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★**Partial differential equations in several complex variables.**

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Generations of students have learned the theory of the $\overline{\partial}$-Neumann problem from the monograph of G. B. Folland and J. J. Kohn [*The Neumann problem for the Cauchy-Riemann complex*, Ann. of Math. Stud., 75, Princeton Univ. Press, Princeton, N.J., 1972; MR0461588]. In the three decades since the publication of that little book, the theory of the multidimensional Cauchy-Riemann equations has undergone tremendous development. Now two prominent researchers in the field, both former students of Kohn, have written a "next generation" volume that may serve both as a text for students and as a reference for workers in the area.

The first three of the twelve chapters introduce background material about multidimensional complex analysis: the biholomorphic inequivalence of the ball and the polydisc, the Cauchy and Bochner-Martinelli integral representations, holomorphic extension phenomena, pseudoconvexity, and the Levi problem. The next three chapters are devoted to the Hilbert space approach to solvability and regularity of the $\overline{\partial}$-equation: the L^2 existence theory on pseudoconvex domains, the $\frac{1}{2}$-subelliptic estimate for the $\overline{\partial}$-Neumann problem on strongly pseudoconvex domains, Sobolev estimates for the $\overline{\partial}$-Neumann problem on more general pseudoconvex domains, boundary regularity of biholomorphic mappings, irregularity of the Bergman projection on worm domains.

"The second half of the book is intended as a self-contained introduction to the tangential Cauchy-Riemann equations", according to the authors' preface. Chapter 7 introduces the tangential Cauchy-Riemann complex and discusses Lewy's equation. Chapter 8 proves a $\frac{1}{2}$-subelliptic estimate and local regularity for $\Box_b$ under condition $Y(q)$, while Chapter 9 establishes the L^2 existence theory of $\overline{\partial}_b$ on pseudoconvex boundaries. Then the theme changes to integral representations: Chapter 10 constructs a fundamental solution for $\Box_b$ on the Heisenberg group, and Chapter 11 uses integral formulas to study L^p and Hölder estimates for solutions of $\overline{\partial}$ and $\overline{\partial}_b$ on strictly convex domains. The concluding chapter addresses the embeddability of abstract CR structures.

Compared to a text on the same subject by S. G. Krantz [*Partial differential equations and complex analysis*, CRC, Boca Raton, FL, 1992; MR1207812], the book under review has less preparatory background about partial differential equations but a much more extensive account of contemporary researches on $\overline{\partial}$ and $\overline{\partial}_b$. Anyone planning to do research in this area will want to have a copy of the book. *Harold P. Boas*