Midwest Several Complex Variables Conference

University of Notre Dame, April 29 – May 1, 2022

TALKS

David Barrett (University of Michigan)

Spectral theory of skew projections, with application to holomorphic reproducing kernels

Abstract: The talk will begin with a discussion of the role of singular value decomposition in the unitary equivalence problem for non-self-adjoint projections and related operators. This will be followed by an examination of applications of those results to the study of the Cauchy and Leray projections onto Hardy spaces.

Eric Bedford (Institute for Mathematical Sciences at Stony Brook)

Complex analysis in the dynamics of Hénon mappings

Abstract: I will discuss how the iteration of complex Hénon mappings leads to some interesting problems in analysis.

Debraj Chakrabarti (Central Michigan University)

Projection operators onto L^p-Bergman spaces

Abstract: For $1 \leq p \leq \infty$, the L^p-Bergman space $A^p(\Omega)$ of a domain in $\mathbb{C}^n$ is the Banach space of all p-th power integrable holomorphic functions. It is natural to ask whether there is a natural bounded linear projection from $L^p(\Omega)$ to $A^p(\Omega)$, analogous to the well-known orthogonal Bergman projection for $p = 2$. Traditionally, one obtains such a projection operator as a continuous extension of the L^2 Bergman projection, when the Bergman projection satisfies L^p-estimates. However, as it has been known since the work of Barrett (1984) on his nonpseudoconvex worm domain, that such L^p-estimates on the Bergman projection need not exist in general. We therefore try to understand the fundamental structural differences between the L^p extension of the Bergman projection and projection operators better behaved on L^p, such as the Szegő projection. In the context of Reinhardt domains, we propose a possible approach to constructing projection operators onto L^p-Bergman spaces, which can be expected to have better mapping properties. This is ongoing joint work with Luke Edholm.

Anne-Katrin Gallagher (Gallagher Tool & Instrument)

On plurisubharmonic defining functions

Abstract: I will present results pertaining to the question of existence of plurisubharmonic defining functions for smoothly bounded, pseudoconvex domains. This talk is based on joint work with Tobias Harz.

Phillip Harrington (University of Arkansas)

The Diederich-Fornæss Index and regularity of defining functions

Abstract: For a pseudoconvex domain Ω, the Diederich-Fornæss Index is the supremum over all exponents $0 < \eta < 1$ such that there exists a defining function ρ for Ω such that $-(-\rho)^\eta$ is strictly plurisubharmonic on Ω. If the defining functions under consideration are merely Lipschitz, Berndtsson and Charpentier (2000) have shown that this suffices to prove regularity for the Bergman Projection in Sobolev Spaces $W^s(\Omega)$ for $0 \leq s < \frac{\eta}{2}$. When the defining functions can be chosen to be smooth, Kohn (1999) pioneered an alternative approach to obtain improved estimates for the Bergman Projection. In this talk, we will examine the qualitative differences between these two approaches.

Joseph Kohn (Princeton University)

Subelliptic estimates on weakly pseudoconvex CR manifolds

Abstract: Let $M \subset \mathbb{C}^{n+1}$ be the germ at the origin of the manifold defined by:

$$r(z_1, \ldots, z_{n+1}) = 2\mathbf{Re}\, z_{n+1} + \sum_1^n |h_j(z_1, \ldots, z_n)|^2,$$

where the h_j are holomorphic functions in a neighborhood of $0 \in \mathbb{C}^n$. Further we assume that the variety defined by $h_1 = \ldots = h_n = 0$ is the origin. Then there exist a neighborhood U of the origin in $\mathbb{C}^{n+1}$, $\varepsilon > 0$, and $C > 0$ such that the following subelliptic estimate holds:

$$\|\varphi\|_\varepsilon^2 \leq C\big(\|\overline{\partial}_b\varphi\|^2 + \|\overline{\partial}_b^*\varphi\|^2\big) = CQ_b(\varphi, \varphi),$$

for all forms $\varphi = \sum_1^n \varphi_j d\bar{z}_j$ with $\varphi_j \in C_0^\infty(U \cap M)$. One method to prove such estimates is to use subelliptic multipliers. A rough description of this method is as follows: for the above a subelliptic multiplier is a holomorphic function $f(z_1, \ldots, z_n)$ such that for some $\delta > 0$ and $C > 0$ we have: $\|f\varphi\|_\delta^2 \leq CQ_b(\varphi, \varphi)$ and we show that 1 is a subelliptic multiplier. The set of multipliers $\mathcal{M}$ is an ideal with the properties that $\sqrt{\mathcal{M}} = \mathcal{M}$ and that if $dh_1 \wedge \ldots \wedge dh_n = g\, dz_1 \wedge \ldots \wedge dz_n$ then $g \in \mathcal{M}$ ($|g|^2$ is the determinant of the Levi form of M). Furthermore, if $f_j \in \mathcal{M}$ with $1 \leq j \leq n$ then the coefficients of $df_1 \wedge \ldots \wedge df_j \wedge dh_{k_1} \wedge \ldots \wedge dh_{k_{n-j}}$ are in $\mathcal{M}$. Then the algorithm consists of inductively forming the sequence of ideals $\mathcal{J}_1 \subset \ldots \subset \mathcal{J}_m$ with $\mathcal{J}_1 = \sqrt{(g)}$ and $J_{j+1} = \sqrt{\mathcal{S}}_j$, where $\mathcal{S}_j$ consists of the coefficients of $df_1 \wedge \ldots \wedge df_n$ with $f_i \in \mathcal{J}_j$. Then there exists an m such that $1 \in \mathcal{J}_m$. In this algorithm ε depends only on m, n, and the number of roots used to arrive at 1. Catlin proved that ε depends only on n and the D'Angelo type of M at the origin

(equivalently it depends only on n and the multiplicity of the ideal $(h_1, \ldots, h_n)$). D'Angelo found a sequence of CR manifolds $M_k \subset \mathbb{C}^3$ for which the D'Angelo type is independent of k but the above algorithm leads to an ε that does depend on k. This shows that the algorithm described above is not effective. Effective algorithms were found by Siu and also in the recent work of Kim and Zaitsev.

Here I will describe a variant of the subelliptic multiplier approach via microlocal analysis which avoids the direct use of determinants and minimizes the number of roots. This approach is effective and should lead to proving Hölder and L^p estimates. I will illustrate how this approach works for the D'Angelo manifolds M_k which are defined by the above defining functions r with: $h_1(z_1, z_2) = z_1^p$ and $h_2(z_1, z_2) = z_2^q + z_2 z_1^k$.

Christine Laurent-Thiébaut (Université Grenoble Alpes)

Solving the Cauchy-Riemann equation with support conditions, pseudoconvexity and holomorphic approximation

Abstract: This is a joint work with Mei-Chi Shaw. We will give some characterization of pseudoconvex domains which satisfy Runge or Mergelyan holomorphic approximation by means of the Dolbeault cohomology with compact support or prescribed support in the domain. To get some sufficient geometrical condition, we relate our characterization with the Dolbeault cohomology in some special family of support in the complement of the domain.

László Lempert (Purdue University)

The principle of least action in spaces of plurisubharmonic functions

Abstract: Consider a compact Kähler manifold (X, ω). Starting with Mabuchi, various spaces of ω–plurisubharmonic functions on X have been endowed with metrics, and geodesics of these metrics have been studied. Here we adopt a broader perspective. We introduce a class of Lagrangians on spaces of ω–plurisubharmonic functions, and describe properties of the action functionals associated with these Lagrangians.

Duong Hong Phong (Columbia University)

L^∞ estimates for the Monge-Ampère and other fully non-linear equations in complex geometry

Abstract: A priori estimates are essential for the understanding of partial differential equations, and of these, L^∞ estimates are particularly important as they are also needed for other estimates. The key L^∞ estimates were obtained by S.T. Yau in 1976 for the Monge-Ampère equation for the Calabi conjecture, and sharp estimates obtained later in 1998 by Kolodziej using pluripotential theory. It had been a long-standing question whether a PDE proof of these estimates was possible. We provide a positive answer to this question, and derive as a consequence sharp estimates for general classes of fully non-linear equations. This is joint work with B. Guo and F. Tong.

Emil Straube (Texas A&M University)

A sufficient condition for global regularity revisited: DF-index 1 and regularity

Abstract: We revisit a sufficient condition for global regularity in the $\overline{\partial}$–Neumann problem due to the second author. The condition involves a certain L^2 estimate of the one-form usually denoted by α. Because the required bound is L^2–type, rather than pointwise, it combines nicely with recent work of the first author and Yum that shows how to express the DF-index via α. As a result, we obtain that for domains in $\mathbb{C}^2$, DF-index 1 implies global regularity in the $\overline{\partial}$–Neumann problem. This is joint work with Bingyuan Liu.

Liz Vivas (Ohio State University)

Bergman space dimensions on Riemann surfaces

Abstract: Given a Riemann surface X, a line bundle L, and a domain D in X, one can define a generalized Bergman space of holomorphic sections of L restricted to D. I will explain results that relate the dimension of this Bergman space with potential theory properties of D in X. This is joint work with Anne Katrin Gallagher and Purvi Gupta.

Paul Yang (Princeton University)

A sphere theorem in CR geometry

Abstract: We call a 3-D CR structure (M, θ, J) universally embeddable if each finite cover is embeddable. In a joint work with Jeffrey Case, we show that such a CR structure of positive CR Yamabe invariant and positive total Q-prime curvature is contact diffeomorphic to the standard 3-sphere with its standard contact structure.

Yuan Yuan (Syracuse University)

Local holomorphic maps preserving (p, p)-forms

Abstract: The holomorphic maps between Kähler manifolds preserving (p, p)-forms are a natural generalization of holomorphic isometric embeddings. I will discuss (non)-existence of such maps between different types of complex space forms and explain how to reduce this problem to other geometric problems. This is joint work with Shan Tan Chan.