Partial Differential Equations (Math 40750) – Fall 2018

MWF 3:00-3:50, HAYE 231

Instructor: Mei-Chi Shaw

Course Description: Partial differential equations (PDEs) have long been the backbone of mathematical analysis and other scientific fields. They describe various fascinating and useful phenomena in physics, mathematical biology, mathematical finance/economics etc. Modern PDEs are often motivated by scientific discoveries that require new mathematical tools, like Fourier series, Fourier transforms, real analysis, linear operator theory and other methods. In mathematics, PDEs have greatly advanced real and complex analysis and geometry.

This course provides an introduction to the most important PDEs, including heat, wave and Laplace equations. The course should be useful to any science or engineering student who are interested in theoretical research. It is also essential for mathematics students whose interests are in analysis and geometry.

Prerequisite: Some basic knowledge of ODEs, like Math 30650 or its equivalent.

Textbook: Walter A. Strauss, *Partial Differential Equations, An Introduction*, Second Edition, Wiley 2008.

Reference Books: For additional readings the following books are recommended:

[**1**] G. B. Folland, *Fourier Analysis and its Applications*, American Mathematical Society, Third Edition. (There is more emphasis on Fourier Analysis.)

[**2**] Mark A. Pinsky, *Partial Differential Equations and Boundary Value Problems with Applications*, American Mathematical Society, Third Edition. (It provides a more detailed presentation of the Separation of Variables Method and Fourier Analysis.)

HOMEWORK: Each week homework will be assigned.

EXAMS:

Midterm Exam: October 10, Wednesday 3:00-5:00 pm

Final Exam: December 11, Tuesday, 4:15-6:15 pm

GRADING: Your course grade will be computed as follows:
Midterm Exam: 40%
Final Exam: 40%
Homework/Project: 20%

Honor Code: The exams are under the honor code. The honor code does not apply to homework. So you may discuss the homework with other classmates but do not copy each other.

Office Hours: There will be regular office hours every Wednesday, 12:00-2:00 pm (or by appointments).

Absence from Exams: A student who is absent from an examination without an official excuse shall receive a grade of zero for that examination. A student who is officially excused will not be penalized. If you miss a test for any reason, call the instructor or the Mathematics Department as soon as possible.

Syllabus – Math 40750, Partial Differential Equations

Part A. We will cover most sections of the following chapters from the textbook by W. Strauss.

- **Chapter 1/Where PDEs Come From**
  - 1.1 What is a Partial Differential Equation?
  - 1.2 First-Order Linear Equations
  - 1.3 Flows, Vibrations, and Diffusions
  - 1.4 Initial and Boundary Conditions
  - 1.5 Well-Posed Problems
  - 1.6 Types of Second-Order Equations

- **Chapter 2/Waves and Diffusions**
  - 2.1 The Wave Equation
  - 2.2 Causality and Energy
  - 2.3 The Diffusion Equation
  - 2.4 Diffusion on the Whole Line
  - 2.5 Comparison of Waves and Diffusions

- **Chapter 4/Boundary Problems**
  - 4.1 Separation of Variables, The Dirichlet Condition
  - 4.2 The Neumann Condition
  - 4.3 The Robin Condition

- **Chapter 5/Fourier Series**
  - 5.1 The Coefficients
  - 5.2 Even, Odd, Periodic, and Complex Functions
  - 5.3 Orthogonality and General Fourier Series
  - 5.4 Completeness
  - 5.5 Completeness and the Gibbs Phenomenon
  - 5.6 Inhomogeneous Boundary Conditions

- **Chapter 6/Harmonic Functions**
  - 6.1 Laplaces Equation
  - 6.2 Rectangles and Cubes
  - 6.3 Poissons Formula
  - 6.4 Circles, Wedges, and Annuli

Part B. This part is based on lecture notes.

- **Fourier Transforms and the Laplace Equation**
  - 7.1 Fourier transform.
  - 7.2 Laplaces Equation, L^2 approach.
  - 7.3 Regularity of the weak solution.
  - 7.4 Spectral theory for the Dirichlet and Neumann problem.
  - 7.5 Can one hear the shape of a drum?