Mei-Chi Shaw

Math 80350, Topics in Complex Analysis

Fall 2020, MWF 10:30-11:20 am (The class will meet online)

Prerequisite: Basic knowledge of one complex variable.

Holomorphic functions in one or several complex variables are defined as functions satisfying the homogeneous Cauchy-Riemann equations. Function theory in several complex variables is fundamentally different from one complex variable. Its development started at the beginning of the twentieth century, from some important observations by Poincaré and Hartogs. The purpose of this course is to study holomorphic function theory in several complex variables.

In one and several complex variables, the Cauchy-Riemann equations play central role. In this course, we will first study the basic properties of the theory for the Caucchy-Riemann equations and the boundary complex in the complex Euclidean spaces or Stein manifolds. Next we will study the Cauchy-Riemann equations in the geometric setting, i.e., on manifolds with positive curvature using the Bochner's technique. We will make this course as self-contained as possible.

We will start from reviewing the Cauchy-Riemann equation in one complex variable and discuss the function theory on Riemann surfaces. Then we will introduce the main topics in several complex variables.

In particular, we will cover the following topics:

1. An introduction to Several Complex Variables.
2. Hilbert space techniques.
3. The Cauchy-Riemann equations and the Levi problem.
4. Tangential Cauchy-Riemann equations.
5. Hermitian and Kähler manifolds.
6. Bochner Techniques and function theory on complex manifolds.

References

1. So-Chin Chen and Mei-Chi Shaw, *Partial Differential Equations in Several Complex Variables*, American Math. Society-International Press, Studies in Advanced Mathematics, Volume 19, Providence, R.I. 2001. ISBN: 978-0-8218-2961-5

2. Steven S. Krantz, *Function Theory of Several Complex Variables, 2nd edition*, Wadswort Belmont, California 1992. ISBNs: 978-0-8218-2724-6