Project 1. Japanese Designs: New and Old

ITOKI corporation is a company that originated in Japan. (It was founded there in 1890.) It is now multinational with an office furniture branch ITOKIdesígn in New York City. One of its products is illustrated below. The key element is a polygonally shaped module about 15 inches high. Several such modules can be arranged in a variety of ways to provide seating for different purposes. The two modular arrangements shown in Figure 1 involve five and

Figure 1

seven such modules respectively. The shape of the module is to an important extent determined by the quadrilateral at the top. What exactly is its shape? What geometry makes it possible to come up with the various different arrangements? Is it a quadrilateral that can be constructed precisely with straightedge and compass? The explanation of how this is done follows. Study it carefully.

The answer begins with Figure 2.10 on page 21 of the text. This figure is reproduced in Figure 2. Figure 3 below is directly related to Figure 2. Problem 13 on page 45 is also relevant. By applying the conclusion of this problem to Figure 3, we get that $\frac{a}{s} = \tan 18° = \frac{\sin 18°}{\cos 18°} = \frac{\sqrt{5}-1}{4} \cdot \sqrt{\frac{8}{5+\sqrt{5}}}$. The last term can be simplified. Start by squaring both sides to get

Figure 2
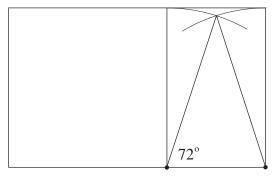
$(\tan 18°)^2 = \frac{5-2\sqrt{5}+1}{16} \cdot \frac{8}{5+\sqrt{5}} = \frac{3-\sqrt{5}}{5+\sqrt{5}}$. So $\tan 18° = \sqrt{\frac{3-\sqrt{5}}{5+\sqrt{5}}}$. Set $m = \frac{a}{s} = \tan 18°$. By pushing

Figure 3
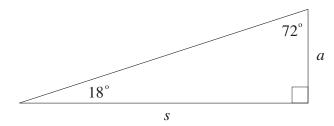
the buttons of a calculator, $m = \tan 18° \approx \sqrt{\frac{0.76393}{7.23607}} \approx 0.32492$. So $a = ms \approx 0.32492s$.

Consider Figure 3 and let $b = \frac{1}{2}(s-a)$. Observe that $b = \frac{1}{2}(s-ms) = \frac{1}{2}(1-m)s \approx$ 0.33754s. Take a square of side s and turn to Figure 4. Draw the horizontal and vertical
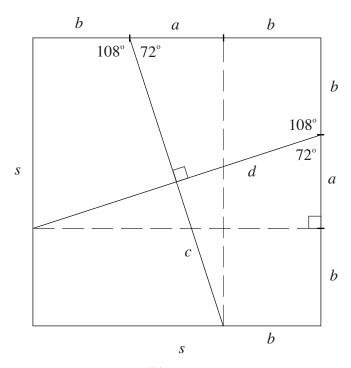

Figure 4

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dashed lines as indicated and place the triangle of Figure 3 into the square as shown. Draw in the slating segment from the base of the square to the top as shown. The two slanting segments partition the square into four quadrilaterals. The next statement is valid for any quadrilateral. Verify it.

Proposition. The interior angles of any quadrilateral add up to 360°.

When applied to Figure 4, this tells us that the two slanting segments in Figure 4 intersect at right angles. Check that the sequence $108°, b, 90°, a + b, 72°$ of consecutive angle, side, angle, side, angle occurs in all four of the quadrilaterals. Refer to Figure 5 and notice that there is only one way to complete the diagram to a quadrilateral. This means that the four
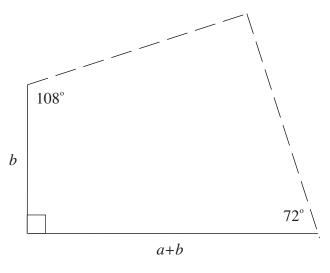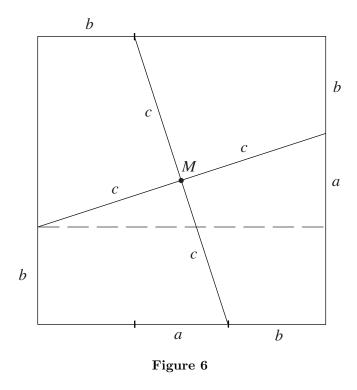

Figure 5

quadrilaterals are identical. All are given by Figure 5. This is the quadrilateral of the module of the ITOKI design.

Return to Figure 4 and let c and d be the indicated lengths of the two segments. For the quadrilateral on the lower right the segment labelled c is the side opposite the angle 72°. For the quadrilateral on the upper right, the side opposite the angle 72° is the segment labelled d. Since the two quadrilaterals are identical, it follows that the lengths c and d are equal. Let M be the point of intersection of the two slanting segments. From the observation just made, we know that each of the four segments from M to the side of the square is c units long. Refer to Figure 6. Notice that $a^2 + s^2 = (2c)^2$, and therefore that
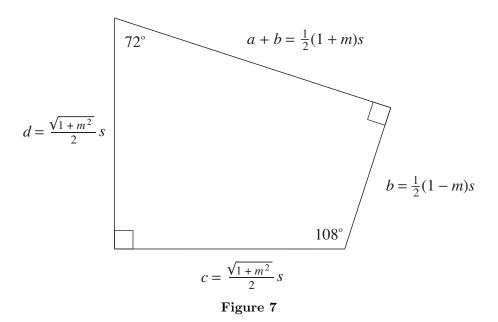
$$d = c = \sqrt{\tfrac{1}{4}\left(a^2 + s^2\right)} = \tfrac{1}{2}\sqrt{m^2 s^2 + s^2} = \tfrac{\sqrt{1+m^2}}{2}\, s \approx 0.52573s.$$

We have determined that the lengths of the sides of the ITOKI quadrilateral in terms of the length of the side s of the square and the constant $m = \tan 18° \approx 0.32492$ are

$$a = ms, \ \ b = \tfrac{1}{2}(1 - m)s, \ \ a + b = \tfrac{1}{2}(1 + m)s \ \ \text{and} \ \ c = d = \tfrac{\sqrt{1+m^2}}{2}s.$$

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Figure 6

The relevant dimensions of the quadrilateral that is the basis of the module of ITOKIdesígn are summarized in Figure 7. It turns out that the point of intersection M is the midpoint

In the figure:
$$72°$$
$$a + b = \tfrac{1}{2}(1 + m)s$$
$$d = \frac{\sqrt{1 + m^2}}{2}s$$
$$b = \tfrac{1}{2}(1 - m)s$$
$$108°$$
$$c = \frac{\sqrt{1 + m^2}}{2}s$$

Figure 7

of the square. Use similar triangles to verify this fact (even though it will not be relevant in the remainder of the discussion).

Figures 8 and 9 show the ITOKI quadrilateral in the context of two of the several
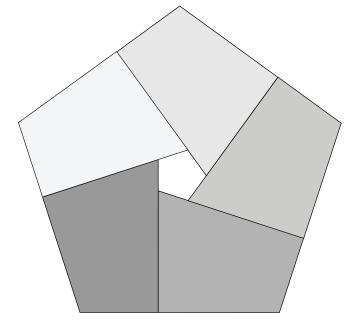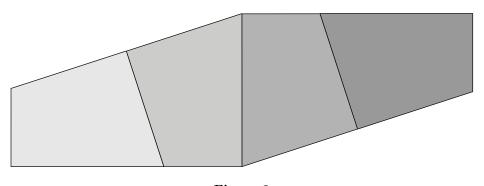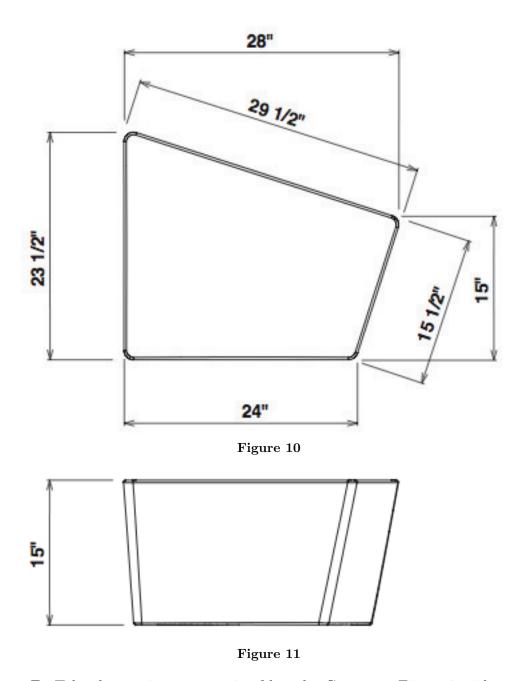
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Figure 8

Figure 9

different configurations that ITOKIdesígn suggests. This modular design by the firm won an Innovation Award in 2010.

The company includes the diagram of Figure 10 in its prospectus. It shows the top surface of the module with rounded corners and it provides its dimensions. Figure 11 shows a vertical section of the ITOKI module that corresponds to Figure 10.

Project 1A. Study the diagram of Figure 10. What do the listed dimensions imply about the angles that the sides make at the upper left and lower right of the diagram? Assess the accuracy of the dimensions of the diagram of Figure 10 by comparing them with the exact data of Figure 7. Start by choosing an appropriate s in inches. (What choice for s results in the length of 24 inches for the bottom side?) Can you suggest more accurate alternatives—in whole inches or whole and half inches—to the numbers that Figure 10 provides?

Figure 10

Figure 11

Project 1B. Take the centimeter as unit of length. Construct Figure 3 with straightedge and compass, taking $s = 10$. Then go on and construct the square of Figure 4, as well as the four ITOKI quadrilaterals inside it. Cut twelve such quadrilaterals from sheets of paper and make models of the flat upper surfaces of the two configurations of Figure 1.

Project 1C. Continue with a square of side $s = 10$ in centimeters and repeat the constructions of Figures 3 and 4, this time with the angles $30°, 60°,$ and $120°$ (in place of $18°, 72°,$ and $108°$). Cut as many copies of the resulting quadrilateral as necessary from sheets of paper and use them to model the flat upper surfaces of the two designs that are analogous to those of Figure 1.

Interesting hexagonal geometries were created by the firm Shigeru Ban Architects in 2010 for the club house of the spectacular Haesley Nine Bridges golf course in Korea. See Figure 12 and http://www.core.form-ula.com/2011/03/10/shigeru-ban-nine-bridges/. A similar

Figure 12

Figure 13

design was used for the fruit bowl depicted in Figure 13.

Shigeru Ban has become famous for his designs of shelters for victims of natural disasters in places like Rwanda, Turkey, India, China, Haiti, and Japan. His cardboard cathedral in Christchurch, New Zealand, built after the devastating earthquake of 2011 caused major damage to the Anglican cathedral, has become a tourist attraction. See

http://www.christchurchnz.com/destinations/christchurch/cardboard-cathedral/

Mr. Ban is also known for more conventional projects, like the club house of Haesley Nine Bridges golf course in Korea already mentioned, the Pompidou Center's satellite museum in Metz, France, with its roof that is inspired by a woven bamboo hat, and the new 33,000 square foot home of the Aspen Art Museum (it opened in August, 2014) with its woven exterior wood screen that shields the building from the sun. See

http://aspenartmuseum.org/

http://www.archdaily.com/546446/aspen-art-museum-shigeru-ban-architects/

Shigeru Ban was awarded the prestigious Pritzker Architecture Prize in 2014. See

http://www.pritzkerprize.com

He takes his fellow architects to task for not putting their expertise to work for a greater social good and asserts "I'm not saying I'm against building monuments, but I'm thinking we can work more for the public, and continues "Architects are not building temporary housing because we are too busy building for the privileged people."

Project 1D. Explore these and other innovative designs by this award winning firm on

http://www.shigerubanarchitects.com

Pick out one design that you find compelling, present some images of it, and write a short essay that describes it and explains why you find it interesting. Also watch

https//www.youtube.com/watch?v=IjHlyKT_Uug and

https://www.youtube.com/watch?v=lEazoVuwQ2s

and include comments about each of them in your write up.

Some beautiful pentagonal designs appear in Japanese art. The formal presentation of a gift by placing it on a wooden or lacquered tray and covering it with a square cloth became a common ceremony in Japan during the Edo or Tokugawa period (1615–1867). Since ceremonial procedures, including the giving of gifts, permeated all facets of Japanese life, these square cloths were often elaborately decorated. Wealthy families commissioned

Figure 14

famous artists to create them. Known as *fukusas*, they were often preserved for generations as treasured heirlooms. A fukusa was generally hand woven of silk, hand painted and signed by the artist, and then embroidered with threads of pure gold, platinum, or silver. *The Coral Crane* is a wonderful silk fukusa. The richly embroidered Mandarin crane represents good fortune, a fitting theme for the cover of a gift. The family crest, a stylized image of a Japanese Bellflower, or Kikyo, appears in golden threads on the other side of the fukusa.

Project 1E. Construct the pentagonal design pictured in Figure 15 below with straightedge and compass. Base your construction on a circle of radius 10 cm (and include the construction of the small circular arcs of the petals).
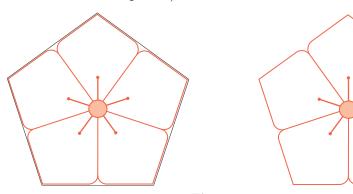

Figure 15

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For a final example of an interesting geometric construction we'll return to the U.S., to Chicago, in particular. In an article in 2004, Ted Whalen, a research engineer at North-
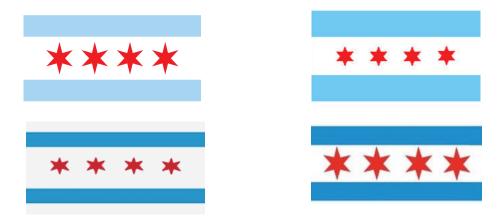

Figure 16

western University at the time, pointed out that the shapes of the four six pointed red stars of the flag of the City of Chicago vary significantly from one rendition of the flag to another.
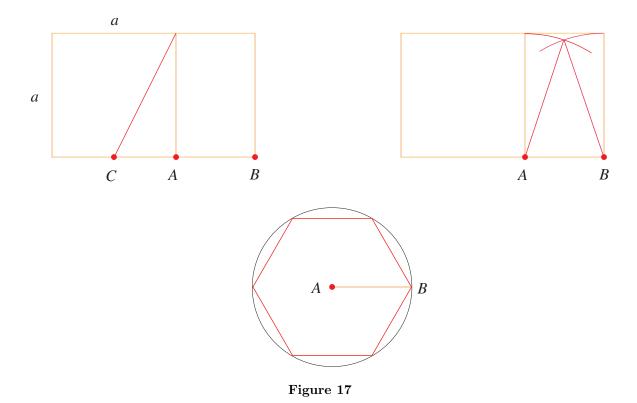

Figure 17

Figure 16 shows four versions of the flag with stars that are thicker, thinner, longer, and shorter. He suggested that this was an unsatisfactory state of affairs and that there should be a standard shape. He proposed the design outlined in the sequence of diagrams in Figure 17.

Putting the components together results in the six pointed star of Figure 18. This beautiful construction combines the golden rectangle with the regular hexagon. The star that it gives rise to is pleasing to the eye and steers a middle ground between thick and thin and short
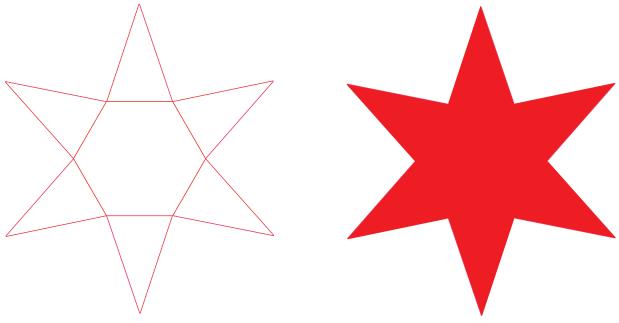

Figure 18

and long.

Project 1F. Construct a golden rectangle and in particular an angle of 72°. Then construct a hexagon with side length 4 centimeters and complete it to a construction of Ted Whalen's Chicago star. Compute the distance between two opposite tips of the star you have constructed. Compute the distance between two adjacent tips.

By the way, Chicago has a very important place in the history of architecture. For example, skyscrapers were first built in Chicago (and not New York City).