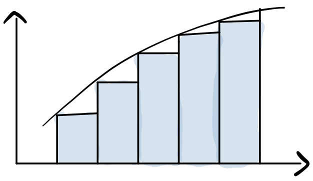
hw 2

9.4.

" If $f(x)$ is increasing on $[a,b]$, then left Riemann sum is an underestimate. "

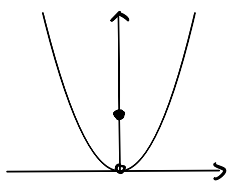


• Always.

9.5

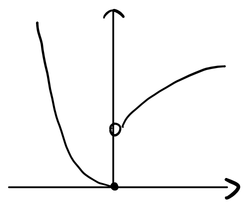
" If $f(x)$ has a discontinuity at $x=0$, then $\int_{-1}^{1} f(x)\,dx$ does not exist. "

• sometimes



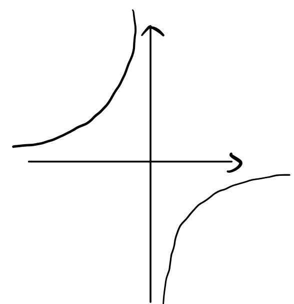
point / removable dis.

2 sides limit $\exists$ and same



jump dis.

2 sides limit $\exists$, but different



asymptotic dis.

2 sides limit $\not\exists$.

9.7.

$$\int \sec^2 t \tan^3 t \, dt$$

$$= \int (1+\tan^2 t)\tan^3 t \, dt$$

$u = \tan t \qquad du = \sec^2 x \, dx$

Let $y = \arctan x \iff x = \tan y$

$\Rightarrow x^2 = \frac{\sin^2 y}{\cos^2 y}$, $\quad x^2+1 = \frac{\cancel{\sin^2 y + \cos^2 y}\ 1}{\cos^2 y}$

$\Rightarrow \frac{1}{x^2+1} = \cos^2 y$

$\Rightarrow \frac{1}{\sqrt{x^2+1}} = \cos y$

$= \cos \arctan x$

$\downarrow$

$\cos(\arctan u) = \frac{1}{\sqrt{1+u^2}}$

$$= \int (1+u^2)u^3 \frac{1}{\sec^2 x} \, du$$

$$= \int (1+u^2)u^3 \cos^2 x \, du$$

$$= \int (1+u^2)u^3 \underline{\cos^2(\arctan u)} \, du$$

$$= \int (1+\cancel{u^2})u^3 \cdot \frac{1}{\cancel{1+u^2}} \, du$$

$= \int u^3 \, du$

$= \frac{1}{4} u^4 + C$

$= \frac{1}{4} \tan^4 t + C$

9 12

$\int \frac{x}{1+x^4} \, dx$

$u = x^2$, $du = 2x \, dx$

$= \int \frac{\cancel{x}}{1+u^2} \frac{1}{2\cancel{x}} \, du$

$= \frac{1}{2} \int \frac{1}{1+u^2} \, du$

$= \frac{1}{2} \arctan u + C$

$= \frac{1}{2} \arctan x^2 + C$

Integration by parts

$\int u\,dv = uv - \int v\,du$

Strategy :

Logarithmic functions

Inverse trig f.

Arithmetic (polynomials)

Trig f.

Exponential f.

Practice problem.

a) $x^3 + 3x^2 + 5$

u polynomial

b) e^{3x}

exponential f.

$\Rightarrow$ u

c) e^{-x^2}

exponential f.

$\Rightarrow$ u

d) $\cos(3x)$

Trig f.

2. a) $\int x^2 e^x\,dx$

$= \int x^2\,de^x$

$= x^2 e^x - \int e^x \, d(x^2)$

$= x^2 e^x - 2\int e^x \, x \, dx$

$= x^2 e^x - 2\int x \, de^x$

$= x^2 e^x - 2\left(xe^x - \int e^x \, dx\right)$

$= x^2 e^x - 2xe^x + 2e^x + C$

b) $\int \ln(2x+1)\, dx$

$u = 2x+1 \Rightarrow du = 2\,dx$

$\Rightarrow \int \ln u \, \frac{1}{2} \, du$

$= \frac{1}{2}\int \ln u \, du$

integration by parts

$= \frac{1}{2}\left(u \ln u - \int u \, d(\ln u)\right)$

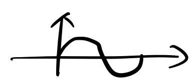
$= \frac{1}{2} u \ln u - \frac{1}{2}\int u \cdot \frac{1}{u} \, du$

$= \frac{1}{2} u \ln u - \frac{1}{2} u + C$

$= \frac{1}{2}(2x+1)\ln(2x+1) - \frac{1}{2}(2x+1) + C$

3.

a) $\int_0^{\frac{\pi}{3}} \cos x \sin x \, dx$



Let $\sin x = u$, $du = \cos x \, dx$

$\sin 0 = 0 \quad \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\Rightarrow u \in [0, \frac{\sqrt{3}}{2}]$

$= \int_0^{\frac{\sqrt{3}}{2}} \overset{u}{\cancel{\sin x}} \, d \, \overset{u}{\cancel{\sin x}}$

$= \frac{1}{2} u^2 \Big|_0^{\frac{\sqrt{3}}{2}}$

$= \frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$

b) $\int_1^e \frac{\ln t}{t^2} \, dt$

$= \int_1^e \ln t \, d\left(-\frac{1}{t}\right)$

$- \int -\frac{1}{t} \, d\ln t$

$= \int \frac{1}{t}\frac{1}{t} dt$

$= -\ln t \cdot \frac{1}{t} + \int_1^e \frac{1}{t}\frac{1}{t} \, dt$

$= -\ln t \cdot \frac{1}{t}\Big|_1^e - \frac{1}{t}\Big|_1^e$

$$= -\underbrace{\frac{\ln e}{e}}_{\frac{1}{e}} + 0 - \frac{1}{e} + 1 = 1 - \frac{2}{e}$$

c) $\int_0^{\pi} \sin(2\theta)\, e^{4\theta}\, d\theta$

$$= \frac{1}{2}\int_0^{\pi} e^{4\theta} (-d \cos 2\theta)$$

$$= -\frac{1}{2}\int_0^{\pi} e^{4\theta}\, d \cos 2\theta$$

$$= -\frac{1}{2}\left(e^{4\theta} \cos 2\theta \Big|_0^{\pi} - \int_0^{\pi} \cos 2\theta \, d e^{4\theta} \right)$$

$$= -\frac{1}{2} e^{4\theta} \cos 2\theta \Big|_0^{\pi} + \frac{1}{2} \cdot 4 \int_0^{\pi} \cos 2\theta \, e^{4\theta} d\theta$$

$$= -\frac{1}{2} e^{4\pi} + \frac{1}{2} + \frac{2}{2}\int_0^{\pi} e^{4\theta}\, d \sin 2\theta$$

$$= -\frac{1}{2} e^{4\pi} + \frac{1}{2} + \left(\cancelto{0}{e^{4\theta} \sin 2\theta \Big|_0^{\pi}} - \int_0^{\pi} \sin 2\theta \, d e^{4\theta} \right)$$

$$= -\frac{1}{2} e^{4\pi} + \frac{1}{2} - 4\int_0^{\pi} \sin 2\theta \, e^{4\theta} d\theta$$

$$I := \int_0^{\pi} \sin 2\theta \, e^{4\theta} d\theta$$

$$\Rightarrow \quad I = -\frac{1}{2} e^{4\pi} + \frac{1}{2} - 4I$$

$$\Rightarrow \quad I = \frac{1}{5}\left(\frac{1}{2} - \frac{1}{2} e^{4\pi} \right)$$