BOOK OF ABSTRACTS

87^{th} **Midwest PDE Seminar**
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Conference Organizers:
Qing Han (qhan@nd.edu),
Alex Himonas (himonas@nd.edu),
Bei Hu (b1hu@nd.edu)

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Main Talks

Jerry Bona (University of Illinois, Chicago)
"Nonlinearity, Dispersion and Dissipaton"
In this lecture, we intend to investigate the interaction between nonlinearity, dispersion and dissipation in a little more detail than heretofore.

Hongqiu Chen (University of Memphis)
"The long wavelength limit of Periodic solutions of water wave models"
It has been common practice for decades to approximate localized solutions of evolution equations set on unbounded spatial domains by solutions of associated periodic initial-value problems. We mention work of Zabusky and Kruskal in the early 1960's. We don't doubt that this practice goes back further, though not much further as computers of large enough scale to handle even one-space and one-time dimension problems were not available much earlier. Despite this method's long history, rigorous theory with error estimates is sparse. It is our purpose to provide theory with error estimates here. This is done in the context of long-crested, surface water wave models. The two models we analyse are both unidirectional, long-wave models. While the theory is concretely attached to these models, one could formulate a more general theory if so desired. The first model is the RLW-BBM model and the second 5 BBM-KdV. The work is joint with J. Bona, Y. Hong, M. Panthee and M. Scialom

Peter Constantin (Princeton University)
"Pressure, Intermittency, Singularity"
I will describe conditions for regularity of solutions of three dimensional incompressible Navier-Stokes equations based on the pressure and on structure functions.

Carlos Kenig (University of Chicago)
"New channels of energy for wave equations, new non-radiative solutions and soliton resolution"
We will discuss the role of non-radiative solutions to nonlinear wave equations, in connection with soliton resolution. Using new channels of energy estimates we characterize all radial non-radiative solutions for a general class of nonlinear wave equations. This is joint work with C.Collot, T. Duyckaerts and F. Merle.

Barbara Keyfitz (Ohio State University)
"Singular Shocks, A Conservation Law Model in Chromatography, and Geometric Singular Perturbation Theory"
The theory of two-component chromatography is a well-known source of interesting problems in the field of hyperbolic conservation laws in a single space variable. A recent discovery is that some chromatographic systems may have solutions with very low regularity. Such solutions have also been around for some time, and are variously known as singular shocks or delta shocks. The theory of conservation laws has expanded to describe candidates for solutions for some equations that exhibit singular shocks, and to determine in what sense they may be said to satisfy the equations, for example as limits of approximations. But no genuine applications were known before this. Furthermore, the chromatography example has resisted the

theory developed so far. In this talk, I will advocate for a relatively new approach. Singular perturbation theory, long a mainstay of classical applied mathematics, has been put on a new footing by an approach developed by Fenichel in the 1970's and since then extended by many other researchers. This approach replaces the formalism of classical constructions with manifold and dynamical systems theory. It was first applied to singular shocks in conservation laws by Stephen Schecter. Using geometric singular perturbation theory, Ting-Hao Hsu, Martin Krupa, Charis Tsikkou and I have recently made progress on finding a singular shock structure for these unusual chromatography equations.

Adrian Lam (Ohio State University)
"A Hamilton-Jacobi approach for nonlocal competition models of many species"
The evolution of dispersal is a classical problem in evolutionary biology. The main question is to define the fittest dispersal rate for a population in a bounded domain. From the point of view of adaptive evolution, Perthame and Souganidis formulated a nonlocal competition model, in which the population is structured by space and a phenotypic trait variable, with the trait acting directly on the dispersal rate. For the small mutation limit, it was shown that the equilibrium population concentrates in the trait variable associated with the lowest dispersal rate, while remaining regular in the spatial variables. In this talk, we discuss the time-dependent solutions which exhibit moving Dirac concentrations in a fast timescale. We will derive, using the notion of Floquet bundles of parabolic equations, the constrained Hamilton-Jacobi equation describing the trajectory of the moving Dirac concentration.

Marta Lewicka (University of Pittsburgh)
"The Monge-Ampere system: flexibility in arbitrary dimension and codimension"
In this talk, we will be concerned with weak solutions obtained by means of convex integration, to the Monge-Ampere system. This system is a natural extension of the Monge-Ampere equation $\det \nabla^2 v = f$ in dimension $d = 2$, to the strong version of the linearization of the full Riemann curvature tensor in arbitrary dimension d, and it arises in relation to: (i) the problem of existence of isometric immersions, and (ii) the context of nonlinear elasticity. Our main technical ingredient consists of the "stage" construction, in which we achieve the Holder regularity $\mathcal{C}^{1,\alpha}$ of solutions to the aforementioned system, approximating its any subsolution, with $\alpha < \frac{1}{1+d(d+1)/k}$ where d is an arbitrary dimension and $k \geq 1$ is an arbitrary codimension in the problem. When $d = 2$ and $k = 1$, we recover the previous result by Lewicka-Pakzad for the Monge-Ampere equation. Our construction can be translated to the isometric immersion problem, where it is consistent for $k = 1$ with a result by Conti-Delellis-Szekelyhidi, and for large k it quantifies a result by Kallen.

Shuwang Li (IIT – Chicago)
"PDE modeling and computation of fluid-structure interaction problems"
In this talk, I will use vesicle dynamics as an example to demonstrate two modeling approaches and discuss challenges in computation. The first approach is based on a classical sharp interface description. I will show the derivation of the model and corresponding interface (boundary) conditions. In the second part, I will present a phase field model by focusing on vesicle expanding or shrinkage, induced by osmotic pressure that arises due to a chemical potential gradient. The model consists of an Allen-Cahn equation, which describes the phase field evolution; a Cahn-Hilliard equation, which simulates the fluid concentration; and a Stokes equation, which

describes the motion of the vesicle. Conditions for vesicle growth or shrinkage are analyzed via the common tangent construction in osmotic energy. Finally, I will show some interesting numerical results.

Yanyan Li (Rutgers University)
"Some recent results on conformally invariant equations"
We will present some recent work on conformally invariant fully nonlinear elliptic equations. This includes results on Liouville type theorems, first and second derivertive estimates, existence and nonexistence of smooth solutions to conformally invariaint equations.

Natasa Pavlovic (University of Texas – Austin)
"Two tales of a rigorous Derivation of the Hamiltonian Structure"
Many mathematical works have focused on understanding the manner in which the dynamics of a nonlinear equation arises as an effective equation.
• For example, the cubic nonlinear Schrödinger equation (NLS) is an effective equation for a system of N bosons interacting pairwise via a delta or approximate delta potential. In this talk, we will advance a new perspective on deriving an effective equation, which focuses on structure. In particular, we will show how the Hamiltonian structure for the cubic NLS in any dimension arises from corresponding structure at the N-particle level.
• On the other hand, the Vlasov equation in any spatial dimension has long been known to be an infinite-dimensional Hamiltonian system whose bracket structure is of Lie-Poisson type. In parallel, it is classical that the Vlasov equation is a mean-field limit for a pairwise interacting Newtonian system. Motivated by this knowledge, we provide a rigorous derivation of the Hamiltonian structure of the Vlasov equation, both the Hamiltonian functional and Poisson bracket, directly from the many-body problem. This work settles a question of Marsden, Morrison, and Weinstein on providing a "statistical basis" for the bracket structure of the Vlasov equation. The talk is based on joint works with Dana Mendelson, Joseph Miller, Andrea Nahmod, Matthew Rosenzweig and Gigliola Staffilani.

Monica Visan (University of California – Los Angeles)
"The derivative nonlinear Schrodinger equation"
We will discuss the derivative nonlinear Schrodinger equation, how some inherent instabilities have hindered the study of this equation, and how we were able to demonstrate global well-posedness in the natural scale-invariant space. This is joint work with Ben Harrop-Griffiths, Rowan Killip, and Maria Ntekoume.

Lu Wang (Yale University)
"A mean curvature flow approach to density of minimal cones"
Minimal cones are models for singularities in minimal submanifolds, as well as stationary solutions to the mean curvature flow. In this talk, I will explain how to utilize mean curvature flow to yield near optimal estimates on density of topologically nontrivial minimal cones. This is joint with Jacob Bernstein.

Yu Yuan (University of Washington)
"Hessian estimate for the sigma-2 equation in dimension four"
We present joint work with Ravi Shankar on Hessian estimate and interior regularity for the sigma-2 equation in four dimensions. Our method also provides respectively a Hessian estimate for smooth solutions satisfying a dynamic semi-convexity condition in higher dimensions, and a new proof for the corresponding three dimensional results, which were obtained in the 2007 joint work with Warren. The two dimensional result for the sigma-2 or Monge-Ampere equation was achieved by Heinz in the 1950s. Irregular solutions to the sigma-3 (Monge-Ampere) equation in dimension three were first constructed by Pogorelov in the 1970s.

Short Talks

Lakhdari Abderrahmane (University of Tunis El Manar)
"Multiplicity of solutions for a problem in double weighted Orlicz-Sobolev spaces and its spectrum"
We first analyze the existence of three weak solutions to a nonlinear elliptic problem driven by the weighted ϕ-Laplacian operator with Dirichlet boundary conditions, and then we look for sequences of variational eigenvalues while assuming the $\Delta 2$-condition. Following that, we investigate the existence of eigenvalues without assuming the $\Delta 2$-condition, which renders the space connected to the weighted ϕ-Laplacian non-reflexive, non-separable, and the functionals re- lated to the examined problem not C^1. Technical proofs combined with a Lagrange multipliers type and the Ljusternik-Schnirelmann argument yield our conclusions.

Jeaheang Bang (University of Texas at San Antonio)
"Rigidity of Solutions to the Stationary Navier-Stokes Equations in High Dimensions"
Consider the stationary Navier-Stokes equations. Motivated by its special scaling property, one can study several scaling-invariant classes of solutions to examine various aspects of a stationary solution in general. Sverak and Tsai in 2011, 1998 proved that any stationary self-similar solutions (a special scaling-invariant class) are trivial in $\mathbb{R}^n, n \geq 4$. In this talk, I will present our recent work on this topic. We extended their result to a most general scaling-invariant class by proving that any stationary solutions satisfying $|u(x)| \leq C/|x|$ in $\mathbb{R}^n \setminus \{0\}, n \geq 4$, are trivial. Here, we do not assume any type of self-similarity. It has implications for Liouville-type theorems in higher dimensions and the stationary regularity problem. (The five-dimensional stationary regularity problem is related to the famous three dimensional non-stationary regularity problem in terms of scaling dimension.) Our main idea is to multiply the stationary Navier-Stokes equations or the head pressure equation with various quantities, integrate it over an annulus and study the behavior of the boundary integrals using the main assumption. This is a joint work with Changfeng Gui, Hao Liu, Yun Wang and Chunjing Xie.

Joon Do Chang (University of Memphis)
"Local Well-Posedness of the BBM Equation"
The BBM Equation (aka Benjamin Mahony Equation) is a nonlinear dispersive equation (alternative model for the well known KdV equation) derived to study and model long wavelength water waves with small amplitude. This equation was first introduced by T.B. Benjamin, Jerry L. Bona, and J. Mahony in 1972 whom showed the global well-posedness of his equation in H^k

for $k > 0$. In this presentation, we shall study the BBM equation with the initial conditions $u(0,x) = u_0(x)$ in H^s for $0 \le s \le 1$. In particular, I will talk about the local well-posedness result of J. L. Bona and N. Tzetkov published in 2008. If time permits, I will also briefly talk about Mahendra Panthee's local ill-posedness result of the BBM equation for $s < 0$ and discuss about some open problems. This will be an expository talk.

Luan M. Doan (University of Notre Dame)
"The heat kernels, the coherent state transforms on the spheres, and the large-N limit problems"
The complexification of the sphere $S^{N-1}(\sqrt{N})$ centered at the origin of $\mathbb{R}^N$ with radius $\sqrt{N}$ is the *quadric*

$$Q^{N-1}(\sqrt{N}) = \{\mathbf{z} = (z_1, \ldots, z_N) \in \mathbb{C}^N : z_1^2 + \cdots + z_N^2 = N\}.$$

A "coherent state" transform (or Segal–Bargman transform) at time $T > 0$ is a unitary map from the space of square-integrable functions on $S^{N-1}(\sqrt{N})$ with respect to its volume measure onto the space of *holomorphic* square integrable functions on $Q^{N-1}(\sqrt{N})$ with respect to its heat-kernel measure at time T. One such unitary map is described via convolution with the *spherical* heat kernel.

As N tends to infinity, there are three convergence phenomena: the normalized volume measure on $S^{N-1}(\sqrt{N})$ converges to the unit-time Gaussian measure on $\mathbb{R}^\infty$, the heat-kernel measure on $Q^{N-1}(\sqrt{N})$ converges to a certain Gaussian measure on $\mathbb{C}^\infty$ depending on T, and the Segal–Bargmann transform tends to an exponential operator described via the Hermite differential operator.

Pamela Guerrero (University of Memphis)
"Study of a Modified BBM Equation"
Considered here is the initial value problem for the modified BBM equation:

$$u_t + u_x - u_{xxt} + u^2 u_x = 0, x \in \mathbb{R}, t > 0$$
$$u(x,0) = \varphi(x).$$

We show that when the initial data φ lies in Sobolev space $H^s(\mathbb{R})$ for $s > 1/3$, the problem is locally well-posed, then the global well-posedness follows from apriori estimates of the H^s norm $\|u(\cdot,t)\|_{H^s}$ of the solution $u(x,t)$ for all $t > 0$.

Curtis Holliman (The Catholic University of America)
"Blow-up conditions for generalizations of the Camassa-Holm Equation"
We will examine some new blow-up results for a particular generalization of the Camassa-Holm and Novikov's equation in Sobolev spaces.

John Holmes (The Ohio State University)
"Continuity of the data-to-solution map for conservation laws"
In 1975, Kato proved the well-posedness (existence, uniqueness, and continuous dependence of the solution on the initial data) for the Cauchy problem corresponding to a general system of symmetric hyperbolic equations. At the end of the paper, he showed via the Burgers equation, that one should not expect that the data-to-solution map be Hölder continuous for any Hölder exponent. Since his result, many mathematicians have refined his ideas while investigating the

continuity of the data-to-solution map for similar equations (Euler, Camassa-Holm, others) and proved that for these problems the data-to-solution map is not even uniformly continuous. In this talk we will explore the development of these results, and we will use the ideas introduced by Kato and other mathematicians to prove that, for a general class of conservation laws, the data-to-solution map is not uniformly continuous.

Jeongsu Kyeong (Temple University)
"Multilayer potentials associated with the higher-order Cauchy-Riemann operator in Uniformly Rectifiable domains"
The classical Cauchy integral operator is one of the most famous and most studied singular integral operator in mathematics. In this talk, I will be presenting a higher-order analogue of the existing theory for the classical Cauchy operator, in which the salient role of the Cauchy-Riemann operator $\overline{\partial}$ is now played by natural powers of this. A central role will be played by integral representation formulas, jump relations and higher-order Fatou-type theorems. This is joint work with Irina Mitrea (Temple University), Dorina Mitrea and Marius Mitrea (Baylor University).

Thierry Laurens (University of California - Los Angeles)
"Sharp well-posedness for the Benjamin–Ono equation"
We will discuss a proof of sharp well-posedness for the Benjamin–Ono equation in the class of H^s spaces, on both the line and the circle. This result was previously unknown on the line, while on the circle it was obtained recently by Gérard, Kappeler, and Topalov. This is joint work with Rowan Killip and Monica Visan.

Nicholas Lohr (Northwestern University)
"Semiclassical measures of eigenfunctions of the hydrogen atom"
We begin by defining semiclassical measures and reviewing some of their basic properties. We continue by giving an overview of the results in this field, including results of Macià, Jakobson, and Zelditch. The main part of the talk will be discussing the idea of the proof behind the recent characterization of all the semiclassical measures of the hydrogen atom.

Ilya Marchenko (University of Notre Dame)
"Asymptotic Expansions at Infinity up to Arbitrary Order for Solutions of Special Lagrangian Equations"
We study solutions of a family of nonlinear elliptic equations which are known to be asymptotic to quadratic polynomials at infinity. We characterize the remainders in those asymptotic expansions using the Kelvin transform. We also show that, in an appropriate sense, any solution to an equation in this family can be obtained from an approximate solution, which is constructed in a mechanical way and that any such approximate solution that we construct corresponds to an actual solution.

GiangVuThanh Nguyen (Old Dominion University)
"Asymptotic expansion of a singular potential near the nematic-isotropic phase transition point in the Landau-de Gennes theory"

The Landau-de Gennes theory is a type of continuum theory that describes nematic liquid crystal configurations in the framework of the Q-tensor order parameter. In the free energy, there is a singular bulk potential which is considered as a natural enforcement of a physical constraint on the eigenvalues of symmetric, traceless Q-tensors. In this talk we shall discuss some analytic properties related to this singular potential. More specifically, we study the asymptotic expansion of this singular potential (up to fourth order) near the nematic-isotropic phase transition point.

Mashud MD Parvez (Old Dominion University)
"The Coupling Conduction Effects on Natural Convection Flow along a Vertical Flat Plate with Joule Heating and Heat Generation"
The out-turn of heat generation and Joule heating on natural convection flow through a vertical flat plate have been investigated in the given article. Joule heating and heat conduction due to wall thickness 'b' are esteemed as well in this analysis. With the intent to obtain similarity solutions to the problem being constituted, the evolved equations are made dimensionless employing appropriate transformations. The non-dimensional equations are then modified into non-linear equations by bringing into being a non-similarity transformation. The out-turn non-linear alike equations confine with their commensurate boundary conditions formed on conduction and convection are solved numerically applying the finite difference method accompanied by Newton's linearization approximation. The numerical outcomes in terms of the skin friction coefficient, velocity, temperature, and surface temperature profiles are shown both graphically and in tabular forms for the different values of the parameters correlated with the problem.

Brian Reyes (University of Notre Dame)
"On the Cauchy problem of the modified b-family"
We consider the Cauchy problem of the modified b-family of equations and study its well-posedness in Sobolev and analytic spaces. Also, we study the evolution of the uniform radius of analyticity for global solutions.

Amir Sagiv (Columbia University)
"Effective Gaps in Floquet Hamiltonians"
Floquet topological insulators are an emerging category of materials whose properties are transformed by time-periodic forcing. Can their properties be understood from their first-principles continuum PDE models? Experimentally, graphene is known to transform into an insulator under a time-periodic driving. A spectral gap, however, is conjectured to not form. How do we reconcile these two facts? We show that the original Schrödinger equation has an "effective gap" - a new and physically-relevant relaxation of a spectral gap.

Ryan Thompson (University of North Georgia)
"Classical Solutions of the Fornberg-Whitham Equation"
In this talk, we will show well-posedness in $C^1(\mathbb{R})$ (a.k.a. classical solutions) of the Fornberg-Whitham equation. To achieve this objective, we study its weak formulation under a Lagrangian framework. Applying the fundamental theorem of ordinary differential equations to a generated semi-linear system, we then construct a unique solution to the equation that is continuously dependent on the initial data. These results improve upon others in Sobolev and Besov spaces.

Feride Tigley (The Ohio State University)
"Integrating evolution equations using Fredholm determinants"
We outline the construction of special functions in terms of Fredholm determinants to solve boundary value problems of the string spectral problem. Our motivation is that the string spectral problem is related to the spectral equations in Lax pairs of at least three nonlinear evolution equations from mathematical physics.

Wenchuan Tian (UCSB)
"Compactness conjecture for closed Riemannian 3-manifold with non-negative scalar curvature"
Gromov and Sormani conjectured that under suitable geometric uniform bounds, sequences of closed 3-dimensional Riemannian manifold with nonnegative scalar curvature converges in certain sense.

Specifically, let $\{(M_j^3, g_j)\}_{j=1}^{\infty}$ be a sequence of compact 3-dimensional Riemannian manifold without boundary. Assume that each (M_j^3, g_j) has non-negative scalar curvature. Also assume that the volume and diameter are uniformly bounded above, and that the area of minimal surface is uniformly bounded away from zero. Then the conjecture states that there exists a subsequence that converges in the intrinsic flat sense to the limit space which has nonnegative scalar curvature in the generalized sense.

In this talk, we discuss two special cases. One case is the sequence of rotationally symmetric 3-manifold. In this case we confirm the conjecture. The other case is the sequence of warped product $\mathbb{S}^2 \times \mathbb{S}^1$. In this case we discuss some interesting examples.

Ruoyu Wang (Northwestern University)
"Damped waves with singular damping on manifolds"
We will discuss a new damped wave semigroup for damping exhibiting H́older-type blowup near a hypersurface of a compact manifold. We will use this semigroup to prove a sharp energy decay result for singular damping on the torus, where the optimal rate of energy decay explicitly depends on the singularity of the damping. We also show that no finite time extinction could happen under this setting. This is a joint work with Perry Kleinhenz.

Fangchi Yan (Virginia Tech)
"The Schrödinger-Korteweg-De Vries system on the half-line"
Using the Fokas unified transform method (UTM), the well-posedness of the initial-boundary value problem (ibvp) for the Schrödinger-Korteweg-de Vries system on the half-line is studied for initial data $(u_0, v_0)(x)$ in spatial Sobolev spaces $H^{s_1}(0,\infty) \times H^{s_2}(0,\infty)$, $s_1 > 0$, $s_2 > -3/4$, and boundary data $(g_0, h_0)(t)$ in the temporal Sobolev spaces suggested by the time regularity of the Cauchy problem for the corresponding linear system. First, linear estimates in Bourgain spaces are derived by utilizing the Fokas solution formula of the ibvp for the forced linear system. Then, using these and the needed bilinear/trilinear estimates, it is shown that the iteration map defined by the Fokas solution formula is a contraction in an appropriate solution space.

Jincheng Yang (University of Chicago)
"Layer separation, anomalous dissipation, and drag force in the inviscid limit for 3D NSE"
We provide an unconditional upper bound for the boundary layer separation, anomalous dissipation, and the work of drag force in the zero viscosity limit, of Leray–Hopf weak solutions to the 3D incompressible Navier-Stokes equation in a smooth bounded domain. Layer separation refers to the discrepancy between a (turbulent) low-viscosity Leray–Hopf solution and a fixed (laminar) regular Euler solution with similar initial conditions and body force. In addition, we show the boundedness of the drag coefficient with a high Reynolds number.