Commutative Algebra and its Interaction with Algebraic Geometry
SLMath/CMND 2023 Summer School

May 22 – June 2, 2023

TALK ABSTRACTS

Giulio Caviglia (Purdue University)
"Bounds on the number of generators of prime ideals"
Let S be a polynomial ring over any field k, and let P in S be a non-degenerate homogeneous prime ideal of height h. When k is algebraically closed, a classical result attributed to Castelnuovo establishes an upper bound on the number of linearly independent quadric contained in P which only depends on h. We significantly extend this result by proving that the number of minimal generators of P in any degree j can be bounded above by an explicit function that only depends on j and h. In addition to providing a bound for generators in any degree j, not just for quadrics, our techniques allow us to drop the assumption that k is algebraically closed. By means of standard techniques, we also obtain analogous upper bounds on higher graded Betti numbers of any radical ideal.

This is a joint work with Alessandro De Stefani.

David Eisenbud (University of California - Berkeley)
"Infinite Free Resolutions"
I will survey some of the history of the study of free resolutions, and talk about some recent results from my joint work with Hai Long Dao around the question: When does the residue field of a local ring appear as a direct summand of a syzygy?

Eloísa Grifo (University of Nebraska - Lincoln)
"Symbolic Powers II"
In this talk, we will continue to discuss symbolic powers and their ubiquity in commutative algebra. We will revisit the Zariski-Nagata theorem from a differential operators perspective, and contrast the counterexamples to the containment problem from the previous talk with some positive results.

Jack Jeffries (University of Nebraska - Lincoln)
"Differential operators and singularities"
As discussed in Anurag's lectures, the ring of differential operators over a polynomial ring has many nice homological and representation-theoretic properties; these yield striking applications to finiteness properties of local cohomology modules. More generally, one can associate to any K-algebra a ring of differential operators. Outside of the polynomial case, rings of differential operators are much less well understood, and most of the results from the polynomial case fail in general. In this talk we will survey some results, both positive and negative, on the extent to which differential operators over singular rings behave as in the regular case.

Claudia Miller (Syracuse University)
"Differential graded algebra resolutions"
Differential graded (dg) algebra structures on complexes, and more particularly on free resolutions, have played a key role in commutative algebra. Some of the history will be mentioned, from its origins in algebraic topology to the motivations for having such structures on minimal free resolutions. In addition, we will discuss some general constructions producing (mostly non-minimal) resolutions with dg algebra
structures. Time permitting, we may very briefly mention the related roles of A-infinity structures and Massey operations.

Jonathan Montaño (Arizona State University)
"Multiplicities in Commutative Algebra: Past, Present, and Future"
Multiplicities of algebras have their roots in intersection theory in the first half of the 20th century with the work of several authors including Hilbert, Noether, and Van der Waerden. In the 1950s developments by Samuel, Serre, and Rees brought multiplicities to the interest of commutative algebraists. Since then multiplicities has become a very active research theme in commutative algebra with connections with other fields such as algebraic combinatorics, convex geometry, and singularity theory. In this talk I will survey the history and some results in this topic, and will include some open problems along the way.

Alexandra Seceleanu (University of Wisconsin - Madison)
"Symbolic Powers I"
This talk aims to introduce symbolic powers of ideals focusing on geometric reasons for studying them. The classical Zariski-Nagata theorem characterizing symbolic powers as higher order vanishing ideals, interpolation problems, and some examples illustrating the subtlety of the containment problem between the symbolic and ordinary powers of an ideal will be discussed.

Bernd Ulrich (Purdue University)
"Liaison and Residual Intersection"
This will be a survey about the theory of (complete intersection) linkage and residual intersection. After reviewing results about linkage, we will focus on the more general notion of residual intersection. Residual intersections arise naturally in intersection theory, enumerative geometry, the study of Rees rings and integral closures, for instance. Of interest are the dimension and depth, the canonical module, and the defining equations of residual intersections.

Uli Walther (Purdue University)
"Computing local cohomology of polynomial rings of characteristic zero"
Suppose $R=KK[x\_1,...,x\_n]$ is a polynomial ring over a field KK of characteristic zero. If I is an ideal in R, then the local cohomology modules $H^j\_I(R)$ can be computed (at least in principle) in Macaulay2. In this talk we discuss the basic ideas that go into this (Groebner basis driven) algorithm. This involves a tour through the land of D-modules, with specific focus on the concept of Bernstein—Sato polynomials attached to elements f of R (which we will define and discuss).