries.

D Appendix D

In this appendix, we provide details on the derivations to establish the (structural) bilateral import-demand equation, the (structural) bilateral import-unit value equation, and then the (extended F/BW) reduced-form estimation equation (40) (which builds upon reduced-form equation (38)).

D.1 Bilateral Import Demand

Because the price index $\int_{\varphi_{ij}^*}^{\infty} p_{ij}^c(\varphi)^{1-\sigma}\mu_{ij}(\varphi)d\varphi$ in equation (23) is not observable, we cannot use equation (23) to estimate the parameters of the model. To make progress, we express the observable average cost-insurance-freight (or cif) import unit value $\overline{p}_{ijt}^c$ as the ratio of two unobservable price indexes ($\tilde{p}_{ij}^c$, $\grave{p}_{ij}^c$), as noted in equation (24):

$$\overline{p}_{ij}^c \equiv \frac{X_{ij}^D}{Q_{ij}^D} = \frac{\int_{\varphi_{ij}^*}^{\infty} p_{ij}^c(\varphi)^{1-\sigma}\mu_{ij}(\varphi)d\varphi}{\int_{\varphi_{ij}^*}^{\infty} p_{ij}^c(\varphi)^{-\sigma}\mu_{ij}(\varphi)d\varphi} \equiv \frac{\tilde{p}_{ij}^c}{\grave{p}_{ij}^c}. \tag{D.1}$$

In what follows, we use the theoretical model to obtain analytical expressions for each of the unobserved price indexes, $\tilde{p}_{ij}^c$ and $\grave{p}_{ij}^c$. We then show that, by combining these two expressions in conjunction with the Pareto distribution (and allowing for deviations from Pareto, e.g., e_{ij}^{P1}, etc.), we can express nominal bilateral import demand as a function of the observable bilateral import unit value ($\overline{p}_{ij}^c$), e_{ij}^{P1}, etc.

We proceed in several steps. The first step is to solve for firm-level (bilateral) prices $p_{ij}^c(\varphi)$ as functions of the productivity threshold φ_{ij}^*. Recalling $q_{ij}(\varphi)/\tau_{ij} = c_{ij}(\varphi)$ and $p_{ij}^c(\varphi) = \tau_{ij}p_{ij}(\varphi)$, we can use optimal demand equation (2) and optimal pricing rule (6) to show:

$$\frac{q_{ij}(\varphi)}{q_{ij}(\varphi_{ij}^*)} = \left(\frac{\varphi}{\varphi_{ij}^*}\right)^{\sigma\left(\frac{\gamma}{\sigma+\gamma}\right)}. \tag{D.2}$$

Substituting into equation (D.2) using equation (8) for $q_{ij}(\varphi_{ij}^*)$ yields:

$$q_{ij}(\varphi) = \left[\left(\frac{\gamma}{\sigma+\gamma}\right)(\sigma-1)f_{ij}\right]^{\frac{\gamma}{1+\gamma}} (\varphi_{ij}^*)^{-\left(\frac{\gamma}{1+\gamma}\right)\left(\frac{\gamma}{\sigma+\gamma}\right)(\sigma-1)} \varphi^{\sigma\left(\frac{\gamma}{\sigma+\gamma}\right)}. \tag{D.3}$$

Substituting equation (D.3) for $q_{ij}(\varphi)$ into optimal pricing rule (6) yields:

$$p_{ij}(\varphi) = \left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)\left[\left(\frac{\gamma}{\sigma+\gamma}\right)(\sigma-1)f_{ij}\right]^{\frac{1}{1+\gamma}} (\varphi_{ij}^*)^{-\left(\frac{1}{1+\gamma}\right)\left(\frac{\gamma}{\sigma+\gamma}\right)(\sigma-1)} \tilde{w}_i \varphi^{-\frac{\gamma}{\sigma+\gamma}} \tag{D.4}$$

where recall that $\tilde{w}_i = w_i/A_i$.

In the second step, we compute the two unobservable average prices $\tilde{p}_{ij}^c$ and $\grave{p}_{ij}^c$ and show that the observable import unit value $\overline{p}_{ij}^c$ is proportional to the optimal price of the break-even exporter, $p_{ij}^c(\varphi_{ij}^*)$. Using equation (D.4), optimal pricing function (6), the Pareto distribution assumption allowing deviations from Pareto, and recalling $p_{ij}^c(\varphi) = \tau_{ij} p_{ij}(\varphi)$, we can solve for:

$$\tilde{p}_{ij}^c = \left[\frac{\theta(\sigma+\gamma)}{\theta(\sigma+\gamma)-\gamma(\sigma-1)}\right] p_{ij}^c(\varphi_{ij}^*)^{1-\sigma} e_{ij}^{P1} \tag{D.5}$$

where $e_{ij}^P \neq 1$ implies deviations from the Pareto distribution for $\tilde{p}_{ij}^c$. Using equation (D.1), optimal pricing function (6), and the Pareto distribution assumption allowing deviations e_{ij}^{P2}, we can solve for:

$$\grave{p}_{ij}^c = \left[\frac{\theta(\sigma+\gamma)}{\theta(\sigma+\gamma)-\gamma\sigma}\right] p_{ij}^c(\varphi_{ij}^*)^{-\sigma} e_{ij}^{P2}. \tag{D.6}$$

Using these results and equation (D.1), we obtain:

$$\overline{p}_{ij}^c = \left[\frac{\theta(\sigma+\gamma)-\gamma\sigma}{\theta(\sigma+\gamma)-\gamma(\sigma-1)}\right] p_{ij}^c(\varphi_{ij}^*) \left(\frac{e_{ij}^{P1}}{e_{ij}^{P2}}\right) \tag{D.7}$$

which shows that observable $\overline{p}_{ij}^c$ is proportional to the optimal price of the zero-cutoff-profit exporter and Pareto deviations.

The third step is straightforward. We can rewrite equation (D.7) with $p_{ij}^c(\varphi_{ij}^*)$ as a function of the observable price import unit value $\overline{p}_{ij}^c$:

$$p_{ij}^c(\varphi_{ij}^*) = \left[\frac{\theta(\sigma+\gamma)-\gamma\sigma+\gamma}{\theta(\sigma+\gamma)-\gamma\sigma}\right] \overline{p}_{ij}^c \left(\frac{e_{ij}^{P2}}{e_{ij}^{P1}}\right) \tag{D.8}$$

and substitute this last result into equation (25) to obtain:

$$\tilde{p}_{ij}^c = \left[\frac{\theta(\sigma+\gamma)}{\theta(\sigma+\gamma)-\gamma\sigma}\right] \left[\frac{\theta(\sigma+\gamma)-\gamma\sigma}{\theta(\sigma+\gamma)-\gamma(\sigma-1)}\right]^{\sigma-1} (\overline{p}_{ij}^c)^{1-\sigma} \left(\frac{e_{ij}^{P2}}{e_{ij}^{P1}}\right)^{1-\sigma} e_{ij}^{P1}. \tag{D.9}$$

We can now use this last result to express the aggregate nominal bilateral import demand, defined in equation (23), as a share of total expenditure as follows:

$$\frac{X_{ij}^D}{E_j} = k_2 M_{ij} P_j^{\sigma-1} (\overline{p}_{ij}^c)^{1-\sigma} \left(\frac{e_{ij}^{P2}}{e_{ij}^{P1}}\right)^{1-\sigma} e_{ij}^{P1} \tag{D.10}$$

where k_2 is a constant that depends only on the structural parameters σ, γ, and θ:

$$k_2 = \left[\frac{\theta(\sigma+\gamma)}{\theta(\sigma+\gamma)-\gamma(\sigma-1)}\right] \left[\frac{\theta(\sigma+\gamma)-\gamma\sigma}{\theta(\sigma+\gamma)-\gamma(\sigma-1)}\right]^{\sigma-1}.$$

In the fourth step, we remove the productivity threshold, φ^*_{ij}, using equation (8) and the mass of firms, M_{ij}, using an extended version of equation (11) allowing deviations from Pareto to yield:

$$\frac{X^D_{ij}}{E_j} = k_3 A_i^{1+\theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)} L_i w_i^{-\theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)} b_i^{-\theta\left(\frac{1+\gamma}{\gamma}\right)} E_j^{\theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{1}{\sigma-1}\right)}$$
$$P_j^{(\sigma-1)+\theta\left(\frac{1+\gamma}{\gamma}\right)} \tau_{ij}^{-\theta\left(\frac{1+\gamma}{\gamma}\right)} f_{ij}^{\frac{-\theta\left(\frac{1+\gamma}{\gamma}\right)}{\frac{1+\gamma}{\sigma+\gamma}(\sigma-1)}} (\overline{p}^c_{ij})^{1-\sigma} e^D_{ij} \tag{D.11}$$

where k_3 is a constant that depends only on the structural parameters σ, γ, θ, δ, and f^e:

$$k_3 = \frac{k_2}{\delta f^e}\left[\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)\right]^{\theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)}\left[\frac{\gamma}{\sigma+\gamma}\left((\sigma-1)\right)\right]^{\frac{-\theta\left(\frac{1+\gamma}{\gamma}\right)}{\frac{1+\gamma}{\sigma+\gamma}(\sigma-1)}}$$

and $e^D_{ij} \equiv e^{P1}_{ij}\left(\frac{e^{P2}_{ij}}{e^{P1}_{ij}}\right)^{1-\sigma} e^{P3}_{ij}$. While the first two RHS terms of e^D_{ij} were motivated above, the Pareto deviation e^{P3}_{ij} is associated with the mass of firms in the presence of deviations from Pareto. Referring back to section A.5 of Online Appendix A, the mass of firms M_{ij} turns out to be an extension of equation (A.41) with e^{P3}_{ij} appended to the RHS. Importantly, e^{P3}_{ij} has two components, one of which is e^{P4}_{ij} which surfaces because $1-G(\varphi^*_{ij}) = (\varphi^*_{ij})^{-\theta}e^{P4}_{ij}$ in the presence of deviations from Pareto. e^{P4}_{ij} is important for e^{P3}_{ij}, and hence e^D_{ij}, because of its particular influence on small exporters that tend to be near the cutoff productivity, consistent with the evidence that deviations from Pareto tend to surface for small exporters. The superscript D in e^D_{ij} refers to the role of deviations from Pareto on the "demand" side $(s_{ij} \equiv \frac{X^D_{ij}}{E_j})$.

This completes the derivation for the demand-side equation of the empirical model.[53]

[53] In reality, X^D_{ij} on the LHS of equation (D.11) is unobservable. Beginning with Feenstra (1994), empirical implementation has used actual industry-level bilateral trade flows, presumably reflecting (bilateral) partial equilibrium, i.e., $X^D_{ij} = X^S_{ij}$. This literature has not incorporated the general equilibrium considerations addressed in sections 2.4 and 2.5. However, the theoretical trade flows are determined under equilibrium conditions; in particular, the framework in sections 2.4 and 2.5 assumes goods-market clearing $R_i = E_i$, i.e., multilateral trade balances. Observed trade flows may be influenced by deviations from multilateral trade balances; in reality, multilateral trade imbalances exist at the aggregate level and at the industry level. In order to allow for the fact that actual trade flows are not likely to equal equilibrium trade flows, we *could* allow theoretical trade-flow shares to deviate from actual trade-flow shares (labeled s_{ij}) by an error term e^X_{ij}. However, this additional error term is unnecessary to obtain identification for estimation; hence, for simplicity, we ignore it.

D.2 Bilateral Export Supply

The derivations for the bilateral import-unit value equation are largely in the text and use part of section D.1 above. In those derivations, we use a constant k_4, defined as:

$$k_4 \equiv \left[\left(\frac{\gamma}{1+\gamma}\right)\left(\frac{\sigma-1}{\sigma}\right)\right]^{\gamma}\left[\frac{\theta(\sigma+\gamma)}{\theta(\sigma+\gamma)-\gamma\sigma}\right]\left[\frac{\theta(\gamma+\sigma)-\gamma\sigma+\gamma}{\theta(\gamma+\sigma)-\gamma\sigma}\right]^{\gamma}. \tag{D.12}$$

The full solution for the bilateral import-unit value equation in levels is:

$$\begin{aligned}\overline{p}_{ij}^{c} = k_5 A_i^{-1-\left(\frac{\theta-\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)} L_i^{-\frac{1}{1+\gamma}} w_i^{\frac{\gamma}{1+\gamma}+\left(\frac{\theta-\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)} b_i^{\left(\frac{\theta-\gamma}{\gamma}\right)\sigma} \tau_{ij}^{\frac{\theta-\gamma}{\gamma}} f_{ij}^{\frac{\frac{\theta-\gamma}{\gamma}}{\frac{1+\gamma}{\sigma+\gamma}(\sigma-1)}} \\ \times E_j^{\frac{1}{1+\gamma}+\left(\frac{\theta-\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)} P_j^{-\left(\frac{\theta-\gamma}{\gamma}\right)\sigma} s_{ij}^{\frac{1}{1+\gamma}} e_{ij}^{S},\end{aligned} \tag{D.13}$$

where

$$k_5 = \left(\frac{k_4}{\delta f^e}\right)^{-\frac{1}{1+\gamma}}\left[\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)\right]^{\left(\frac{\theta-\gamma}{\gamma}\right)\left(\frac{\sigma}{\sigma-1}\right)}\left[\frac{\gamma}{\sigma+\gamma}(\sigma-1)\right]^{\frac{\frac{\theta-\gamma}{\gamma}}{\frac{1+\gamma}{\sigma+\gamma}(\sigma-1)}}. \tag{D.14}$$

D.3 Reduced-Form Specification

In this subsection, we provide derivations associated with a complete specification of theoretical coefficients based upon the model that are associated with estimating equations (38) or (40); in the interest of brevity, we provide this reduced-form equation where the underlying variables include only Group 1 and Group 2 variables associated with Specification 2, "IMC-Partial." Adapting equations (28) and (29) for this specification, ϵ_{ijt} simplifies to:

$$\epsilon_{ijt} = \Delta^k \ln s_{ijt} + (\sigma-1)\Delta^k \ln \overline{p}_{ijt}^{c} + \theta\left(\frac{1+\gamma}{\gamma}\right)\Delta^k \ln tar_{ijt} + \theta\left(\frac{1+\gamma}{\gamma}\right)\Delta^k \ln trans_{ijt}, \tag{D.15}$$

and adapting equations (36) and (37) for this specification, ψ_{ijt} simplifies to:

$$\psi_{ijt} = -\frac{1}{1+\gamma}\Delta^k \ln s_{ijt} + \Delta^k \ln \overline{p}_{ijt}^{c} - \frac{\theta-\gamma}{\gamma}\Delta^k \ln tar_{ijt} - \frac{\theta-\gamma}{\gamma}\Delta^k \ln trans_{ijt}. \tag{D.16}$$

Taking the product of ϵ_{ijt} and ψ_{ijt} yields an equation with $\epsilon_{ijt}\psi_{ijt}$ on the LHS and 16 products on the RHS. Consolidating common RHS products yields:

$$\begin{aligned}
\epsilon_{ijt}\psi_{ijt} = {} & (\sigma-1)\left(\Delta^k \ln \overline{p}^c_{ijt}\right)^2 - \frac{1}{1+\gamma}\left(\Delta^k \ln s_{ijt}\right)^2 + \left(1-\frac{\sigma-1}{1+\gamma}\right)\left[\left(\Delta^k \ln s_{ijt}\right)\left(\Delta^k \ln \overline{p}^c_{ijt}\right)\right] \\
& + \left[\theta\left(\frac{1+\gamma}{\gamma}\right) - \frac{\theta-\gamma}{\gamma}(\sigma-1)\right]\left[\left(\Delta^k \ln \overline{p}^c_{ijt}\right)\left(\Delta^k \ln tar_{ijt}\right)\right] \\
& - \left(\frac{\theta}{\gamma} + \frac{\theta-\gamma}{\gamma}\right)\left[\left(\Delta^k \ln s_{ijt}\right)\left(\Delta^k \ln tar_{ijt}\right)\right] \\
& - \theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\theta-\gamma}{\gamma}\right)\left(\Delta^k \ln tar_{ijt}\right)^2 \\
& + \left[\theta\left(\frac{1+\gamma}{\gamma}\right) - \frac{\theta-\gamma}{\gamma}(\sigma-1)\right]\left[\left(\Delta^k \ln \overline{p}^c_{ijt}\right)\left(\Delta^k \ln trans_{ijt}\right)\right] \\
& - \left(\frac{\theta}{\gamma} + \frac{\theta-\gamma}{\gamma}\right)\left[\left(\Delta^k \ln s_{ijt}\right)\left(\Delta^k \ln trans_{ijt}\right)\right] \\
& - \theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\theta-\gamma}{\gamma}\right)\left(\Delta^k \ln trans_{ijt}\right)^2 \\
& - \theta\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{\theta-\gamma}{\gamma}\right)\left[\left(\Delta^k \ln tar_{ijt}\right)\left(\Delta^k \ln trans_{ijt}\right)\right]. \qquad \text{(D.17)}
\end{aligned}$$

Rearranging terms to isolate $(\sigma-1)(\Delta^k \ln \overline{p}^c_{ijt})^2$ on the LHS and dividing through by $(\sigma-1)$ yields the estimating equation (where $\xi_{ijt} = \epsilon_{ijt}\psi_{ijt}$):

$$\begin{aligned}
\left(\Delta^k \ln \overline{p}^c_{ijt}\right)^2 = {} & \frac{1}{(1+\gamma)(\sigma-1)}\left(\Delta^k \ln s_{ijt}\right)^2 + \left(\frac{\sigma-\gamma-2}{(1+\gamma)(\sigma-1)}\right)\left[\left(\Delta^k \ln s_{ijt}\right)\left(\Delta^k \ln \overline{p}^c_{ijt}\right)\right] \\
& - \left[\frac{\theta}{\sigma-1}\left(\frac{1+\gamma}{\gamma}\right) - \frac{\theta-\gamma}{\gamma}\right]\left[\left(\Delta^k \ln \overline{p}^c_{ijt}\right)\left(\Delta^k \ln tar_{ijt}\right)\right] \\
& + \left(\frac{\theta}{\gamma(\sigma-1)} + \frac{\theta-\gamma}{\gamma(\sigma-1)}\right)\left[\left(\Delta^k \ln s_{ijt}\right)\left(\Delta^k \ln tar_{ijt}\right)\right] \\
& + \theta\left(\frac{1+\gamma}{\gamma(\sigma-1)}\right)\left(\frac{\theta-\gamma}{\gamma}\right)\left(\Delta^k \ln tar_{ijt}\right)^2 \\
& - \left[\frac{\theta}{\sigma-1}\left(\frac{1+\gamma}{\gamma}\right) - \frac{\theta-\gamma}{\gamma}\right]\left[\left(\Delta^k \ln \overline{p}^c_{ijt}\right)\left(\Delta^k \ln trans_{ijt}\right)\right] \\
& + \left(\frac{\theta}{\gamma(\sigma-1)} + \frac{\theta-\gamma}{\gamma(\sigma-1)}\right)\left[\left(\Delta^k \ln s_{ijt}\right)\left(\Delta^k \ln trans_{ijt}\right)\right] \\
& + \theta\left(\frac{1+\gamma}{\gamma(\sigma-1)}\right)\left(\frac{\theta-\gamma}{\gamma}\right)\left(\Delta^k \ln trans_{ijt}\right)^2 \\
& + \theta\left(\frac{1+\gamma}{\gamma(\sigma-1)}\right)\left(\frac{\theta-\gamma}{\gamma}\right)\left[\left(\Delta^k \ln tar_{ijt}\right)\left(\Delta^k \ln trans_{ijt}\right)\right] + \xi_{ijt}.
\end{aligned}$$

(D.18)

D.4 Moment and Identification Conditions' Derivations

Estimation of equation (40) produces consistent coefficient estimates under two conditions. The first is the moment condition, $\mathbb{E}(\xi_{ijt}) \equiv \mathbb{E}(\epsilon_{ijt}\psi_{ijt}) = 0$; alternatively, the expectation can equal a constant as long as equation (40) includes an intercept (β_0). Recalling $\epsilon_{ijt} \equiv \Delta^k \ln e_{ijt}^{P1} + (1-\sigma)\Delta^k \ln e_{ijt}^{P2} + \Delta^k \ln e_{ijt}^{P3}$ and $\psi_{ijt} \equiv -\frac{1}{1+\gamma}\Delta^k \ln e_{ijt}^{P3} - \frac{1}{1+\gamma}\Delta^k \ln e_{ijt}^{P5}$:

$$\begin{aligned}
\mathbb{E}(\epsilon_{ijt}\psi_{ijt}) = & -\frac{1}{1+\gamma}\left[\text{cov}[\Delta^k \ln e_{ijt}^{P1}, \Delta^k \ln e_{ijt}^{P3}] + [\mathbb{E}(\Delta^k \ln e_{ijt}^{P1})][\mathbb{E}(\Delta^k \ln e_{ijt}^{P3})]\right] \\
& - \frac{1-\sigma}{1+\gamma}\left[\text{cov}[\Delta^k \ln e_{ijt}^{P2}, \Delta^k \ln e_{ijt}^{P3}] + [\mathbb{E}(\Delta^k \ln e_{ijt}^{P2})][\mathbb{E}(\Delta^k \ln e_{ijt}^{P3})]\right] \\
& - \frac{1}{1+\gamma}\left[\text{var}(\Delta^k \ln e_{ijt}^{P3}) + [\mathbb{E}(\Delta^k \ln e_{ijt}^{P3})]^2\right] \\
& - \frac{1}{1+\gamma}\left[\text{cov}[\Delta^k \ln e_{ijt}^{P1}, \Delta^k \ln e_{ijt}^{P5}] + [\mathbb{E}(\Delta^k \ln e_{ijt}^{P1})][\mathbb{E}(\Delta^k \ln e_{ijt}^{P5})]\right] \\
& - \frac{1-\sigma}{1+\gamma}\left[\text{cov}[\Delta^k \ln e_{ijt}^{P2}, \Delta^k \ln e_{ijt}^{P5}] + [\mathbb{E}(\Delta^k \ln e_{ijt}^{P2})][\mathbb{E}(\Delta^k \ln e_{ijt}^{P5})]\right] \\
& - \frac{1}{1+\gamma}\left[\text{cov}[\Delta^k \ln e_{ijt}^{P3}, \Delta^k \ln e_{ijt}^{P5}] + [\mathbb{E}(\Delta^k \ln e_{ijt}^{P3})][\mathbb{E}(\Delta^k \ln e_{ijt}^{P5})]\right] \\
& = -\left(\frac{1}{1+\gamma}\right)\text{var}(\Delta^k \ln e_{ijt}^{P3}) \equiv -4\left(\frac{1}{1+\gamma}\right)\text{var}(\ln e_{ijt}^{P3}) \equiv -4\left(\frac{1}{1+\gamma}\right)\sigma^2_{\ln e_{ij}^{P3}}
\end{aligned} \tag{D.19}$$

where σ still represents the elasticity of substitution in consumption but σ_z^2 represents the variance over time of the variable z. Note that we assume the expected values of the time-differenced (as well as reference-exporter-country differenced) deviations from the Pareto distributions of the underlying variables are zero (e.g., $\mathbb{E}(\Delta^k \ln e_{ijt}^{P1}) = 0$) and the double-differenced deviations have constant variances. Accordingly, we assume the covariances are zero as well (e.g., $\text{cov}[\Delta^k \ln e_{ijt}^{P1}, \Delta^k \ln e_{ijt}^{P3}] = 0$). The moment condition is satisfied as the RHS in the equation above, $-\left(\frac{1}{1+\gamma}\right)\text{var}(\Delta^k \ln e_{ijt}^{P3})$, is a constant.

The second condition necessary for consistent estimates of the coefficients is the identification condition. Following Feenstra (1994), this condition is equation (44) in the text. In the context of our model in section 4.1.5 (ignoring $\Delta \ln f_{ijt}^R$), the necessary condition for identification is equation (45). Equation (45) is obtained by recalling again $\epsilon_{ijt} \equiv \Delta^k \ln e_{ijt}^{P1} + (1-\sigma)\Delta^k \ln e_{ijt}^{P2} + \Delta^k \ln e_{ijt}^{P3}$ and $\psi_{ijt} \equiv -\frac{1}{1+\gamma}\Delta^k \ln e_{ijt}^{P3} - \frac{1}{1+\gamma}\Delta^k \ln e_{ijt}^{P5}$.

Using ϵ_{ijt}:

$$\begin{aligned}\operatorname{var}(\epsilon_{ijt}) &= \operatorname{var}\left[\Delta^k \ln e_{ijt}^{P1} + (1-\sigma)\Delta^k \ln e_{ijt}^{P2} + \Delta^k \ln e_{ijt}^{P3}\right] \\ &= \operatorname{var}(\Delta^k \ln e_{ijt}^{P1}) + (1-\sigma)^2 \operatorname{var}(\Delta^k \ln e_{ijt}^{P2}) + \operatorname{var}(\Delta^k \ln e_{ijt}^{P3}) \\ &\quad + 2\operatorname{cov}\left[\Delta^k \ln e_{ijt}^{P1}, (1-\sigma)\Delta^k \ln e_{ijt}^{P2}\right] \\ &\quad + 2\operatorname{cov}\left[\Delta^k \ln e_{ijt}^{P1}, \Delta^k \ln e_{ijt}^{P3}\right] \\ &\quad + 2\operatorname{cov}\left[(1-\sigma)\Delta^k \ln e_{ijt}^{P2}, \Delta^k \ln e_{ijt}^{P3}\right] \\ &= \sigma^2_{\Delta^k \ln e_{ij}^{P1}} + (1-\sigma)^2 \sigma^2_{\Delta^k \ln e_{ij}^{P2}} + \sigma^2_{\Delta \ln e_{ij}^{P3}}. \end{aligned} \tag{D.20}$$

The latter can be inserted into the LHS of equation (44) to produce the LHS of equation (45). Using ψ_{ijt}:

$$\begin{aligned}\operatorname{var}(\psi_{ijt}) &= \operatorname{var}\left[-\frac{1}{1+\gamma}\Delta^k \ln e_{ijt}^{P3} - \frac{1}{1+\gamma}\Delta^k \ln e_{ijt}^{P5}\right] \\ &= \left(\frac{1}{1+\gamma}\right)^2 \operatorname{var}(\Delta^k \ln e_{ijt}^{P3}) + \left(\frac{1}{1+\gamma}\right)^2 \operatorname{var}(\Delta^k \ln e_{ijt}^{P5}) \\ &\quad + 2\operatorname{cov}\left[-\frac{1}{1+\gamma}\Delta^k \ln e_{ijt}^{P3}, -\frac{1}{1+\gamma}\Delta^k \ln e_{ijt}^{P5}\right] \\ &= \left(\frac{1}{1+\gamma}\right)^2 \left[\sigma^2_{\Delta^k \ln e_{ij}^{P3}} + \sigma^2_{\Delta^k \ln e_{ij}^{P5}}\right]. \end{aligned} \tag{D.21}$$

The latter can be inserted into the RHS of equation (44) to produce the RHS of equation (45).

D.5 Moment and Identification Conditions' Derivations Including $\Delta \ln f_{ijt}^R$

In section 4.1.5, we addressed fixed trade-costs measurement. In that section, we noted that the majority of fixed trade costs are exporter specific or importer specific. However, for estimation, a residual measurement error exists, which we labeled f_{ijt}^R. With the introduction of this additional error term, we can readily modify the moment and identification conditions to accommodate this additional error term. As we will see, this has inconsequential effects on the moment and identification issues addressed earlier.

In this case, we need to redefine ϵ_{ijt} as:

$$\epsilon_{ijt} \equiv \Delta^k \ln e_{ijt}^{P1} + (1-\sigma)\Delta^k \ln e_{ijt}^{P2} + \Delta^k \ln e_{ijt}^{P3} + a^D \Delta \ln f_{ijt}^R \tag{D.22}$$

where a^D is a constant, and redefine ψ_{ijt} as:

$$\psi_{ijt} \equiv -\frac{1}{1+\gamma}\Delta^k \ln e_{ijt}^{P3} - \frac{1}{1+\gamma}\Delta^k \ln e_{ijt}^{P5} + a^S \Delta \ln f_{ijt}^R \tag{D.23}$$

where a^S is another constant. Hence, the moment condition becomes:

$$\begin{aligned}
\mathbb{E}(\epsilon_{ijt}\psi_{ijt}) &= -\frac{1}{1+\gamma}\Big[\text{cov}[\Delta^k \ln e_{ijt}^{P1}, \Delta^k \ln e_{ijt}^{P3}] + [\mathbb{E}(\Delta^k \ln e_{ijt}^{P1})][\mathbb{E}(\Delta^k \ln e_{ijt}^{P3})]\Big] \\
&- \frac{1-\sigma}{1+\gamma}\Big[\text{cov}[\Delta^k \ln e_{ijt}^{P2}, \Delta^k \ln e_{ijt}^{P3}] + [\mathbb{E}(\Delta^k \ln e_{ijt}^{P2})][\mathbb{E}(\Delta^k \ln e_{ijt}^{P3})]\Big] \\
&- \frac{1}{1+\gamma}\Big[\text{var}(\Delta^k \ln e_{ijt}^{P3}) + [\mathbb{E}(\Delta^k \ln e_{ijt}^{P3})]^2\Big] \\
&- \frac{a^D}{1+\gamma}\Big[\text{cov}[\Delta^k \ln f_{ijt}^{R}, \Delta^k \ln e_{ijt}^{P3}] + [\mathbb{E}(\Delta^k \ln f_{ijt}^{R})][\mathbb{E}(\Delta^k \ln e_{ijt}^{P3})]\Big] \\
&- \frac{1}{1+\gamma}\Big[\text{cov}[\Delta^k \ln e_{ijt}^{P1}, \Delta^k \ln e_{ijt}^{P5}] + [\mathbb{E}(\Delta^k \ln e_{ijt}^{P1})][\mathbb{E}(\Delta^k \ln e_{ijt}^{P5})]\Big] \\
&- \frac{1-\sigma}{1+\gamma}\Big[\text{cov}[\Delta^k \ln e_{ijt}^{P2}, \Delta^k \ln e_{ijt}^{P5}] + [\mathbb{E}(\Delta^k \ln e_{ijt}^{P2})][\mathbb{E}(\Delta^k \ln e_{ijt}^{P5})]\Big] \\
&- \frac{1}{1+\gamma}\Big[\text{cov}[\Delta^k \ln e_{ijt}^{P3}, \Delta^k \ln e_{ijt}^{P5}] + [\mathbb{E}(\Delta^k \ln e_{ijt}^{P3})][\mathbb{E}(\Delta^k \ln e_{ijt}^{P5})]\Big] \\
&- \frac{a^D}{1+\gamma}\Big[\text{cov}[\Delta^k \ln f_{ijt}^{R}, \Delta^k \ln e_{ijt}^{P5}] + [\mathbb{E}(\Delta^k \ln f_{ijt}^{R})][\mathbb{E}(\Delta^k \ln e_{ijt}^{P5})]\Big] \\
&+ a^D a^S\Big[\text{var}(\Delta^k \ln f_{ijt}^{R}) + [\mathbb{E}(\Delta^k \ln f_{ijt}^{R})]^2\Big] \\
&+ a^S\Big[\text{cov}[\Delta^k \ln e_{ijt}^{P1}, \Delta^k \ln f_{ijt}^{R}] + [\mathbb{E}(\Delta^k \ln e_{ijt}^{P1})][\mathbb{E}(\Delta^k \ln f_{ijt}^{R})]\Big] \\
&+ (1-\sigma)a^S\Big[\text{cov}[\Delta^k \ln e_{ijt}^{P2}, \Delta^k \ln f_{ijt}^{R}] + [\mathbb{E}(\Delta^k \ln e_{ijt}^{P2})][\mathbb{E}(\Delta^k \ln f_{ijt}^{R})]\Big] \\
&+ a^S\Big[\text{cov}[\Delta^k \ln e_{ijt}^{P3}, \Delta^k \ln f_{ijt}^{R}] + [\mathbb{E}(\Delta^k \ln e_{ijt}^{P3})][\mathbb{E}(\Delta^k \ln f_{ijt}^{R})]\Big] \\
&= -\left(\frac{1}{1+\gamma}\right)\text{var}(\Delta^k \ln e_{ijt}^{P3}) + a^D a^S\,\text{var}(\Delta^k \ln f_{ijt}^{R})
\end{aligned} \tag{D.24}$$

which still satisfies the moment condition.

The second condition necessary for consistent estimates of the coefficients is the identification condition. The extension has inconsequential effects on the identification condition.

Using the redefined ϵ_{ijt} from above:

$$\begin{aligned}
\text{var}(\epsilon_{ijt}) &= \text{var}\left[\Delta^k \ln e^{P1}_{ijt} + (1-\sigma)\Delta^k \ln e^{P2}_{ijt} + \Delta^k \ln e^{P3}_{ijt} + a^D \Delta^k \ln f^R_{ijt}\right] \\
&= \text{var}(\Delta^k \ln e^{P1}_{ijt}) + (1-\sigma)^2 \text{var}(\Delta^k \ln e^{P2}_{ijt}) + \text{var}(\Delta^k \ln e^{P3}_{ijt}) + (a^D)^2 \text{var}(\Delta^k \ln f^R_{ijt}) \\
&\quad + 2\,\text{cov}\left[\Delta^k \ln e^{P1}_{ijt}, (1-\sigma)\Delta^k \ln e^{P2}_{ijt}\right] \\
&\quad + 2\,\text{cov}\left[\Delta^k \ln e^{P1}_{ijt}, \Delta^k \ln e^{P3}_{ijt}\right] \\
&\quad + 2\,\text{cov}\left[(1-\sigma)\Delta^k \ln e^{P2}_{ijt}, \Delta^k \ln e^{P3}_{ijt}\right] \\
&\quad + 2\,\text{cov}\left[\Delta^k \ln e^{P1}_{ijt}, a^D \Delta^k \ln f^R_{ijt}\right] \\
&\quad + 2\,\text{cov}\left[(1-\sigma)\Delta^k \ln e^{P2}_{ijt}, a^D \Delta^k \ln f^R_{ijt}\right] \\
&\quad + 2\,\text{cov}\left[\Delta^k \ln e^{P3}_{ijt}, a^D \Delta^k \ln f^R_{ijt}\right] \\
&= \sigma^2_{\Delta^k \ln e^{P1}_{ij}} + (1-\sigma)^2 \sigma^2_{\Delta^k \ln e^{P2}_{ij}} + \sigma^2_{\Delta \ln e^{P3}_{ij}} + (a^D)^2 \sigma^2_{\Delta^k \ln f^R_{ij}}.
\end{aligned} \tag{D.25}$$

Using the redefined ψ_{ijt} from above:

$$\begin{aligned}
\text{var}(\psi_{ijt}) &= \text{var}\left[-\frac{1}{1+\gamma}\Delta^k \ln e^{P3}_{ijt} - \frac{1}{1+\gamma}\Delta^k \ln e^{P5}_{ijt} + a^S \Delta^k \ln f^R_{ijt}\right] \\
&= \left(\frac{1}{1+\gamma}\right)^2 \text{var}(\Delta^k \ln e^{P3}_{ijt}) + \left(\frac{1}{1+\gamma}\right)^2 \text{var}(\Delta^k \ln e^{P5}_{ijt}) + (a^S)^2 var(\Delta^k \ln f^R_{ijt}) \\
&\quad + 2\,\text{cov}\left[-\frac{1}{1+\gamma}\Delta^k \ln e^{P3}_{ijt}, -\frac{1}{1+\gamma}\Delta^k \ln e^{P5}_{ijt}\right] \\
&\quad + 2\,\text{cov}\left[-\frac{1}{1+\gamma}\Delta^k \ln e^{P3}_{ijt}, a^S \Delta^k \ln f^R_{ijt}\right] \\
&\quad + 2\,\text{cov}\left[-\frac{1}{1+\gamma}\Delta^k \ln e^{P5}_{ijt}, a^S \Delta^k \ln f^R_{ijt}\right] \\
&= \left(\frac{1}{1+\gamma}\right)^2 \sigma^2_{\Delta^k \ln e^{P3}_{ij}} + \left(\frac{1}{1+\gamma}\right)^2 \sigma^2_{\Delta^k \ln e^{P5}_{ij}} + (a^S)^2 \sigma^2_{\Delta^k \ln f^R_{ij}}.
\end{aligned} \tag{D.26}$$

The latter two results can be inserted into the RHS of equation (44) to produce an easily adjusted version of the RHS of equation (45).