

Math 10270 : Final  
May 7, 2018

Name:

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Calculators are allowed, together with an  $8\frac{1}{2}'' \times 11''$  sheet of paper  
with notes in your own handwriting.

**Sign the pledge.** “On my honor, I have neither given nor received  
unauthorized aid on this Exam”:

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1. (10 points.) Explain how to find the midpoint of a line segment using only a straightedge and compass.

Describe each stage carefully and illustrate the steps with sketches.

2. (10 points.) (i) A Gothic arch has a span of 30ft. Find the height of the arch at its highest point.

(ii) Repeat part (i) for a Roman arch of span 30 ft.

(iii) Repeat part (i) for a Gothic acute fifth with span 30 ft.

3. (10 points.) The distance from the focus  $F$  of a parabola to its directrix is 4 units. Suppose the parabola is cut by a line parallel to the directrix and 3 units away. Use the result of Archimedes to determine the area of the parabolic section.

4. (10 points.) A 10 foot ladder makes an angle of  $60^\circ$  with the ground and rests against a frictionless vertical wall. Assume that the ladder itself is weightless, but carries a weight of 100 lb. at its midpoint. Calculate the horizontal frictional force  $H$  applied by the floor to the base of the ladder.

5. (10 points.) (i) Draw an equilateral triangle with one horizontal edge and label the vertices. Describe the six symmetries both in words and using matrix notation.

(ii) Describe the effect of rotating by  $120^\circ$  counterclockwise and then reflecting in the vertical axis.

(ii) Describe the effect of reflecting in the vertical axis and then rotating by  $120^\circ$  counterclockwise.

6. (10 points.) Describe some distinguishing features of Gothic architecture. How were these influenced by a better understanding of forces?

7. (10 points.) (i) Find parametric equations for the line in space which passes through the points  $(1, 2, 3)$  and  $(4, -1, 4)$ .

(ii) For which values of  $r$  does the line intersect the sphere of radius  $r$  with center the origin  $(0, 0, 0)$ ?



8. (10 points.) Two 6 foot high walls are built over the  $xy$ -plane (the ground). The walls start at the point  $(1, 1)$  and extend indefinitely in the region  $y > 1$ . The walls make an angle of  $45^\circ$  with the line  $y = 1$  and the angle between them is  $90^\circ$ .

Suppose that an artist's eye is at the point  $(0, -2, 5)$  and a canvas lies in the  $xz$ -plane. Sketch the image of the two walls on the canvas. (Start by drawing in the vertical corner at the end of the walls.) If the walls are made of horizontal timber frames, sketch in a few of the frames.

9. (10 points.) (i) What is hinging failure in an arch? Draw a sketch.

(ii) Give a brief statement of the Safe Theorem.

(iii) What is the relation of this to the dome of St. Peter's?

10. (10 points.) Consider the hanging truss shown.

(i) Assuming the truss to be symmetric and in equilibrium find the vertical components of the forces in each beam.

(ii) Suppose that the angle  $\theta_1$  is  $45^\circ$ . Find the horizontal components of the forces and the angle  $\theta_2$ .