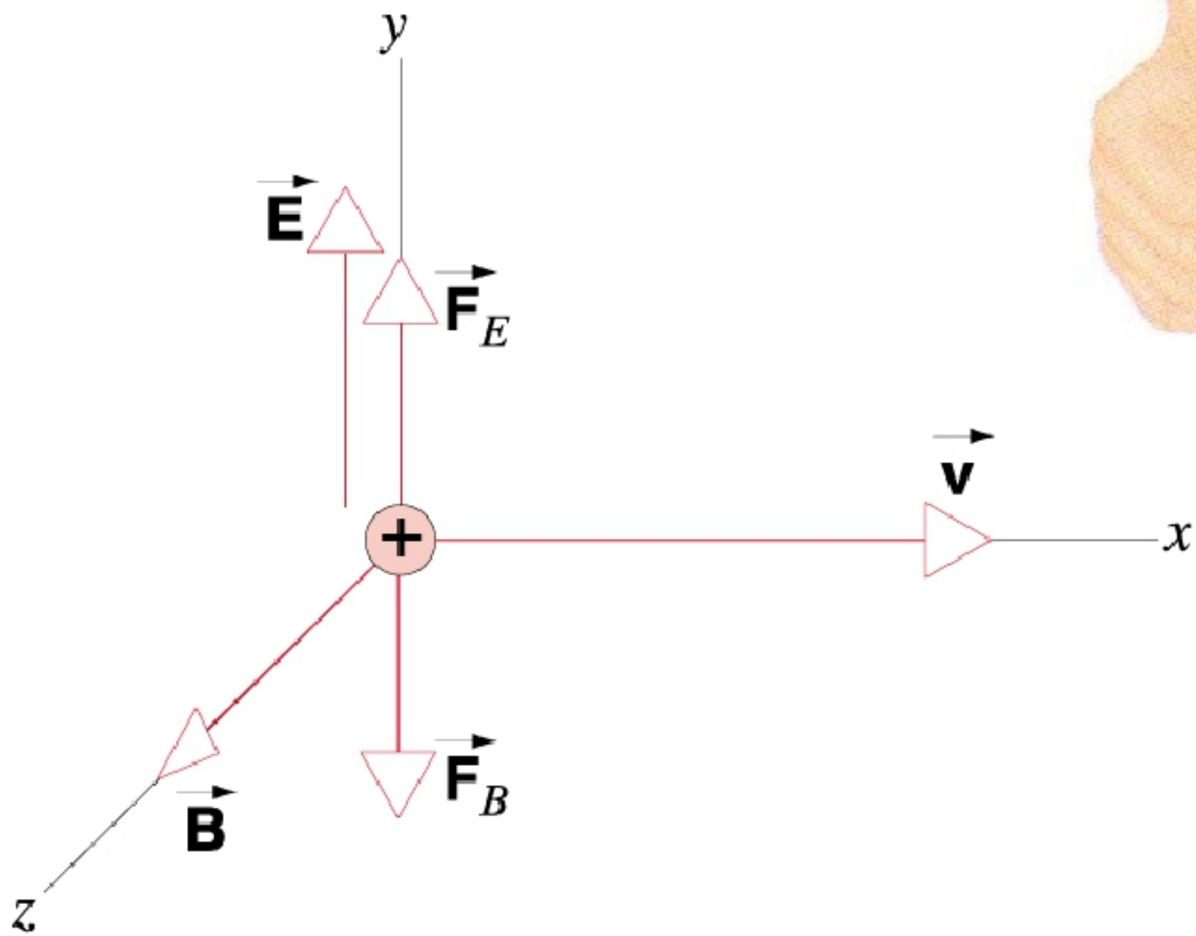
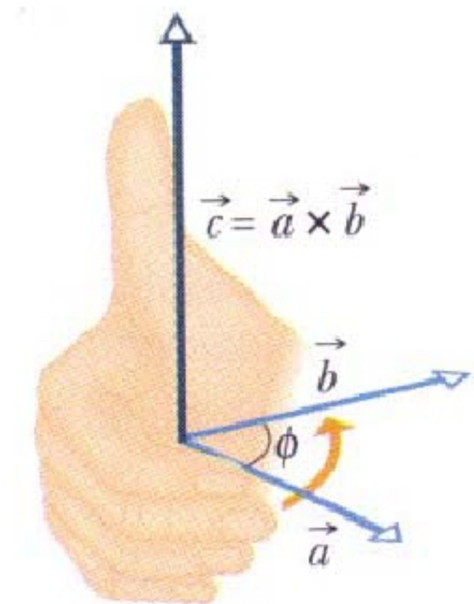
The Hall effect

Ref. Ihn Ch. 10

Reminder: *The Lorentz Force*

$$\mathbf{F} = q[\mathbf{E} + (\mathbf{v} \times \mathbf{B})]$$





Lorentz Force: *Review*

- **Cyclotron Motion**

$$F_B = ma_r \Rightarrow qvB = (mv^2/r)$$

- **Orbit Radius**

$$r = [(mv)/(|q|B)] = [p/(q|B|])$$

A Momentum Filter!!

- **Orbit Frequency**

$$\omega = 2\pi f = (|q|B)/m$$

A Mass Measurement Method!

- **Orbit Energy**

$$K = (\tfrac{1}{2})mv^2 = (q^2B^2R^2)/2m$$

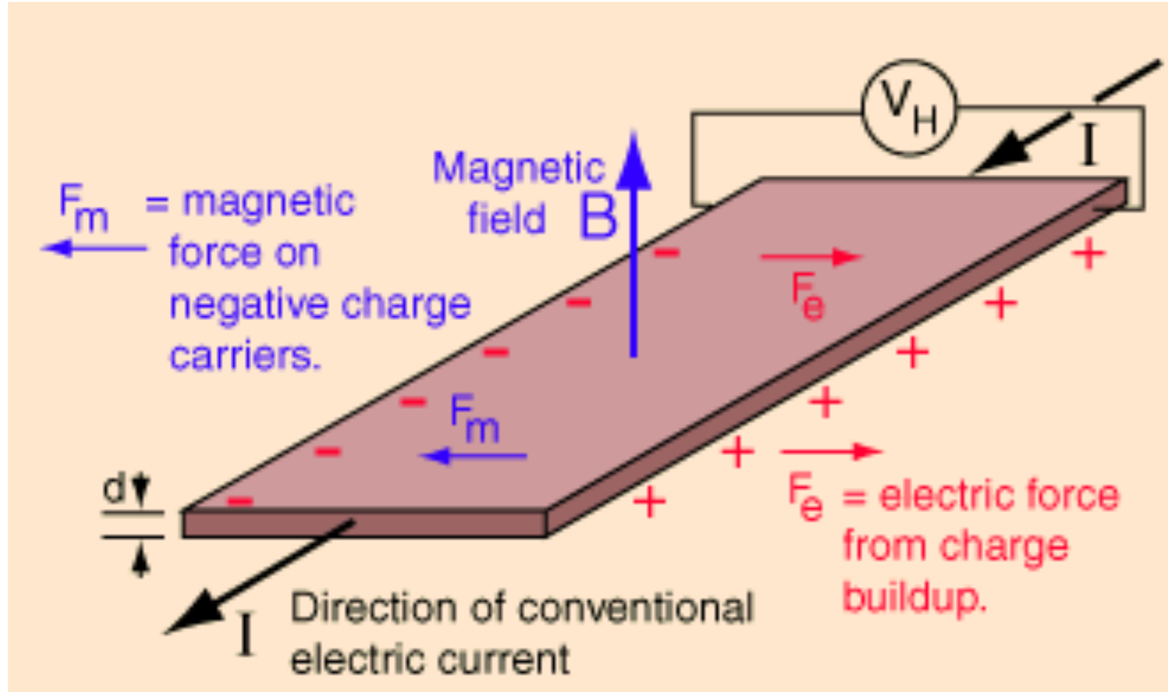
- **Velocity Filter**

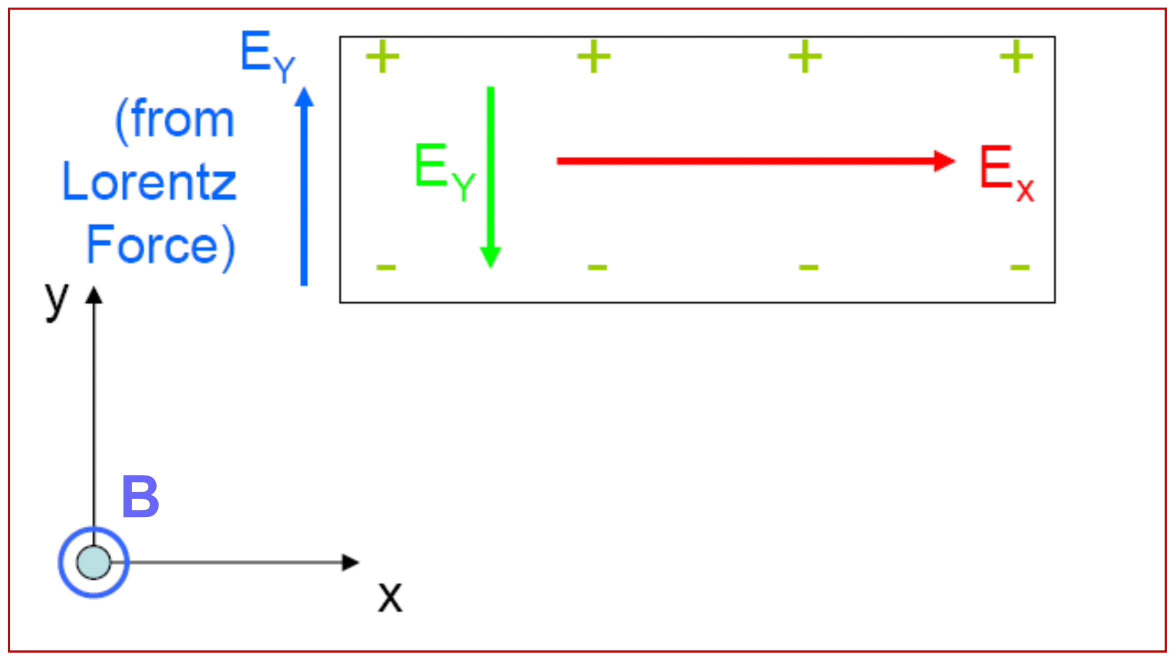
Undeflected trajectories in crossed **E** & **B** fields:

$v = (E/B)$

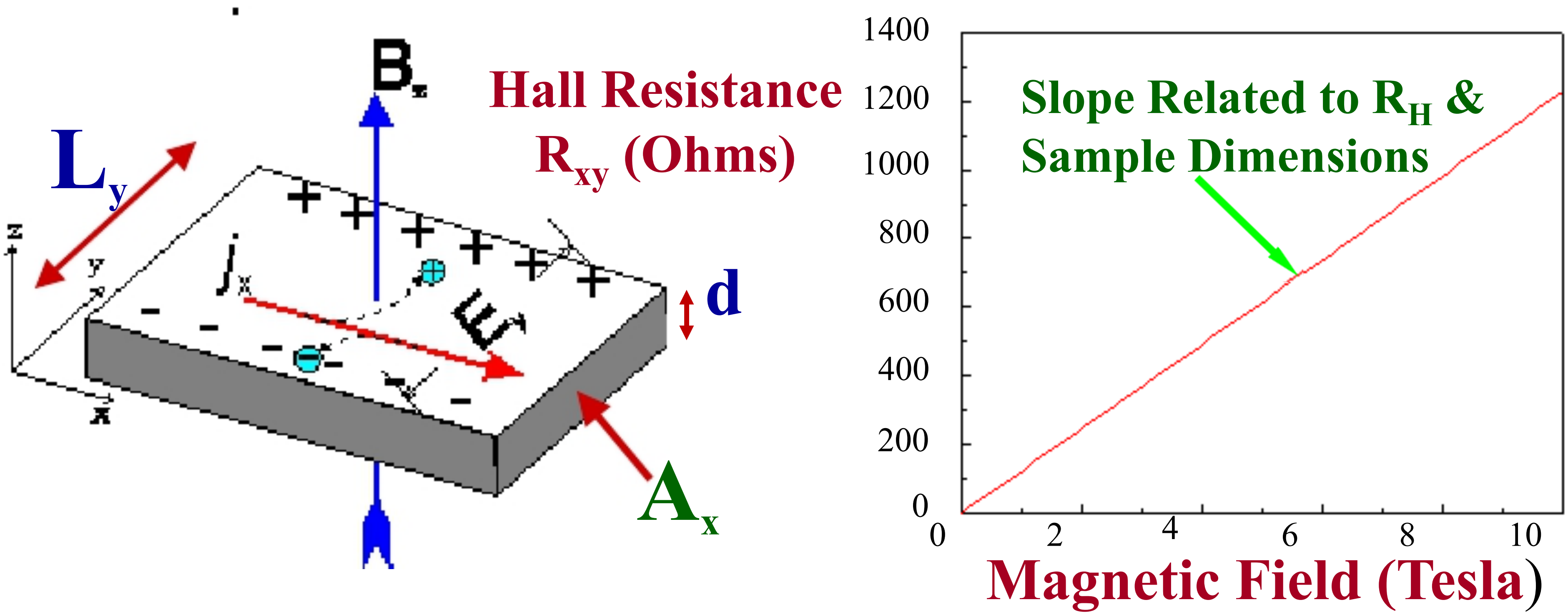
Hall Effect

Electrons move in the $-y$ direction and an electric field component appears in the y direction, E_y. This will continue until the Lorentz force is equal and opposite to the electric force due to the buildup of electrons – that is, a steady condition arises.





The Classical Hall Effect



L_y is the transverse width of the sample

A_x is the transverse cross sectional area

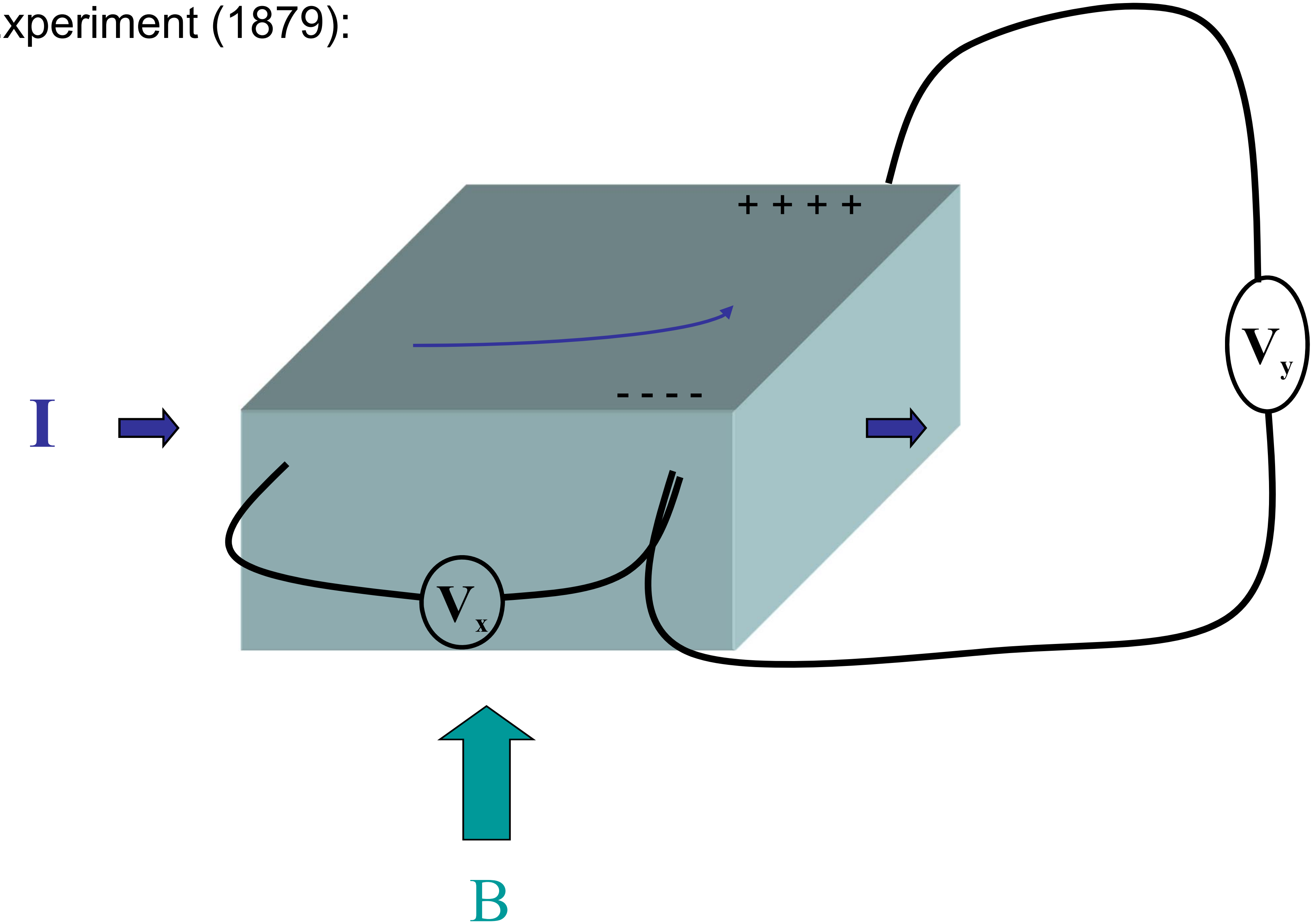
The **Lorentz Force** tends to deflect j_x. However, this sets up an **E-field** which balances that **Lorentz Force**. Balance occurs when

$E_y = v_x B_z = V_y/L_y$. But $j_x = nev_x$ (or $i_x = nev_x A_x$). So

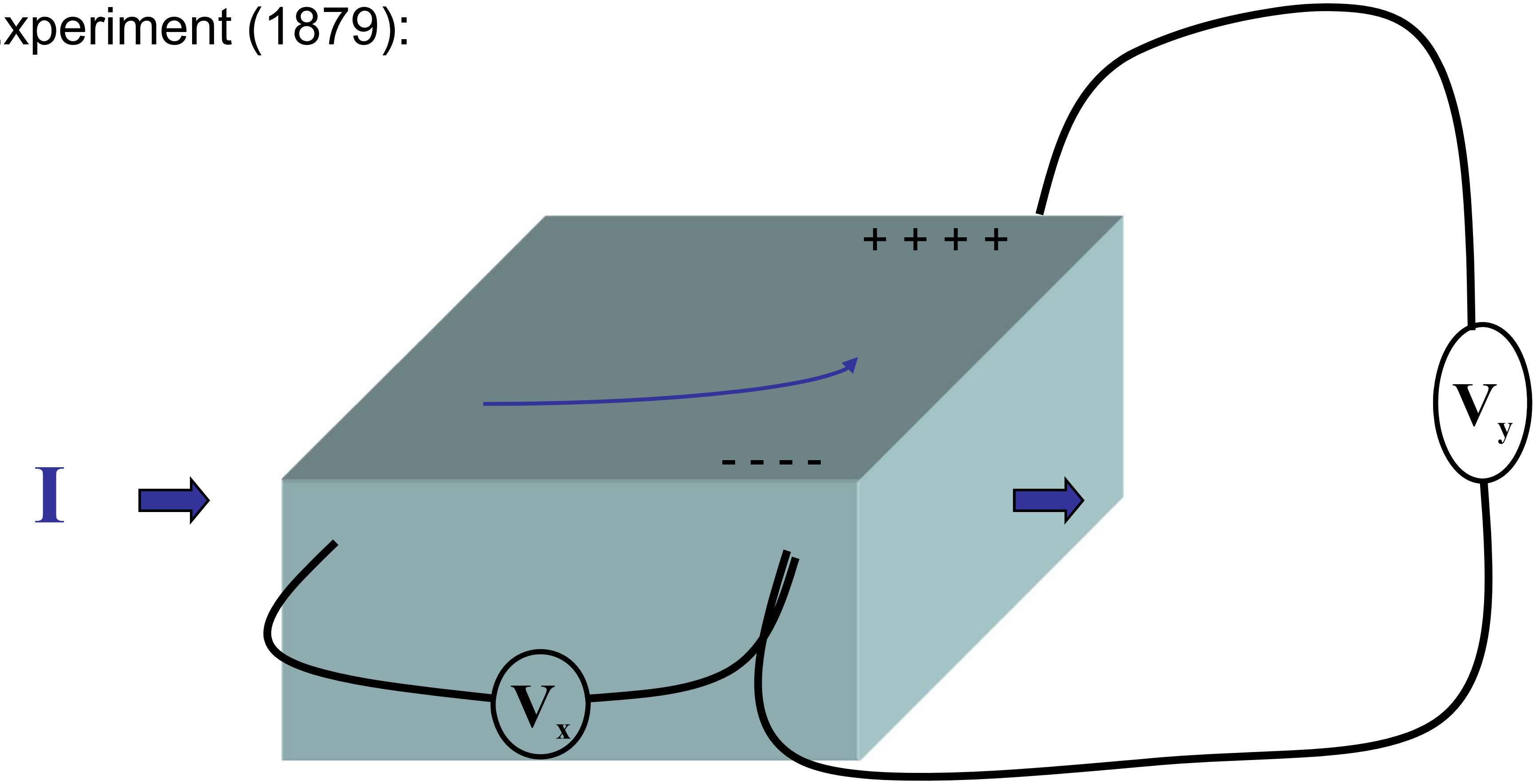
$R_{xy} = V_y / i_x = R_H B_z \times (L_y/A_x)$ where $R_H = 1/ned$

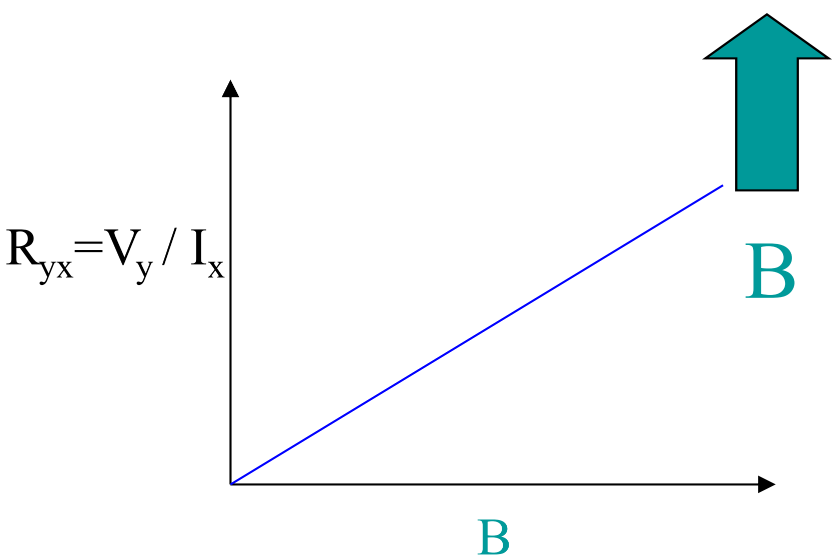
3d-Hall coefficient

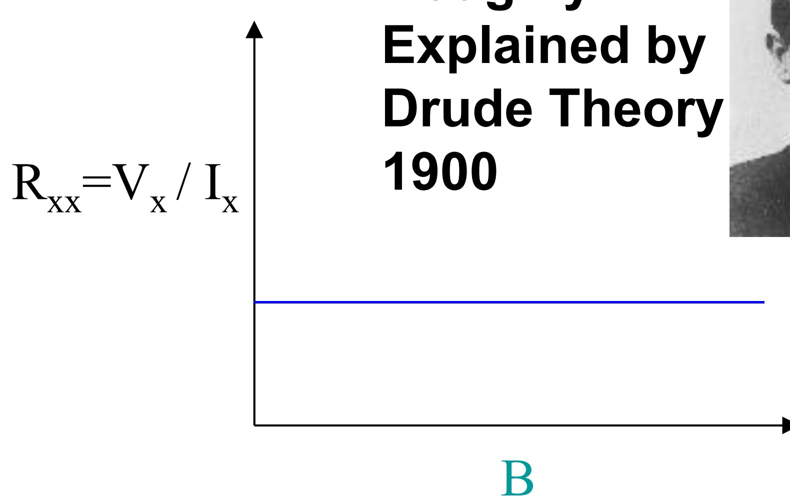
Hall's Experiment (1879):



Hall's Experiment (1879):







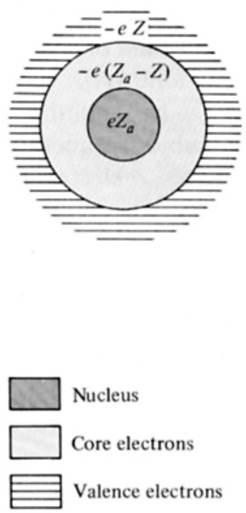
Roughly Explained by Drude Theory 1900

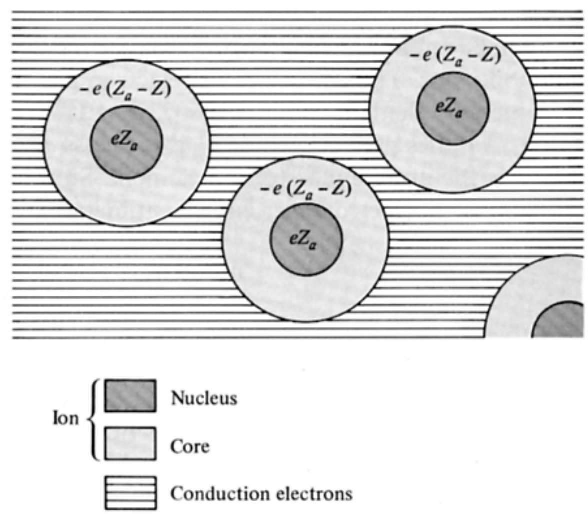
The Drude theory of metals: the free electron theory of metals

Paul Drude (1900): theory of electrical and thermal conduction in a metal
application of the kinetic theory of gases to a metal,
which is considered as a gas of electrons

mobile negatively charged electrons are confined in a metal by attraction to immobile positively charged ions

isolated atom





in a metal

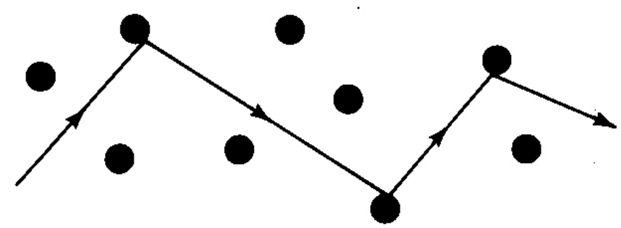
nucleus charge eZ_a
Z <u>valence electrons</u> are weakly bound to the nucleus (participate in chemical reactions)
$Z_a - Z$ <u>core electrons</u> are tightly bound to the nucleus (play much less of a role in chemical reactions)

in a metal – the <u>core electrons</u> remain bound to the nucleus to form the metallic ion
the <u>valence electrons</u> wander far away from their parent atoms
← called <u>conduction electrons</u> or <u>electrons</u>

The basic assumptions of the Drude model

1. between collisions the interaction of a given electron
with the other electrons is neglected ← independent electron approximation
and with the ions is neglected ← free electron approximation

2. collisions are instantaneous events
Drude considered electron scattering off
the impenetrable ion cores



the specific mechanism of the electron scattering is not considered below

3. an electron experiences a collision with a probability per unit time $1/\tau$
dt/τ – probability to undergo a collision within small time dt
randomly picked electron travels for a time τ before the next collision
τ is known as the <u>relaxation time</u>, the <u>collision time</u>, or the <u>mean free time</u>
τ is independent of an electron position and velocity

4. after each collision an electron emerges with a velocity that is randomly directed and with a speed appropriate to the local temperature

DC electrical conductivity of a metal

$V = RI$ Ohm's low

the Drude model provides an estimate for the resistance

introduce characteristics of the metal which are independent on the shape of the wire

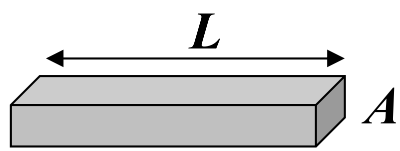
$$\mathbf{E} = \rho \mathbf{j} \qquad \mathbf{j} = \sigma \mathbf{E}$$

$j = I/A$ – the <u>current density</u>

ρ – the <u>resistivity</u>

$R = \rho L/A$ – the <u>resistance</u>

$\sigma = 1/\rho$ – the <u>conductivity</u>



$$\mathbf{j} = -en\mathbf{v}$$

v is the average electron velocity

$$\mathbf{v} = -\frac{e\mathbf{E}}{m}\tau \qquad \mathbf{j} = \left(\frac{ne^2\tau}{m}\right)\mathbf{E}$$

$$\mathbf{j} = \sigma \mathbf{E} \qquad \sigma = \frac{ne^2\tau}{m}$$

$$\tau = \frac{m}{\rho n e^2}$$

at room temperatures
resistivities of metals are typically of the order of microohm centimeters (μohm-cm)
and τ is typically $10^{-14} – 10^{-15}$ s

<u>mean free path</u> $l = v_0 \tau$
v_0 **– the average electron speed**
l **measures the average distance an electron travels between collisions**
estimate for v_0 at Drude's time $1/2\, m v_0^2 = 3/2\, k_B T$ → $v_0 \sim 10^7$ **cm/s** → $l \sim 1 – 10$ **Å**
consistent with Drude's view that collisions are due to electron bumping into ions

at low temperatures very long mean free path can be achieved
$l > 1$ **cm ~ 10^8 interatomic spacings!**
the electrons do not simply bump off the ions!

the Drude model can be applied where
a precise understanding of the scattering mechanism is not required
↓
particular cases: electric conductivity in spatially uniform static magnetic field
and in spatially uniform time-dependent electric field

Very disordered metals and semiconductors

$$\frac{d\mathbf{p}(t)}{dt} = -\frac{\mathbf{p}(t)}{\tau} + \mathbf{f}(t)$$

average momentum ↓ **average velocity** ↓

$$\mathbf{p}(t) = m\mathbf{v}(t)$$

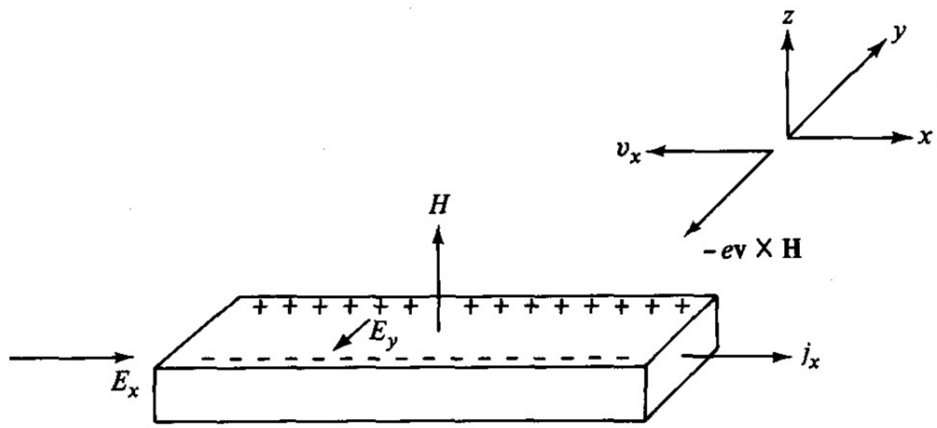
motion under the influence of the force f(*t*) due to spatially uniform electric and/or magnetic fields

<u>equation of motion</u>
for the momentum per electron

electron collisions introduce a frictional damping term for the momentum per electron

Hall effect and magnetoresistance

Edwin Herbert Hall (1879): discovery of the Hall effect



the Hall effect is the electric field developed across two faces of a conductor in the direction **j**×**B** when a current **j** flows across a magnetic field **B**

the Lorentz force $\mathbf{F}_L = -e\mathbf{v} \times B$

in equilibrium $j_y = 0$ → the transverse field (the Hall field) E_y due to the accumulated charges balances the Lorentz force

quantities of interest:

magnetoresistance (transverse magnetoresistance) $R(H) = R_{xx} = \dfrac{V_x}{I_x}$

resistivity $\rho(H) = \rho_{xx} = \dfrac{E_x}{j_x}$

Hall (off-diagonal) resistance $R_{yx} = \dfrac{V_y}{I_x}$

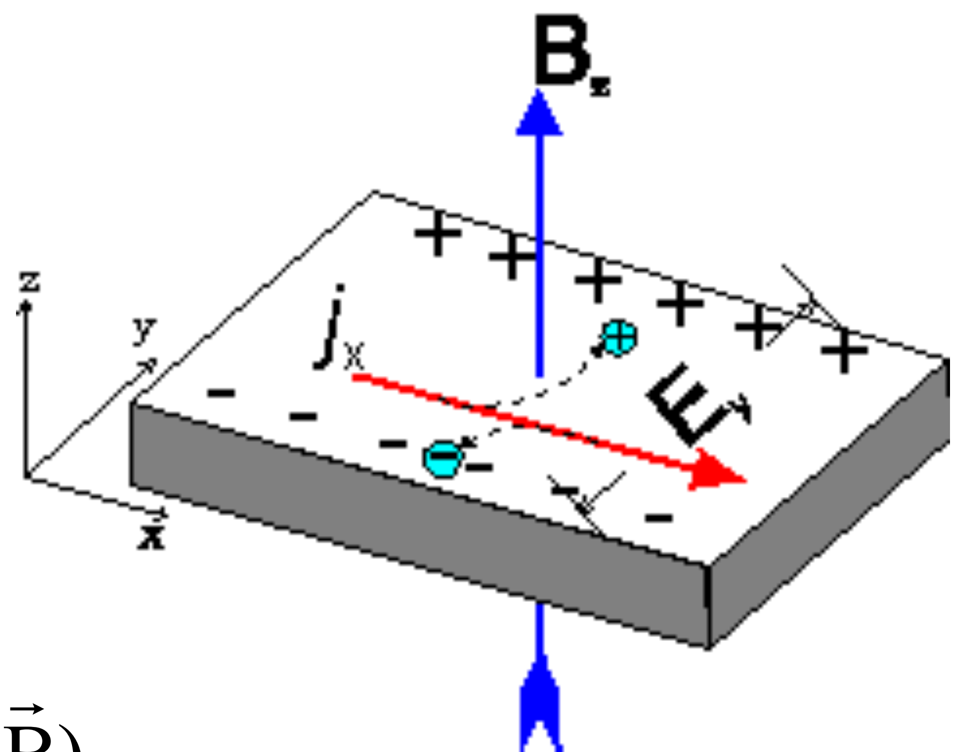
Hall resistivity $\rho_{yx} = \dfrac{E_y}{j_x}$

the Hall coefficient $R_H = \dfrac{E_y}{j_x B}$

R_H → measurement of the sign of the carrier charge

R_H is positive for positive charges and negative for negative charges

Hall Effect



Initially, $\mathrm{v} = v_\mathrm{x}\hat{\mathrm{x}} + v_\mathrm{y}\hat{\mathrm{y}} + v_\mathrm{z}\hat{\mathrm{z}}$

$$\mathrm{E} = E_\mathrm{x}\ \hat{\mathrm{x}}$$

$$\mathrm{B} = B_\mathrm{z}\ \hat{\mathrm{z}}$$

$$\vec{\mathrm{F}} = m\left(\frac{d}{dt} + \frac{1}{\tau}\right)\vec{\mathrm{v}} = -e(\vec{\mathrm{E}} + \vec{\mathrm{v}} \times \vec{\mathrm{B}})$$

m = effective mass

$$F_x = m\left(\frac{d}{dt} + \frac{1}{\tau}\right)v_\mathrm{x} = -e(E_\mathrm{x} + v_\mathrm{y}B)$$

$$F_y = m\left(\frac{d}{dt} + \frac{1}{\tau}\right)v_\mathrm{y} = e(v_\mathrm{x}B)$$

Hall Effect

Steady state condition:

$$\frac{mv_x}{\tau} = -e(E_x + v_y B)$$

$$\frac{mv_y}{\tau} = -e(E_y + v_x B)$$

$$\omega_C = \frac{eB}{m} \quad \Rightarrow \quad \begin{aligned} v_x &= -\frac{e\tau E_x}{m} - \omega_C v_y \tau \\ v_y &= -\frac{e\tau E_y}{m} + \omega_C v_x \tau \end{aligned}$$

Hall Effect

$$v_{\mathrm{y}} = -\frac{e\tau E_{\mathrm{y}}}{m} + \omega_{\mathrm{C}} v_{\mathrm{x}} \tau = 0$$

$$\Rightarrow E_{\mathrm{y}} = m\frac{\omega_{\mathrm{C}} v_{\mathrm{x}}}{e}$$

$$v_{\mathrm{x}} = -\frac{e\tau}{m} E_{\mathrm{x}}$$

$$\Rightarrow E_{\mathrm{x}} = -m\frac{v_{\mathrm{x}}}{e\tau}$$

$$E_y = -\omega_C \tau\, E_{\mathrm{x}} = -\frac{eB\tau}{m} E_{\mathrm{x}}$$

Hall Effect

The Hall coefficient is defined as:

$$R_{\mathrm{H}} = \frac{E_{\mathrm{y}}}{j_{\mathrm{x}} B} = -\frac{\dfrac{eB\tau}{m} E_{\mathrm{x}}}{\dfrac{ne^2\tau}{m} E_{\mathrm{x}} B} = -\frac{1}{ne}$$

Semiconductors: Charge Carrier Density via Hall Effect

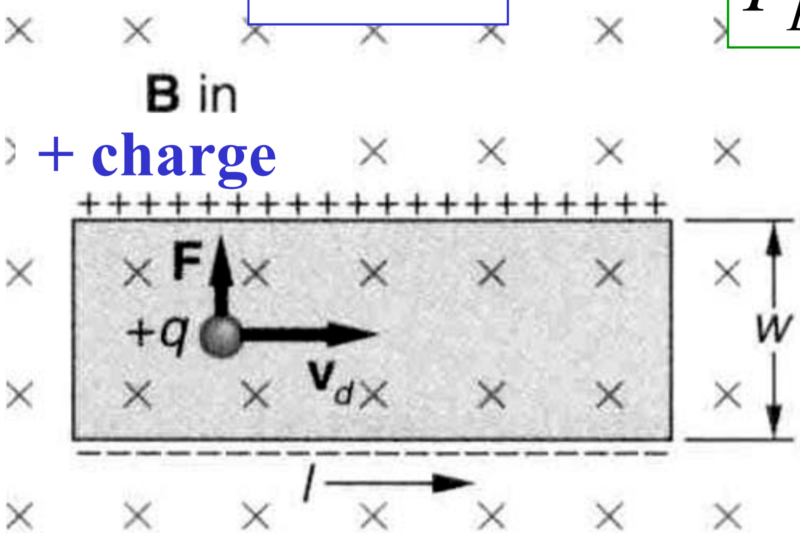
- ***Why is the Hall Effect useful?*** It can determine the **carrier type** (electron vs. hole) & the **carrier density n** for a semiconductor.
- **How?** Place the semiconductor into external **B** field, push current along one axis, & measure the induced Hall voltage $\mathbf{V_H}$ along the perpendicular axis.

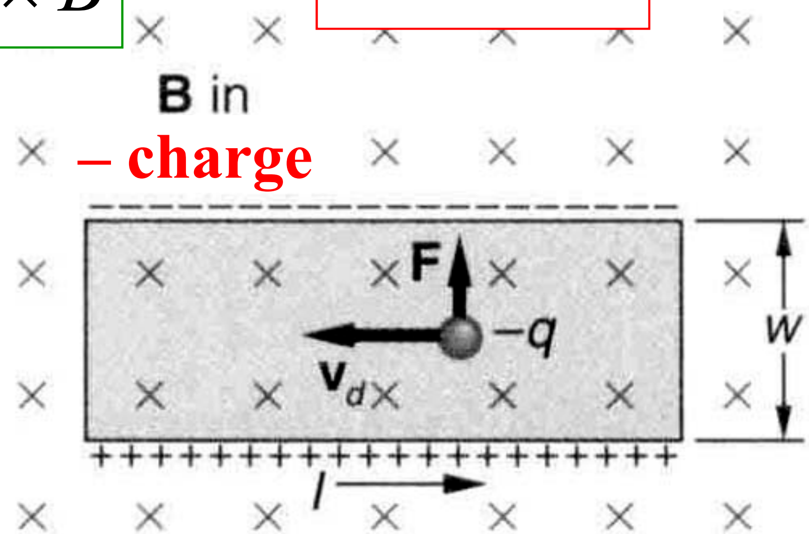
$$\mathbf{n = [(IB)/(qwV_H)]}$$

Hole

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

Electron

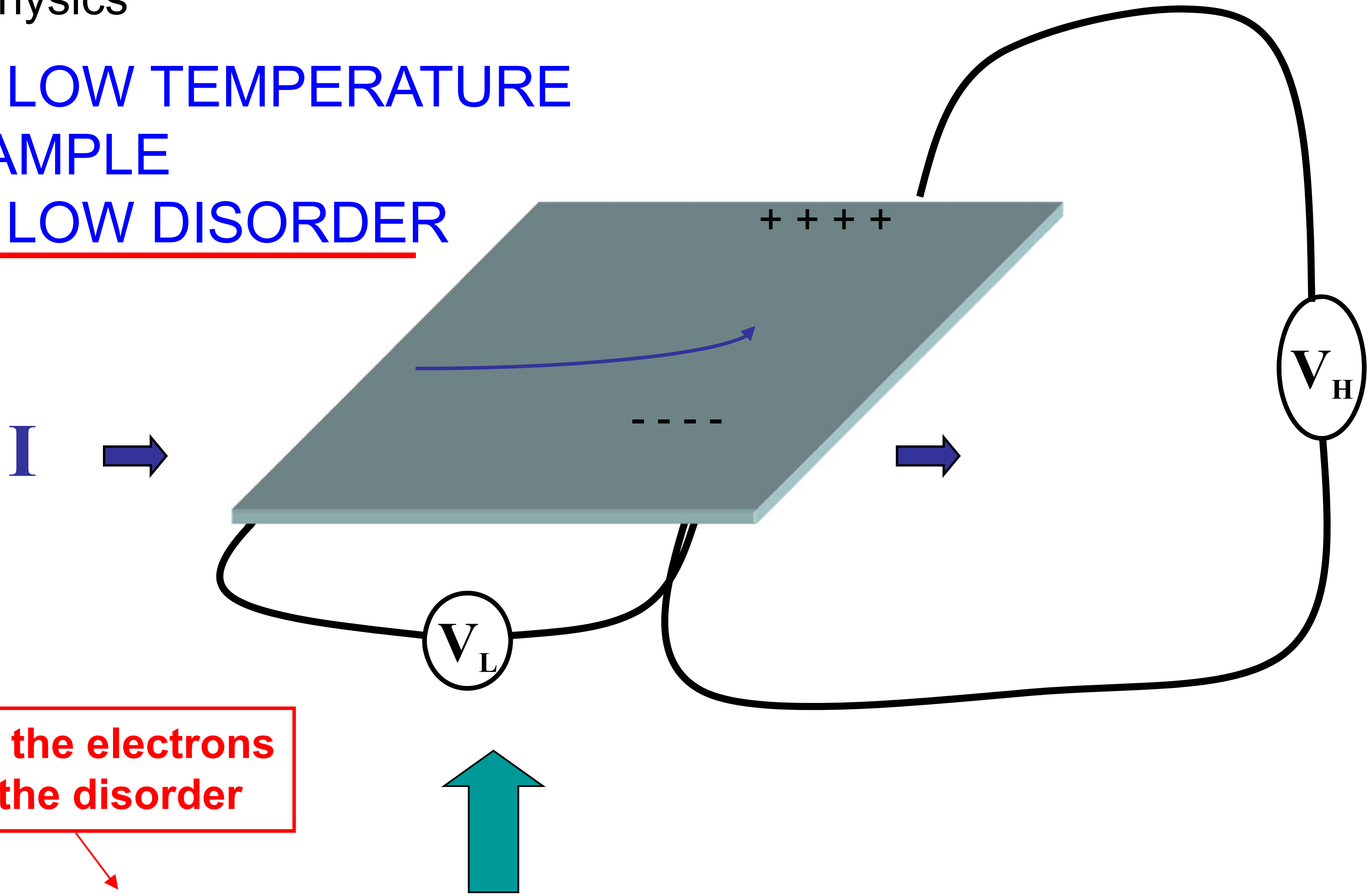




VERY LOW TEMPERATURE

2 D SAMPLE

VERY LOW DISORDER



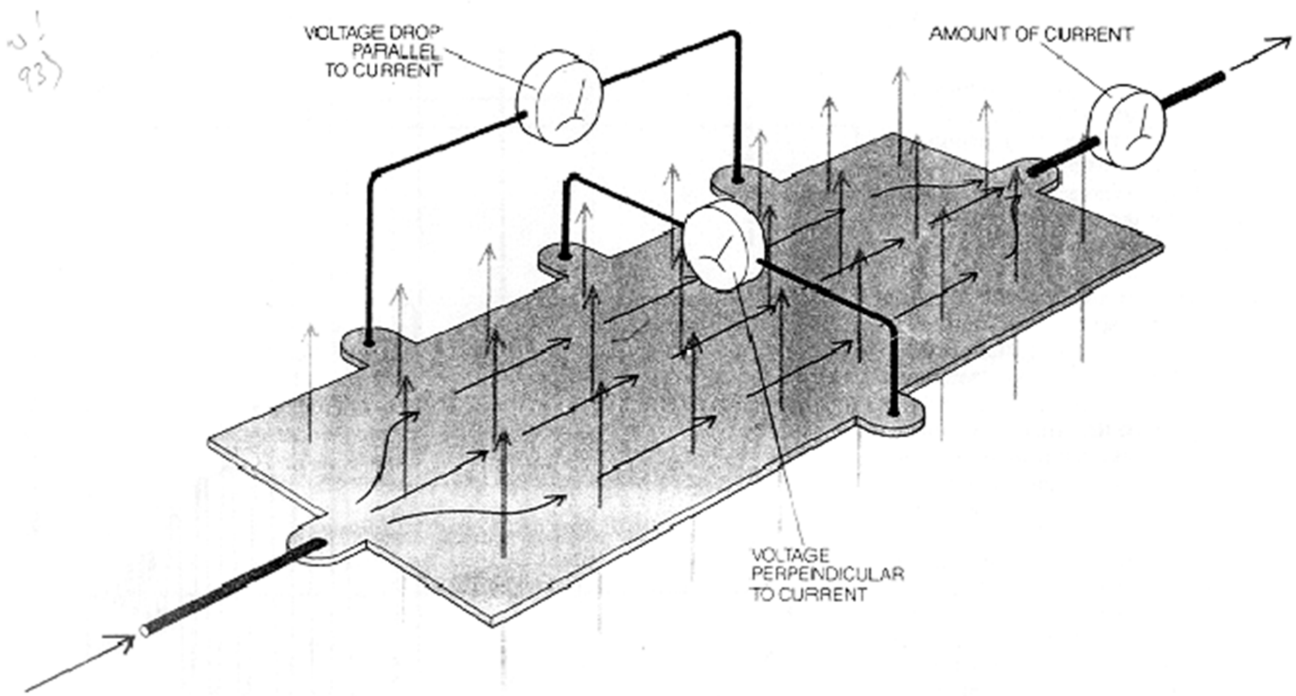
Study the electrons
Not the disorder

SEMICONDUCTOR PHYSICS!

Silicon Technology (i.e., Transistor)

III-V MBE (GaAs)

The 2D Hall Effect



The surface current density is $\mathbf{s_x = v_x n_{2D} q}$, where $\mathbf{n_{2D}}$ is the surface charge density.

$$\mathbf{R_H = 1/n_{2D}e.}$$

$\mathbf{R_{xy} = V_y / i_x = R_H B_z}$. since $\mathbf{s_x = i_x / L_y}$ & $\mathbf{E_y = V_y / L_y}$.
So, $\mathbf{R_{xy}}$ does ***NOT*** depend on the shape of the sample.

measurable quantity – Hall resistance $$R_{xy} = \frac{V_y}{I_x} = -\frac{B}{n_{2D}e}$$

for 3D systems $n_{2D} = nL_z$
for 2D systems $n_{2D} = n$

$$\mathbf{E} = \rho\mathbf{j} \qquad \mathbf{j} = \sigma\mathbf{E}$$

in the presence of magnetic field the resistivity and conductivity tensors become

for 2D: $\rho = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{yx} & \rho_{yy} \end{pmatrix}$ → $\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{yx} & \rho_{yy} \end{pmatrix} \begin{pmatrix} j_x \\ j_y \end{pmatrix}$

$$\sigma_0 E_x = \omega_c \tau j_y + j_x$$
$$\sigma_0 E_y = -\omega_c \tau j_x + j_y$$
$$\sigma_0 = ne^2\tau / m$$

$$E_x = \frac{1}{\sigma_0} j_x + \frac{\omega_c\tau}{\sigma_0} j_y$$
$$E_y = -\frac{\omega_c\tau}{\sigma_0} j_x + \frac{1}{\sigma_0} j_y$$

→ $\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} 1/\sigma_0 & \omega_c\tau/\sigma_0 \\ -\omega_c\tau/\sigma_0 & 1/\sigma_0 \end{pmatrix} \begin{pmatrix} j_x \\ j_y \end{pmatrix}$

$$\rho = \begin{pmatrix} 1/\sigma_0 & \omega_c\tau/\sigma_0 \\ -\omega_c\tau/\sigma_0 & 1/\sigma_0 \end{pmatrix}$$

→

$$\rho_{xx} = \frac{1}{\sigma_0} = \frac{m}{ne^2\tau}$$
$$\rho_{xy} = \frac{\omega_c\tau}{\sigma_0} = \frac{B}{ne}$$

$$\begin{pmatrix} j_x \\ j_y \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

$$\sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{yx} & \rho_{yy} \end{pmatrix}^{-1}$$

$$\sigma_{xx} = \sigma_{yy} = \frac{\sigma_0}{1+(\omega_c\tau)^2}$$

$$\sigma_{xy} = -\sigma_{yx} = \frac{-\sigma_0\omega_c\tau}{1+(\omega_c\tau)^2}$$

$$\sigma_{xx} = \frac{\rho_{xx}}{\rho_{xx}^2 + \rho_{xy}^2}$$

$$\sigma_{xy} = -\frac{\rho_{xy}}{\rho_{xx}^2 + \rho_{xy}^2}$$

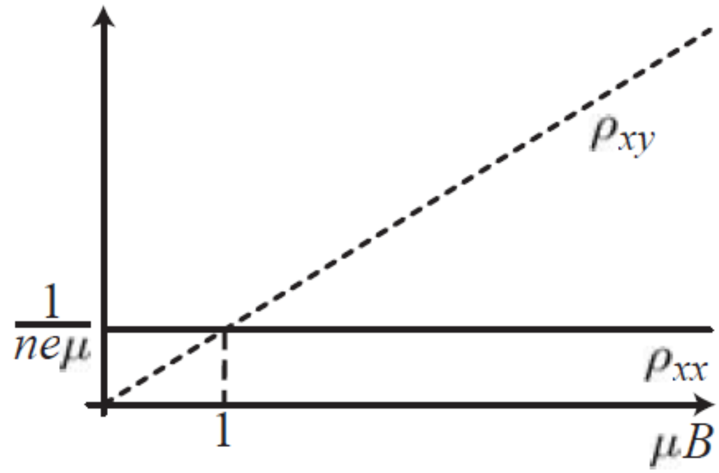
Results of Hall measurements

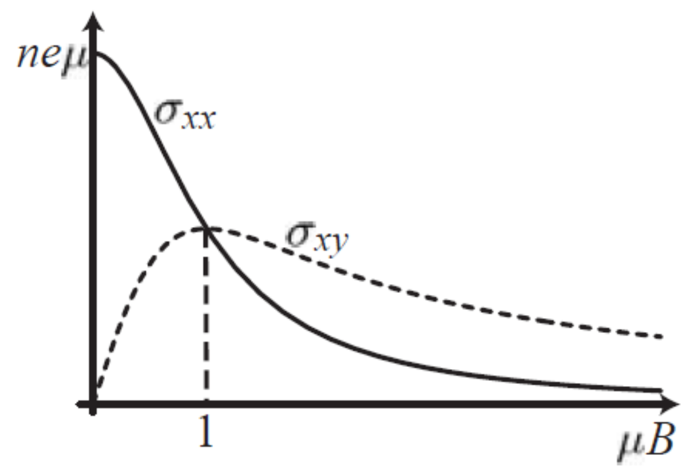
A measurement of the two independent components of the resistivity tensor allows us to determine the density and the mobility (scattering time) of the electron gas.

$$n_{\mathrm{s}} = \frac{1}{|e| \, d\rho_{xy}/dB|_{B=0}}$$

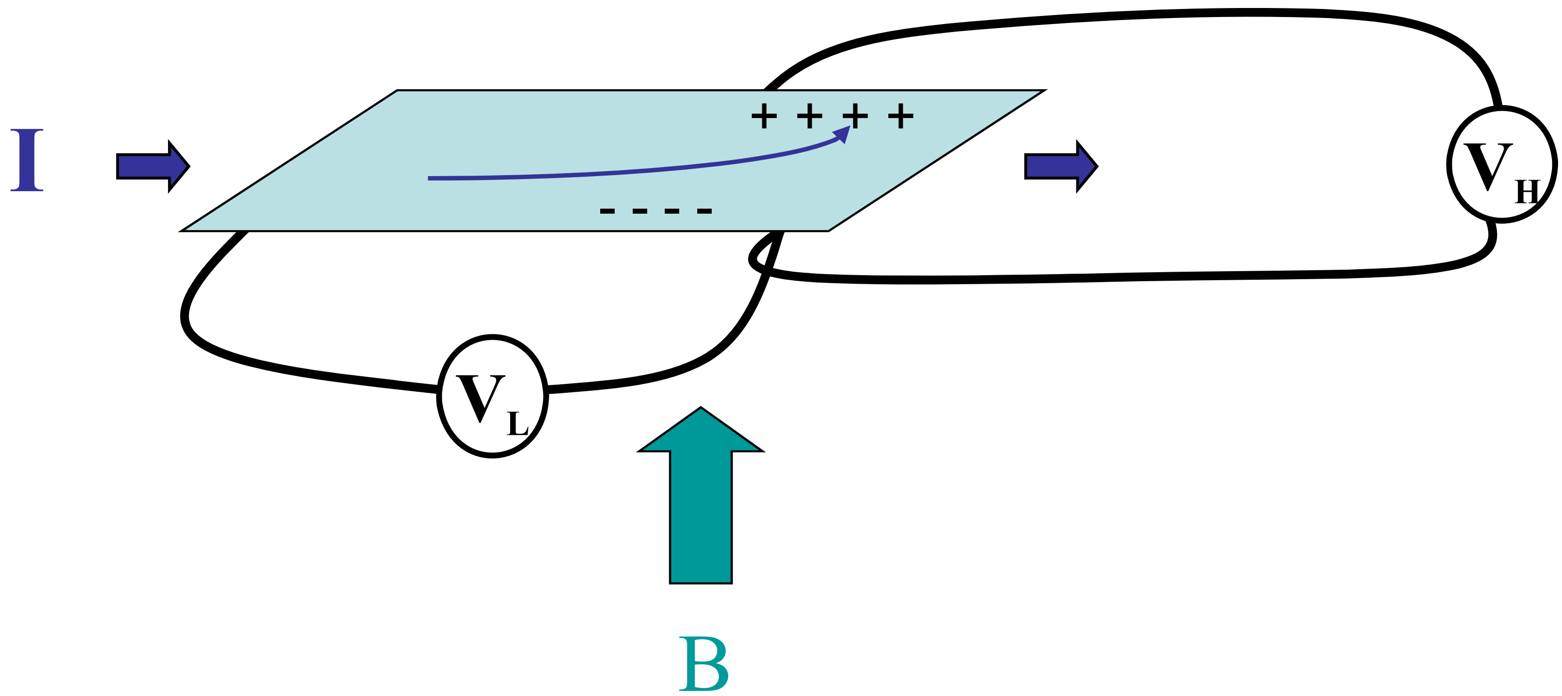
$$\mu = \frac{d\rho_{xy}/dB|_{B=0}}{\rho_{xx}(B=0)}.$$

$$\mu = \frac{|e|\tau}{m^{\star}}$$

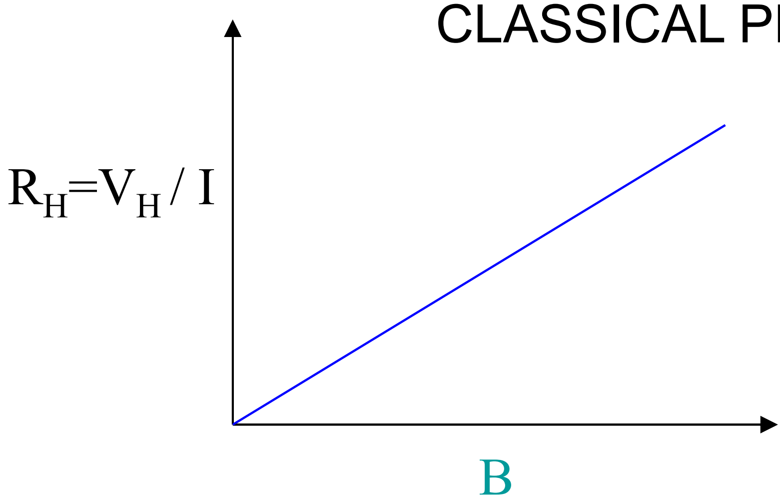


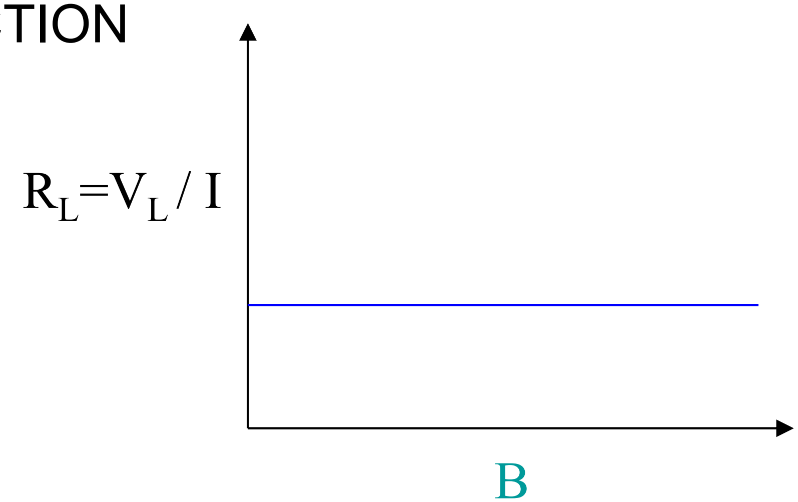


2D Hall Effect (Edwin Hall + 100 Years)



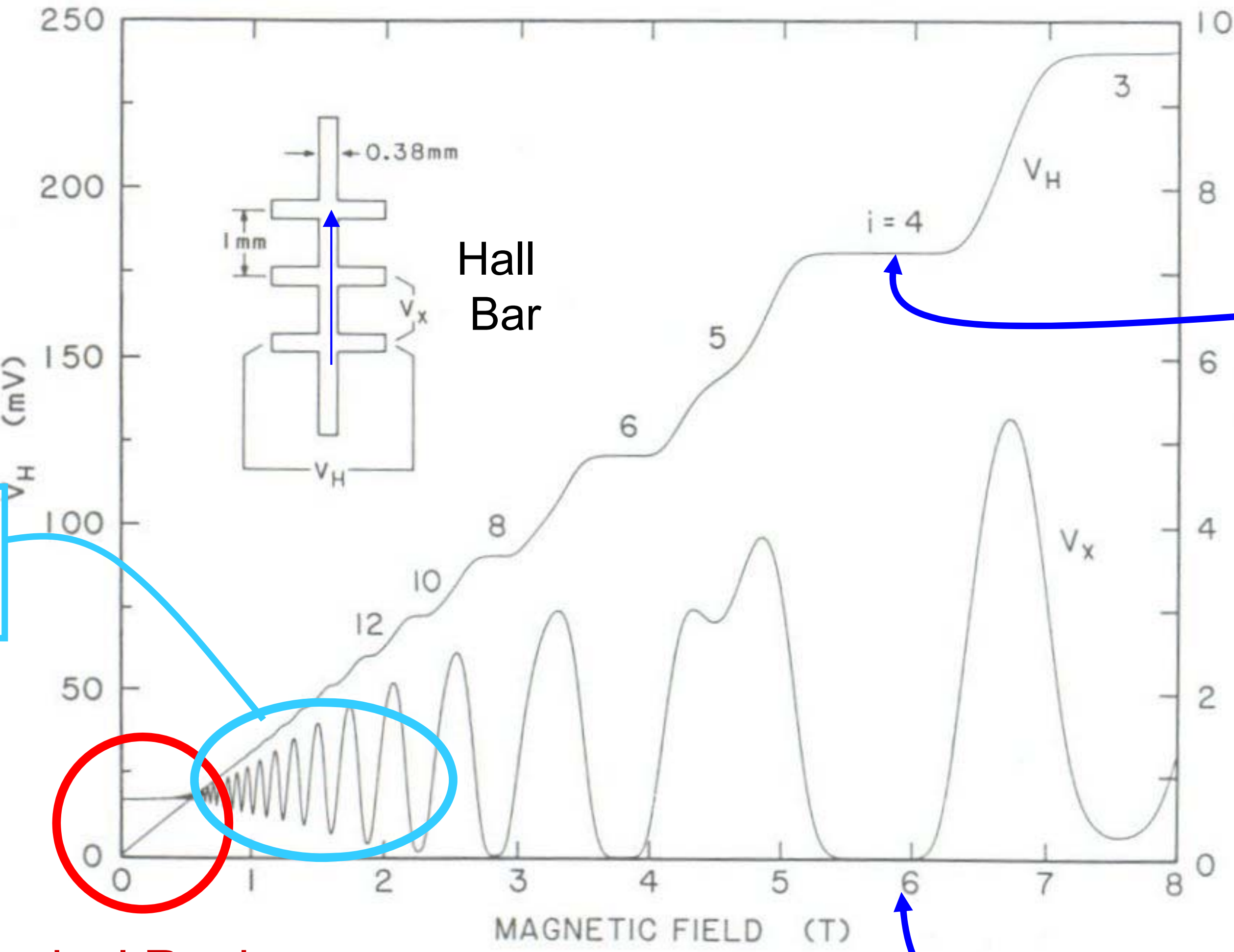
CLASSICAL PREDICTION





(Data: M. Cage)

T=1.2K



Shubnikov deHass

Classical Regime

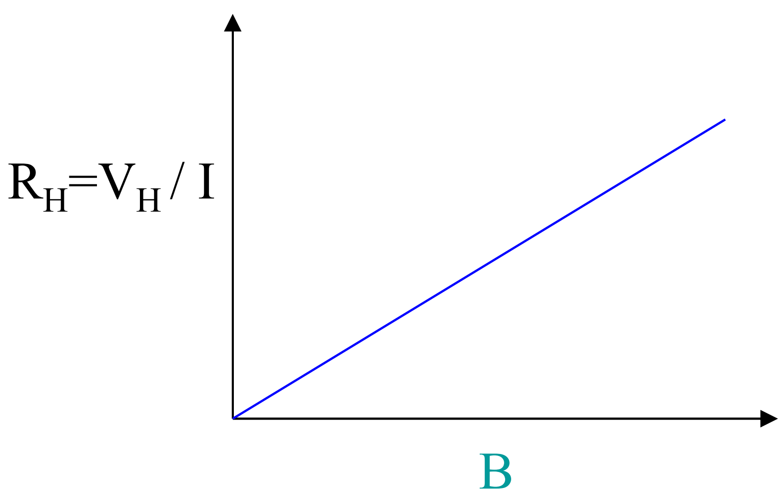
VERY FLAT!

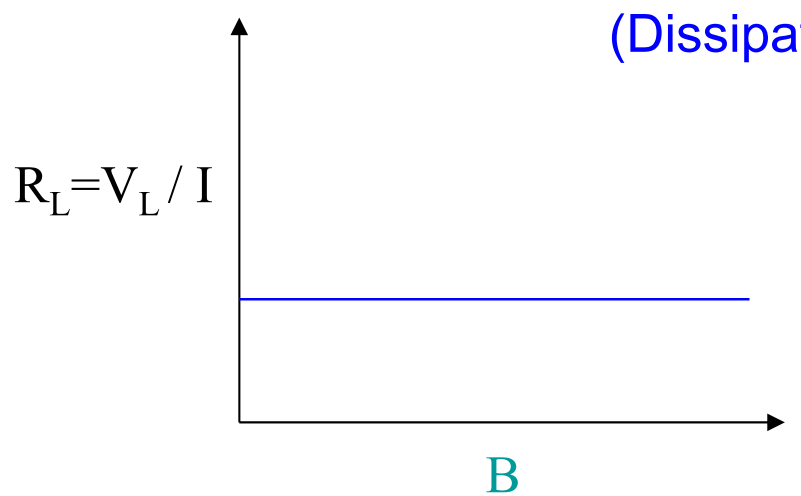
ZERO!
(Dissipationless)

$$R_H = \frac{2\pi\hbar}{e^2}\frac{1}{i}$$

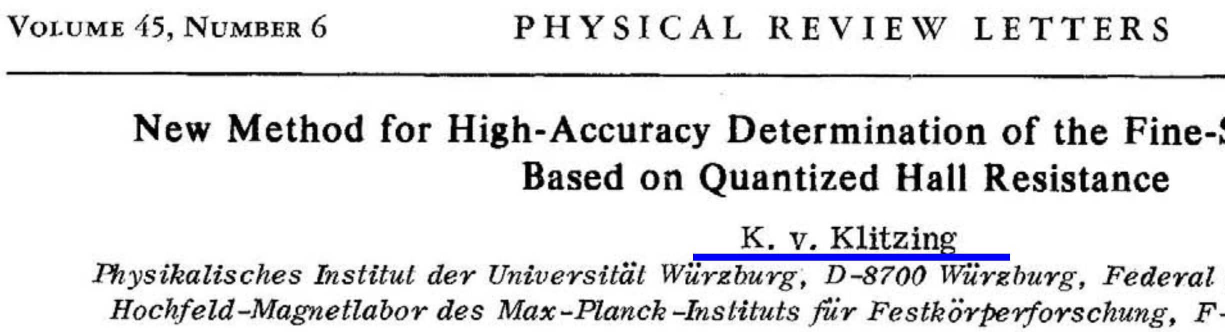
$$= R_K / i$$

i = integer



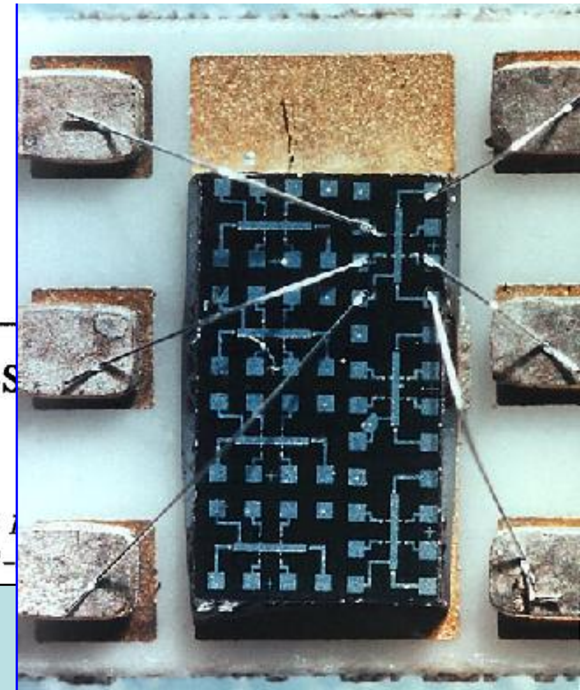


Nobel prize, 1985


VOLUME 45, NUMBER 6 PHYSICAL REVIEW LETTERS

New Method for High-Accuracy Determination of the Fine-S
Based on Quantized Hall Resistance

K. v. Klitzing
Physikalisches Institut der Universität Würzburg, D-8700 Würzburg, Federal
Hochfeld-Magnetlabor des Max-Planck-Institute für Festkörperforschung, F-
Forschungs



Why Quantization?
Why So Precise?
Why doesn't *X* destroy effect?

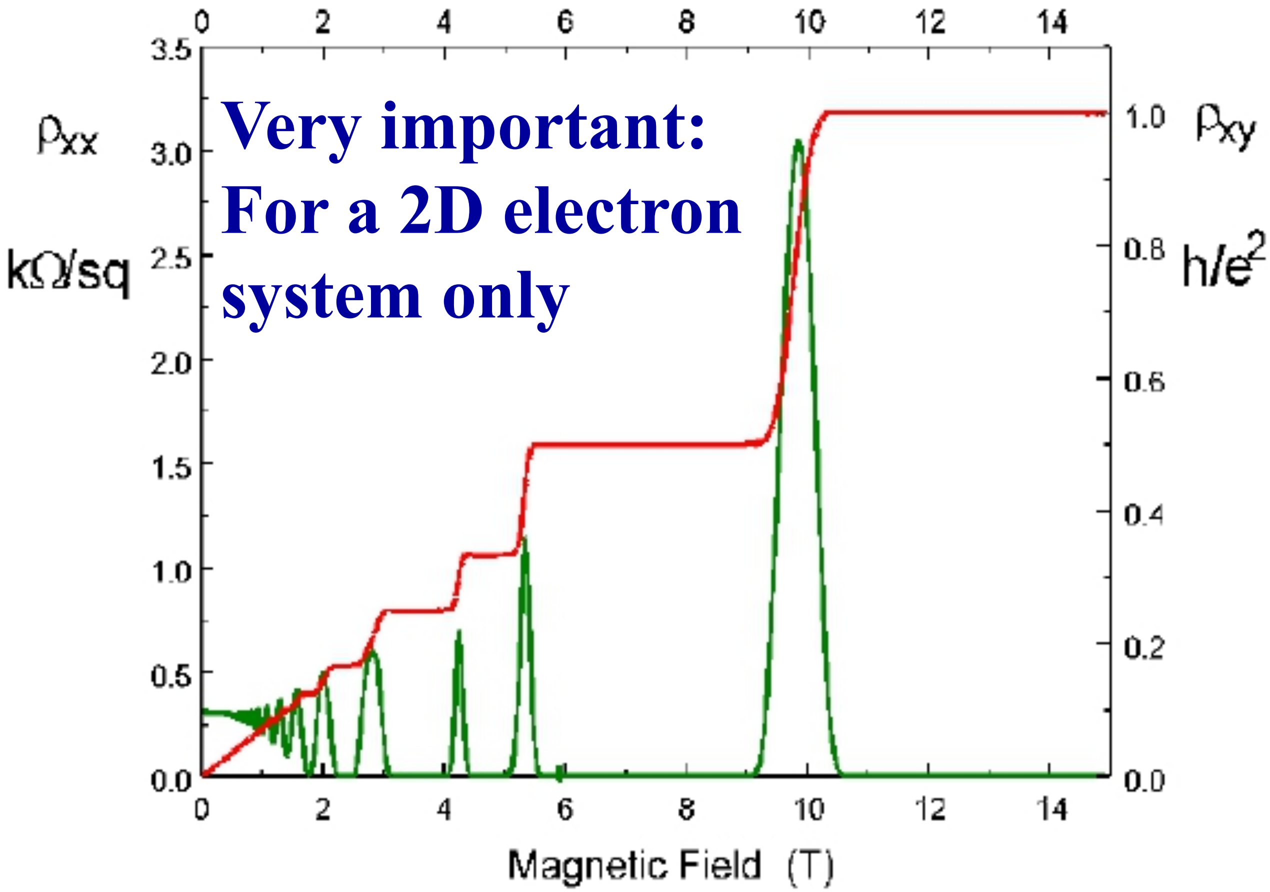
X = Dirt
= Imperfect Sample Shape
= Imperfect Contacts .... etc

Accuracy = better than 1 part in 10^9

Now DEFINES the Ohm

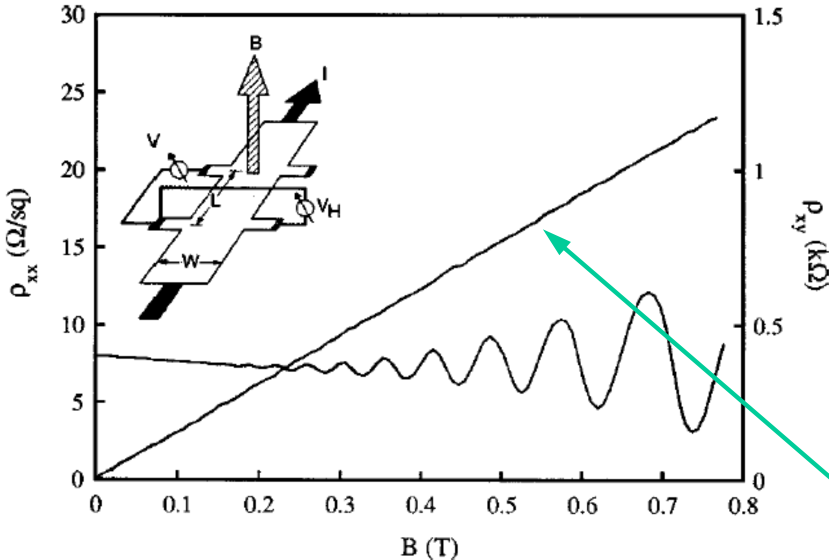
R_K=25812.807449 Ω

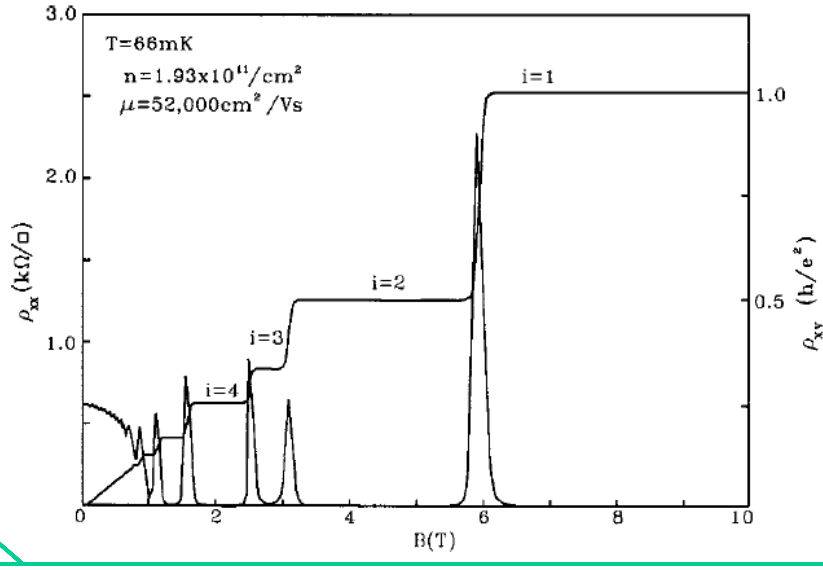
The Integer Quantum Hall Effect

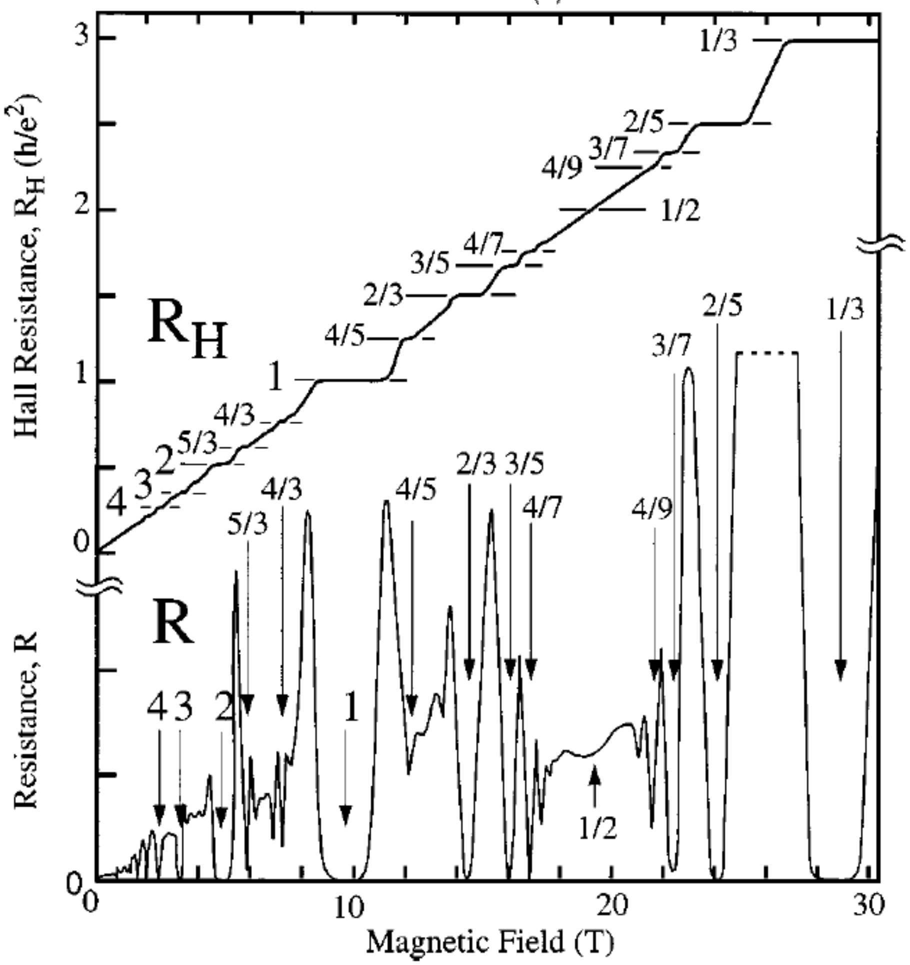


First observed in 1980 by
Klaus von Klitzing
Awarded 1985 Nobel Prize.

Hall Conductance is ***quantized*** in units of e^2/h, or
Hall Resistance $R_{xy} = h/ie^2$, where i is an integer.
The ***quantum of conductance*** h/e^2
is now known as the ***"Klitzing"!!***







from D.C. Tsui, RMP (1999) and from H.L. Stormer, RMP (1999)

Hall resistance $\rho_H = R_H B = -\dfrac{B}{ne}$

weak magnetic fields $\omega_c\tau << 1$

$$\rho_{xx} = \frac{1}{\sigma_0} = \frac{m^*}{ne^2\tau}$$

the Drude model

$$\rho_{xy} = \frac{\omega_c\tau}{\sigma_0} = \frac{B}{ne}$$

the classical Hall effect

strong magnetic fields $\omega_c\tau >> 1$

quantization of Hall resistance $\rho_{xy} = \dfrac{h}{\nu e^2}$

at integer and fractional $\nu = n \Big/ \left(\dfrac{eH}{hc}\right)$

the integer quantum Hall effect

and the fractional quantum Hall effect

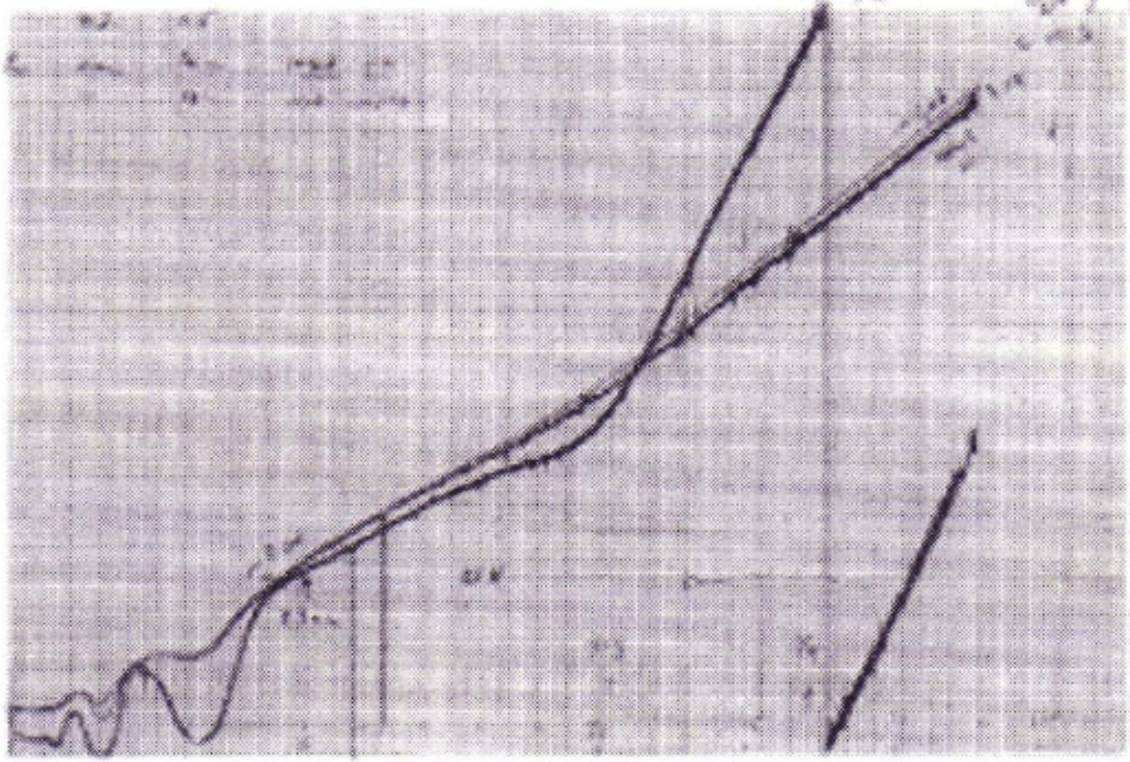
The *Fractional* Quantum Hall Effect

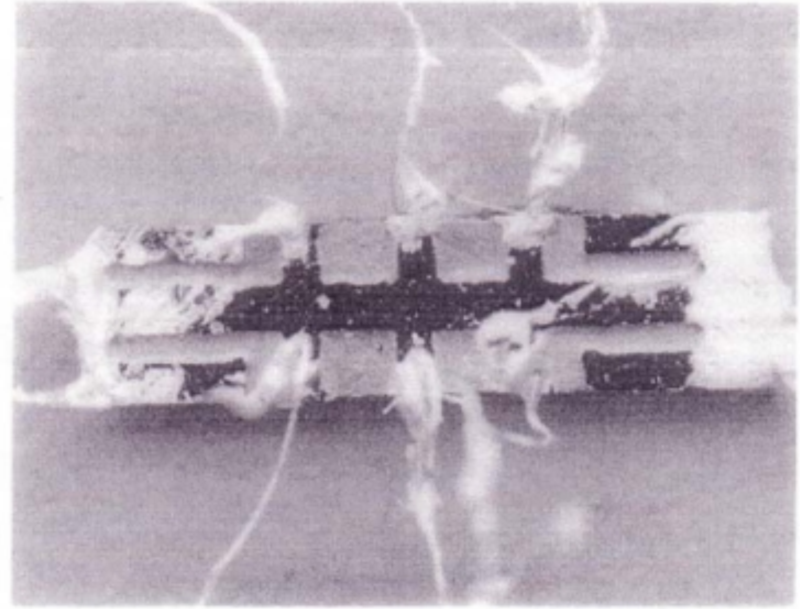
The Royal Swedish Academy of Sciences awarded **The 1998 Nobel Prize in Physics** jointly to Robert B. Laughlin (Stanford), Horst L. Störmer (Columbia & Bell Labs) & Daniel C. Tsui, (Princeton)

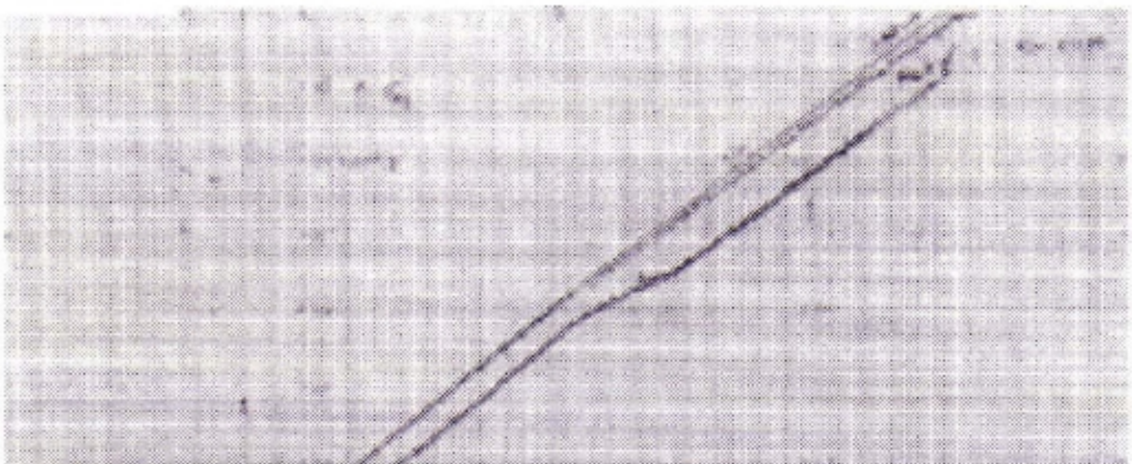
- The researchers were awarded the Nobel Prize for discovering that electrons acting together in strong magnetic fields can form new types of "particles", with charges that are fractions of electron charges.

Citation: "For their discovery of a new form of quantum fluid with fractionally charged excitations"

- Störmer & Tsui made the discovery in 1982 in an experiment using extremely powerful magnetic fields & low temperatures. Within a year of the discovery Laughlin had succeeded in explaining their result. His theory showed that electrons in a powerful magnetic field can condense to form a kind of quantum fluid related to the quantum fluids that occur in superconductivity & liquid helium. What makes these fluids particularly important is that events in a drop of quantum fluid can afford more profound insights into the general inner structure dynamics of matter. The contributions of the three laureates have thus led to yet another breakthrough in our understanding of quantum physics & to the development of new theoretical concepts of significance in many branches of modern physics.







Lightheartedly, Dan Tsui enclosed the distance between B=0 and the position of the last IQHE (~7 cm) between two fingers of one hand and measured the position of the new feature in this unit. He determined it to be three and exclaimed, “quarks!” Although obviously joking, with finely honed intuition, he had hit on the very essence of the data.