Check Your Understanding

Definitions and Proofs

Definition: A topological space X with topology T_X is *Hausdorff* if for every $x, y \in X$ with $x \neq y$, there are $A, B \in T_X$ such that $x \in A$, $y \in B$, and $A \cap B = \emptyset$.

Problem 1: *Is* $\mathbb{N}$ *with* $T_{\mathbb{N}} = \mathcal{P}(\mathbb{N})$ *Hausdorff? Prove you are correct.*

Problem 2: *Prove* $\{\emptyset, \mathbb{N}\}$ *is a topology on* $\mathbb{N}$.

Problem 3: *Is* $\mathbb{N}$ *with* $T_{\mathbb{N}}$ *Hausdorff? Prove you are correct!*

Definition: A subset $X \subset \mathbb{N}$ is cofinite if $\{n \in \mathbb{N} : n \notin X\}$ is finite.

Problem 4: *Is the set of cofinite subsets of* $\mathbb{N}$ *a topology on* $\mathbb{N}$*? Prove you are correct!*