Uncertainty and Change: Survey Evidence of Firms' Subjective Beliefs

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January 11, 2024

Abstract

This paper studies how managers plan under uncertainty. In a new survey panel on German manufacturing firms, we show that *uncertainty reflects change*: Planning incorporates higher subjective uncertainty about future sales growth when the firm has just experienced unusual growth, and more so if the experience was negative. At the quarterly frequency, subjective uncertainty closely tracks the conditional volatility of shocks: Both exhibit an *asymmetric V-shaped* relationship with past growth. In the cross section of firms, however, subjective uncertainty differs from conditional volatility: planning in successful firms—either large or fast-growing—reflects lower subjective uncertainty than in unsuccessful firms even when the size of the shocks is the same.

Keywords: expectation formation, firms, measurement, subjective uncertainty, survey data

JEL codes: C83, D22, E20, E23

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1 Introduction

Firms' planning under uncertainty is a key building block of macroeconomic models. Two channels through which uncertainty affects firm actions have received particular attention. On the one hand, in many business cycle models, firms' cautious behavior in recessions contributes to lower investment and hiring.[1] On the other hand, studies of firm heterogeneity often assume that new firms learn about the viability of their business model over time.[2] A common denominator of both channels is that change in a firm's environment affects decision makers' subjective uncertainty and not simply their forecasts (that is, their conditional expectation). While connections between uncertainty and change are prominent in theory, there is little direct evidence. In particular, we lack panel data on quantitative measures of managers' beliefs—data that are crucial to study how firm planning deals with change and how change shapes perceptions of uncertainty.[3]

This paper characterizes planning under uncertainty in firms, drawing on a new panel data set of German manufacturing firms. Our survey module asks top managers not only for a forecast of one-quarter-ahead sales growth, but also for best and worst case sales growth scenarios. The idea behind this survey design is that managers can directly report scenarios developed as part of their firms' regular planning process. Survey responses confirm that most firms engage in scenario analysis and use results from routine quantitative planning in order to fill out the questionnaire. To summarize how a firm's perceived uncertainty is reflected in its planning, we focus on the *span* between its scenarios, that is, the difference between best and worst case sales growth rates. Our focus in this paper is how this quantitative measure of subjective uncertainty varies over time and across firms.

Our main result is that planning under uncertainty responds strongly to change in the environment of a firm. It provides support for both channels emphasized in the literature, but suggests that both are special cases of a more general principle: A firm's planning reflects higher subjective uncertainty when the firm experiences unusual growth, and more so if the experience is negative. At the quarterly frequency for a given firm, we show that the relationship between span and lagged growth is well described by an asymmetric V with a minimum at zero, a steep negative branch and a flatter but significant positive branch. We find this pattern in a sample without a recession, where most time-series variation is idiosyncratic. In a cross section of mature firms, we show that span is wider not only for fast-growing firms, but also, and even more strongly, for fast-shrinking firms—again an asymmetric V. We conclude that higher uncertainty matters for a firm's planning not only in recessions or at young age, but whenever firms enter unfamiliar territory through experienced change.

How does subjective uncertainty relate to the conditional volatility of shocks experienced by the firm? We show that this relationship depends on the frequency at which change occurs. First, quarterly variation in subjective uncertainty is quite similar to that in the conditional

[1] Bloom (2014), Fernández-Villaverde and Guerrón-Quintana (2020), and Cascaldi-Garcia et al. (2023) survey the literature on business cycle models with time-varying uncertainty following Bloom (2009).

[2] See Hopenhayn (2014) for an overview of models of firm dynamics, including those emphasizing learning, following Jovanovic (1982).

[3] See Bachmann and Carstensen (2022) for an overview of existing data on managers' beliefs.

volatility of forecast errors, estimated by fitting a power GARCH model: Both measures of (conditional) uncertainty exhibit mild persistence and an asymmetric-V-shaped response to lagged growth. In the short run, managers' planning under uncertainty thus reflects their anticipation of the size of future shocks. In this sense, managers appear to understand experienced change.

Over the medium term, by contrast, experienced change goes along with systematic bias in both forecasts and perceptions of uncertainty. Indeed, in firms on either good or bad growth trends, managers' forecasts are consistently too close to the status quo. Moreover, managers' subjective uncertainty cannot be proxied by the conditional volatility of forecast errors: planning in growing firms reflects lower uncertainty than in shrinking firms even when the firms are faced with shocks of the same size. We find the same result for large compared to small firms. Success thus breeds confidence that disconnects planning under uncertainty from conditional volatility.

Our data come from a new module of the *ifo Business Survey*, a long-established survey administered by the Munich-based ifo institute used to develop business sentiment indicators in Germany. The survey is well regarded in the German business community: Questions are typically answered by senior management and there is a high response rate even from large firms.[4] Our analysis uses a quarterly sample over four years from 2013 to 2016 which allows us to explore both time-series and cross-sectional variation in planning behavior. Bachmann, Carstensen, Menkhoff and Schneider (2023) provide an introduction to the data set and further discussion of data quality.

To interpret span and relate it to subjective uncertainty in economic models, we present a simple model of managers performing scenario analysis. Suppose managers face continuous shock distributions, but only have time to think through a finite number of scenarios. They know that their response to shocks will work better when the shock is close to a scenario they have previously thought through. Optimal choice of scenarios therefore "covers" the space of shock realizations, reflecting the shape of the distribution. We show that when the loss function is quadratic and the distribution of shocks belongs to the location-scale family, then span—the difference between the best and worst case scenarios—serves as an index of (subjective) conditional volatility. In particular, an increase in volatility entails an increase in span in the same proportion. Since quadratic cost can be viewed as a second order approximation of a more general loss function, this establishes a close link between span and subjective volatility in a wide range of economic models.

To provide an idea of magnitudes, the mean span between best and worst case quarterly sales growth scenarios is 12.1 percentage points (pp), slightly above the mean absolute forecast error. Firms differ in their average subjective uncertainty: The cross-firm standard deviation of time-averaged span is 7.4 pp. At the same time, we document large time variation in subjective uncertainty for individual firms: The time-series standard deviation of span for the average firm is 5.9 pp. Time-varying span is thus a volatile component of firms' planning pro-

[4] Recent studies that draw on the ifo Business Survey include Bachmann, Elstner and Sims (2013); Bachmann and Elstner (2015); Buchheim and Link (2017); Massenot and Pettinicchi (2018); Bachmann, Born, Elstner and Grimme (2019); Enders, Hünnekes and Müller (2019, 2022).

cesses. In fact, it is almost as volatile as the usual driver of firm planning in economic models, namely, changes in conditional expectations: The average firm's time-series standard deviation of growth forecasts is 7.4 pp. Most of the time-series variation in subjective uncertainty is firm-specific: Time-industry fixed effects explain only a negligible share.

For our medium-term analysis, we regress the time-average of firm-level subjective uncertainty on firm characteristics, including the following two medium-term measures of change in an individual firm's environment over the entire four year sample: *trend* is the firm's unconditional in-sample mean growth rate and *turbulence* is the firm's unconditional in-sample sales growth volatility. Both high and low trend firms perceive more uncertainty: Span in the bottom quartile by trend is 5.9 pp higher than for "normal" firms within the interquartile range; it is 2.2 pp higher in the top quartile. At the same time, high and low trend firms' forecasts are biased towards zero by about 5 pp on average. Controlling for trend, subjective uncertainty is also higher at firms that are smaller and those that experience more turbulence. Sectoral dummy variables do not add much explanatory power.

Turbulence generates not only higher, but also *more variable* subjective uncertainty. Controlling for trend as well as size, the mean span in the top quartile by turbulence is 9.6 pp higher than in the bottom quartile, whereas the time-series standard deviation of span is 5.5 pp higher. In other words, planning at firms that live in a more volatile environment not only uses scenarios that are further apart but also varies those scenarios more over time. This distinction matters because of its implication for behavior such as factor choice: In the presence of adjustment costs or time-to-build, time variation in subjective uncertainty leads firms to respond differently each period. If high-volatility firms simply faced larger iid shocks, they might still behave differently from low-volatility firms, but that behavior would not vary over time. According to our results, theoretical mechanisms that make firms respond to uncertainty should generate both cross-sectional and time-series variation in actions.

To compare subjective uncertainty and the magnitude of forecast errors, we re-estimate our cross-sectional regressions with the average absolute forecast error rather than span on the left-hand side. The key finding is that successful firms—defined as either large or fast-growing—plan with narrower spans even when their absolute forecast errors are of the same magnitude as those of less successful firms. First, large firms with more than 250 employees make similar absolute forecast errors as smaller firms, yet plan with spans that are up to 5 pp narrower, controlling for trend and turbulence. Second, controlling for firm size and turbulence, the absolute forecast error for fast-growing firms is 2.5 pp higher than for normal firms, while span is not significantly different. For shrinking firms, by contrast, both the absolute forecast errors and span increase by about 3 pp.

Moving on to time-series variation, we characterize the dynamics of subjective uncertainty in two steps. First, the V-shaped relationship between subjective uncertainty and past growth is well described by a piecewise linear regression with a minimum at zero. Bad quarters increase uncertainty by more: While a 1 pp lower negative growth rate is followed by 31 basis points (bp) wider span between firms' best and worst case scenarios, a one percentage point higher positive growth rate widens span by only 18 bp; these findings are robust to including controls for firm heterogeneity. Second, we compare time-series variation in subjective uncertainty and conditional volatility estimated using power GARCH models of sales growth. We find that both subjective uncertainty and conditional volatility are mildly persistent, increase with bad past growth and increase somewhat less with good past growth.

We conclude that conditional volatility is a good proxy for subjective uncertainty one quarter ahead: over short horizons, rational expectations about conditional volatility is therefore a sensible assumption. Panel data are crucial here since they allow us to derive a measure of predictable variation in volatility by estimating a GARCH model. We can thus show that span is high for a firm precisely when an econometrician would predict large forecast errors. We note that such a conclusion would not follow from, say, cross-sectional comovement of subjective uncertainty and absolute forecast errors. Indeed, such comovement could arise simply because firms' forecasts are biased more whenever subjective uncertainty is high, which is not consistent with rational expectations. Since we compare subjective uncertainty to the volatility of forecast errors from a statistical model, we rule out that explanation.

Our results provide direct evidence for time variation in subjective uncertainty, as is required for mechanisms from the business cycle literature to operate. At the same time, as in Hassan, Hollander, van Lent and Tahoun (2019) who study political risk by analysing quarterly earnings conference calls, we emphasize large idiosyncratic variation. Mechanisms used to explain recessions should therefore be relevant also for firm dynamics in normal times.[5] They shape the cross-sectional distribution of firms, the key state variable determining the reaction to aggregate shocks in heterogenous-firm models. However, models have to be consistent with a V-shaped relationship between uncertainty and past growth, which rules out uncertainty shocks that affect current growth, but are orthogonal to past growth.[6] Our results further suggest that a model of firms' planning at the quarterly frequency should draw a connection between subjective uncertainty and conditional volatility, such as that traditionally provided by the rational expectations assumption. Firms adjust their planning process based on the experience that high and—even more so—low growth signals larger future surprises.

There are several candidate reasons why uncertainty responds to change quarter-to-quarter. One possibility is a learning process such that uncertainty increases with forecast errors. We indeed document a positive relationship between lagged absolute forecast errors and span. This result is related to Boutros, Ben-David, Graham, Harvey and Payne (2020) who find that executives who provide confidence intervals for stock returns widen those intervals when the realized return falls out of their last interval. However, our managerial forecasts of sales growth also reflect a second force: Uncertainty increases with *predictable* bad change. Indeed, for firm quarters with negative growth, the previous-quarter growth rate is sufficient for pre-

[5] Moreover, our results on high-frequency variation in subjective uncertainty suggest that even a short time-to-build friction could lead to effects of uncertainty on factor choice. With any type of adjustment costs, quarterly variation in uncertainty works like a distortion—a wedge between the marginal product and price of a factor (see, for example, Ilut and Saijo, 2021, for a model of firms facing idiosyncratic risk that clarifies this feature).

[6] Our results thus speak to an active discussion in the literature on the timing of growth and uncertainty movements. Several papers have considered feedback effects from growth to uncertainty (Bachmann and Moscarini, 2012; Fajgelbaum, Schaal and Taschereau-Dumouchel, 2017; Ilut, Kehrig and Schneider, 2018; Baley and Blanco, 2019; Berger and Vavra, 2019; Ludvigson, May and Ng, 2020; Ilut and Valchev, 2023). Our results say that understanding firm dynamics requires either such feedback effects or otherwise uncertainty shocks that are correlated with past growth.

dicting span. This is because predictable low growth realizations increase uncertainty in the same way as growth surprises. A possible explanation is that planning takes into account state variables in addition to growth.[7]

A number of studies use survey measures of firms' uncertainty. Following the pioneering work of Guiso and Parigi (1999), the focus has been on the effect of uncertainty on economic activity; see, for example, Bontempi, Golinelli and Parigi (2010) and Fiori and Scoccianti (2023) for Italy, and Bachmann, Elstner and Sims (2013) and Bachmann, Elstner and Hristov (2017) for Germany. The goal of the present paper is not to study the effect of uncertainty on actions, but instead to characterize how uncertainty varies over time and relates to *past* growth.[8] Altig, Barrero, Bloom, Davis, Meyer and Parker (2022) document properties of one-year-ahead subjective uncertainty in the cross section of firms in business surveys for the United States. In contrast to their paper, our focus is on the dynamics of uncertainty, leveraging our long quarterly panel. The latter is what allows us to compare belief dynamics with a GARCH model and to provide cross sectional results on firms following different growth trends.

Our comparison between subjective and statistical measures of uncertainty is relevant for the large behavioral literature on firm decision making (see Malmendier, 2018, for an overview). The typical application documents systematic biases in forecasts of managers and their relationship to firm actions. Deviations from rational expectations thus typically appear in (conditional) first moments. Ben-David, Graham and Harvey (2013) consider managers' forecast densities for stock returns and show that managers are strongly miscalibrated in that their subjective forecast densities are too narrow, thus questioning rational expectations as a modeling assumption. Gennaioli, Ma and Shleifer (2016) show that managers' expectations are connected to actual firms' investment plans, thus showing that miscalibration has real effects. Ma, Ropele, Sraer and Thesmar (2020) evaluate the aggregate losses from biased forecasts in the structural model of firm dynamics and misallocation developed by David and Venkateswaran (2019). Our results complement this evidence by providing a novel focus on the role of experience for subjective uncertainty. In particular, our medium-term results show how biases in beliefs matter beyond first moments and their effects on actions are likely larger, once effects of uncertainty are taken into account.

The paper is structured as follows. Section 2 explains our new survey questions and provides background on data quality as well as basic summary statistics. Section 3 introduces the raw relationship between uncertainty and change and presents a simple organizing framework. Section 4 studies uncertainty and change in the cross section, while Sections 5 and 6 investigate the time-series variation of subjective uncertainty. Finally, Section 7 compares the dynamic properties of subjective uncertainty and conditional volatility.

[7] For example, in a model with customer capital, a shrinking firm might see the size and/or composition of its future pool of customers and thus its sales growth become more uncertain even when this shrinking occurs in a predictable way. See Gourio and Rudanko (2014) for a model of customer capital.

[8] Our empirical approach relates to the large literature on households that studies the formation of expectations through experience, following Vissing-Jørgensen and Attanasio (2003) and Malmendier and Nagel (2011). Of course, our focus is on uncertainty of firms—not forecasts of households—as a left-hand side variable. There is also an active literature that studies managers' expectations about aggregate variables, such as inflation (see, for New Zealand, Kumar, Afrouzi, Coibion and Gorodnichenko, 2015; Coibion, Gorodnichenko and Kumar, 2018; and, for Italy, Coibion, Gorodnichenko and Ropele, 2020).

2 Data

The ifo Business Survey, run by the Munich-based ifo Institute, is a well-established survey of German businesses. ifo maintains a representative sample of German businesses by replacing exiting firms with new respondents (see Sauer and Wohlrabe, 2020). Responses from the—mostly qualitative—survey are used as inputs for a leading indicator of the German business cycle: The ifo Business Climate Index is widely publicized (see Lehmann, 2022, for its high forecasting quality); it is also part of the EU-harmonized business surveys commissioned by the Directorate General for Economic and Financial Affairs of the European Commission.[9]

In 2012, we designed and added an online module of *quantitative* questions to elicit subjective firm uncertainty. A large majority of the firms participates online in the main ifo Business Survey. All manufacturing firms in the online main survey were invited to participate. An initial pilot wave in December 2012 was met by strong interest. Analysis of text comments submitted by firms further showed that firms had no trouble understanding the questions. The module has now been in the field since 2013, with participation stable between 300 and 400 firms per wave.[10]

A firm in the survey is either a stand-alone firm or a division of a larger conglomerate. For simplicity, we refer to "firms" throughout this paper. Survey questions are about growth in sales. The German term used in the questionnaire, "Umsatz", is a well-defined technical term in profit and loss accounting, translated into English as "sales" or "total revenue." It is commonly used as an accounting statistic at the levels of both a division and an entire firm.

The survey is administered at the beginning of every quarter. Our current sample uses 14 survey waves from 2013:Q2 to 2016:Q3. In fall 2018, we fielded an additional one-time special survey with questions on how firms collect information and arrive at the views expressed in our uncertainty module. We also draw on an additional special survey fielded by ifo in the fall of 2019 that was sent out to all manufacturing firms participating in the main ifo survey.

The composition of respondents to our uncertainty module closely tracks that of the manufacturing firms in the main ifo survey. Indeed, Appendix A shows that it is essentially impossible to predict participation in the uncertainty module. Our data contains a substantial number of large firms: When we measure firm size by the number of employees, the 75th percentile is at about 250 employees. The median firm employs 100 workers while the 25th percentile is at 40.

2.1 Quality of responses

In partnering with ifo, our goal was to develop a high-quality data set that (i) reflects the perception of uncertainty by key decision makers in firms and (ii) allows respondents to draw

[9]Aggregate survey results for Germany are presented at https://www.ifo.de/en/survey/ifo-business-climate-index, the harmonized European results, including the European Economic Sentiment Indicator, can be found here: https://ec.europa.eu/info/business-economy-euro/indicators-statistics/economic-databases/business-and-consumer-surveys_en.

[10]The raw data can be found under: "IBS-IND (2016b): ifo Business Survey Industry 1/1980 - 12/2016, LMU-ifo Economics & Business Data Center, Munich, doi: 10.7805/ebdc-ibs-ind-2016b."

on quantitative analysis they have already done as part of their in-house planning. Special-survey questions provide direct evidence on data quality along both dimensions.[11] We report the findings in the following.

First, in the overwhelming majority of participating manufacturing firms, the survey respondent is a member of top management. Sauer and Wohlrabe (2019) document that 73% of firms mention CEO, CFO or COO, and an additional 13% of survey units refer to a "division head", the natural label for the top executive if the unit surveyed is not a stand-alone firm. For large firms with more than 500 employees, the shares are only slightly lower: 65% CEOs, CFOs or COOs, and 15% division heads. The findings are consistent with an earlier meta-study conducted by ifo about the trade sector (see Abberger, Sauer and Seiler, 2011).

Second, the identity of the responder *within* the firm changes rarely. The special survey fielded by ifo in fall 2019 asks who filled out the questionnaire in the past. 83% of firms indicate the responder is "always the same person", 15% say "mostly the same person", and less than 2% mention a team of people or that the responder "changes frequently". This is important for interpreting our time-series results. In particular, changes in subjective uncertainty will thus almost always reflect changing views of one top decision maker, as opposed to differences in opinions between several executives who take turns filling out the survey.

Third, a large majority of survey respondents in firms of all size classes rely on results from routine quantitative planning when filling out the questionnaire. Our own 2018 special survey contains a series of questions on what information respondents use; its original German version can be found in Appendix B. We first ask whether answers to our uncertainty questions are guided by numbers that the firm has already developed in-house as part of a regular quantitative planning process. The results are summarized in the top row of Table 1, both for all firms and broken down by size class.[12] On average, 80% of firms respond that they use results from quantitative planning. The share is remarkably stable across firm size classes, only small firms report a somewhat lower share.

Fourth, most firms rely on both statistical analysis and *scenario analysis*, that is, thinking about the future in terms of a few concrete—often fairly detailed—scenarios without necessarily attaching probabilities.[13] We know this from a follow-up question to firms that do quantitative planning: Those firms are asked to indicate the importance of either approach on a four point scale, or possibly fill in an alternative approach. The middle and lower rows of Table 1 reports that more than half of firms viewed scenario analysis and statistical analysis as "important" or "very important". Interestingly, firms rely more heavily on scenario analysis than on statistical analysis across all size classes. For firms that routinely compute adverse and favorable scenarios as part of their planning process, filling out the survey thus does not impose an additional forecasting task and is likely to generate more thought-out answers.

[11] Link, Peichl, Roth and Wohlfart (2023) provide additional independent evidence for the case of aggregate economic conditions that, in general, survey answers by firms are of high quality.

[12] In line with the definition by the German Statistical Office, we define firms as "tiny" if they have less than 10 employees, "small" if the number of employees is between 10 and 50, "medium" if the number of employees is between 50 and 250, and "large" if the number of employees exceeds 250.

[13] A well-known example of scenario analysis is bank stress testing: Banks are asked to forecast losses given a detailed set of contingencies, but they are not asked to assign probabilities to those contingencies.

Table 1: Special-survey 2018 answers on quantitative planning

	All obs.	Tiny & Small	Medium	Large
Firms with quantitative sales planning	0.80	0.73	0.80	0.80
Scenario analysis very important or important	0.64	0.57	0.68	0.66
Statistical analysis very important or important	0.52	0.52	0.57	0.47

Notes: The numbers are from the fall 2018 special survey on a sample of 191 firms. The top row presents the share of firms that report that their answers to our uncertainty questions are guided by numbers that the firm has already developed in-house as part of a regular quantitative planning process. Column 1 reports the overall share, while columns 2 to 4 show the share by three size groups. In line with the definition by the German Statistical Office, firms are "tiny" if they have less than 10 employees, "small" if the number of employees is between 10 and 50, "medium" if the number of employees is between 50 and 250, and "large" if the number of employees exceeds 250. The middle and the lower rows contain the results of two follow-up questions for firms that report engaging in regular quantitative planning. We present the shares of firms that consider scenario and statistical analysis, respectively, as "very important" or "important" for their quantitative sales planning. The other answer options were "less important" and "not important." Columns 2 to 4 shows the sum of the shares answering with "very important" or "important" by size group.

2.2 Eliciting subjective uncertainty

The uncertainty module of the ifo Business Survey asks firms, at the beginning of a quarter, a two-part question. Figure 1 displays the sample questionnaire for April 2014 in the original German. In English, the questionnaire reads:

The following questions refer to changes against the previous quarter.

1. *By how much in percentage terms have your sales changed in the first quarter of 2014?*

2. *By how much in percentage terms will your sales change in the second quarter of 2014?*

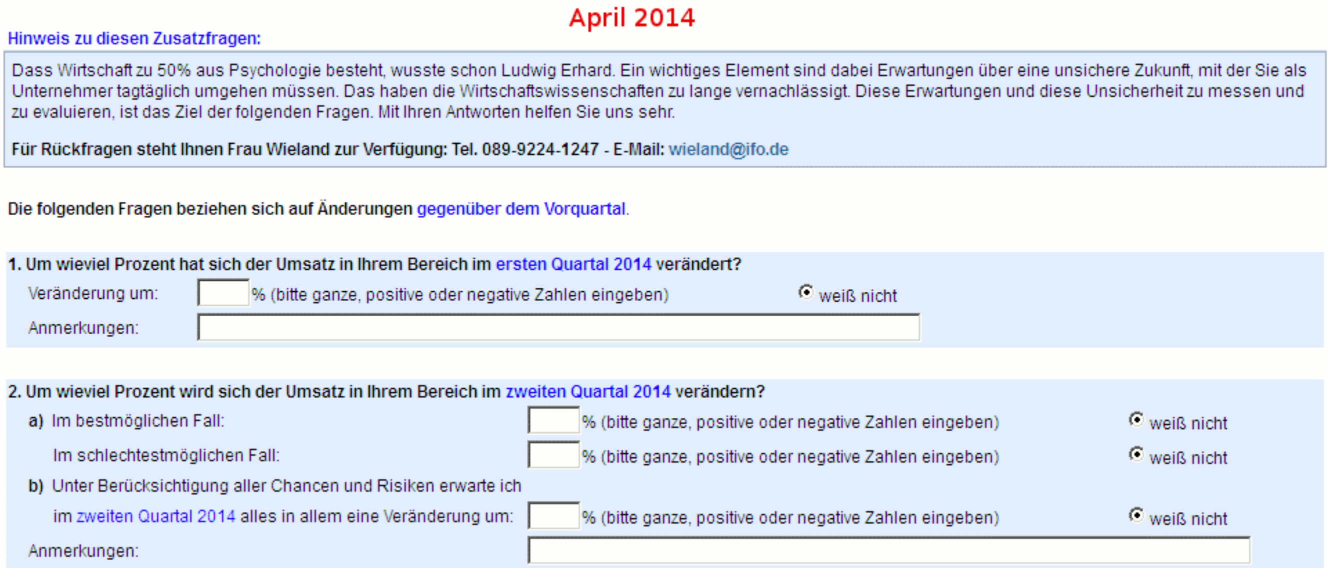
   a. *In the best possible case*:
   In the worst possible case:

   b. *Taking into account all contingencies and risks, I expect for the second quarter of 2014 all in all a change of*:

The questionnaire form contains four boxes for respondents to provide their four numerical answers. Next to every box, there is a reminder to provide positive or negative integers. In addition, respondents are given a "don't know-"option ("weiß nicht" in German) behind the box, as shown in the figure. Finally, underneath both questions 1 and 2, firms are invited to provide free text comments ("Anmerkungen").

To clarify the timing, consider a firm responding in April 2014, that is, in the first two and a half weeks of 2014:Q2. Question 1 asks for the change in sales between 2013:Q4 and 2014:Q1. This is the most recent sales growth realization that the firm has experienced. Question 2 then asks for the firm's outlook over the current quarter 2014:Q2, as compared to the last quarter 2014:Q1. This is the next growth rate realization that the firm expects.

Figure 1: Original survey questionnaire in German



April 2014

Hinweis zu diesen Zusatzfragen:

Dass Wirtschaft zu 50% aus Psychologie besteht, wusste schon Ludwig Erhard. Ein wichtiges Element sind dabei Erwartungen über eine unsichere Zukunft, mit der Sie als Unternehmer tagtäglich umgehen müssen. Das haben die Wirtschaftswissenschaften zu lange vernachlässigt. Diese Erwartungen und diese Unsicherheit zu messen und zu evaluieren, ist das Ziel der folgenden Fragen. Mit Ihren Antworten helfen Sie uns sehr.

Für Rückfragen steht Ihnen Frau Wieland zur Verfügung: Tel. 089-9224-1247 - E-Mail: wieland@ifo.de

Die folgenden Fragen beziehen sich auf Änderungen gegenüber dem Vorquartal.

1. Um wieviel Prozent hat sich der Umsatz in Ihrem Bereich im ersten Quartal 2014 verändert?

Veränderung um: % (bitte ganze, positive oder negative Zahlen eingeben) weiß nicht

Anmerkungen:

2. Um wieviel Prozent wird sich der Umsatz in Ihrem Bereich im zweiten Quartal 2014 verändern?

a) Im bestmöglichen Fall: % (bitte ganze, positive oder negative Zahlen eingeben) weiß nicht

Im schlechtestmöglichen Fall: % (bitte ganze, positive oder negative Zahlen eingeben) weiß nicht

b) Unter Berücksichtigung aller Chancen und Risiken erwarte ich im zweiten Quartal 2014 alles in allem eine Veränderung um: % (bitte ganze, positive oder negative Zahlen eingeben) weiß nicht

Anmerkungen:

Notes: Original questionnaire from ifo's online module on subjective uncertainty in German; screenshot from April 2014.

Our quantitative measure of subjective uncertainty is the *span* between the best and worst case scenarios for sales growth that firms provide in response to question 2.a. A firm's *forecast error* is the difference between its actual sales growth in the current quarter and its expected growth rate at the beginning of that quarter, that is, its answer to part 2.b. At the beginning of every quarter, firms cannot perfectly predict the flow of sales over the entire quarter; forecast errors capture the mistakes they make. We note that in order to observe a forecast error for a firm, we need to observe the firm in two consecutive survey waves.

Sample construction. Our baseline sample consists of 400 firms and 2,762 firm-quarter observations from 14 quarters. We describe the sample construction in detail in Appendix C. Briefly, we first focus on firms that have at least five firm-quarter observations for realized previous-quarter sales growth rate (question 1). We then retain firms that also provide sensible answers to the second question on forecasts. In both steps, text comments provided by firms are useful to assess outliers and to drop firms unwilling or unable to provide quarterly forecasts. 191 of the 400 firms in the baseline sample filled out the fall 2018 special survey.

To study sectoral effects, we form 14 *industries* that are based on two-digit manufacturing codes, but are aggregated further to ensure sufficiently sized populations of surveyed firms. Details are in Appendix D, where we also show the distribution of firms across industries. Our baseline sample contains at least 60 firms per industry.

The main ifo Business Survey requests that firms ignore seasonal fluctuations in their answers. Consistent with this, we observe only negligible seasonal effects in our data. Indeed, we can compare the aggregate sales growth rates measured in our survey—and thus deseasonalized by the individual firms—with a seasonally adjusted time series of manufacturing sales growth rates measured by the Federal Statistical Office, Destatis, through an unrelated survey. The time-series correlation between the Destatis series and our series is 0.76. We thus treat the variables below as seasonally adjusted at the individual firm level.

2.3 Span as a measure of subjective uncertainty

The premise behind our survey module is that when firms perceive more uncertainty, they contemplate positive and negative scenarios that are further apart, and hence report a larger span. In this section, we provide a simple model that relates scenarios chosen by a firm during scenario analysis to their perceived uncertainty. The purpose of the model is to (i) illustrate a plausible thought process underlying survey answers, and (ii) show how to relate span to belief measures in applications. In particular, we argue that for a large and flexible set of distribution families, span is proportional to standard deviation and can therefore serve as an index of subjective conditional volatility.

There are two dates. At date 1, the firm knows that a stochastic growth rate g will realize at date 2. In order to prepare a contingent plan, managers think through a finite number of n scenarios for sales growth, collected in a vector $\hat{g}$ in ascending order, that is, $\hat{g}_{i-1} < \hat{g}_i$ for $i = 2, ..., n$. Span is the difference between the best and worst scenarios, or $\hat{g}_n - \hat{g}_1$. Managers know that their response will work relatively well as long as the realized growth rate is fairly close to a scenario they have thought through. We formalize this idea via a quadratic cost function that depends on the distance between the realized growth rate and the closest scenario. We can think of this objective as a second-order approximation of a more general cost function—second order is the lowest order needed to capture risk effects.

The firm chooses the vector of scenarios $\hat{g}$ to minimize expected cost, given its knowledge of the distribution of g:

$$\min_{\hat{g}} E\left[\min_{1\leq i\leq n}\left(g-\hat{g}_i\right)^2\right]$$

Here, the outer minimum operator reflects that managers' ex ante choice of scenarios minimizes expected cost. The minimum operator inside the expectation captures that, ex post, costs depend on distance to the closest scenario, or equivalently, the minimum distance over all scenarios. The parameter n captures the number of scenarios a firm has the capacity to consider.

The following proposition shows how span responds to the distribution of growth rates, for the location-scale families of distributions that are common in both empirical work and economic modeling.[14]

Proposition. Suppose g can be written as $g = \mu + \sigma\varepsilon$ where ε has a continuous density with mean zero and unit variance: Then the optimal span is a linear function of σ, that is, $\hat{g}_n - \hat{g}_1 = \sigma\left(\hat{s}_n - \hat{s}_1\right)$, where $\left(\hat{s}_n - \hat{s}_1\right)$ is the optimal span for $\sigma = 1$.

The proof is in Appendix E. Intuitively, managers would like to optimally "cover" the

[14] Examples of the location-scale family are the normal, Laplace and t-distributions as well as their generalizations such as the exponential power distribution and the asymmetric power distribution.

outcome space of g in order to minimize cost. With quadratic cost, first-order conditions for the optimal vector of scenarios describe conditions on the first two moments, mean and variance. Since the distribution belongs to a location-scale family, the solution can be expressed in terms of "standardized" scenarios $\hat{s}_i = (\hat{g}_i - \mu) / \sigma$ that solve the moment conditions for $\sigma = 1$ and $\mu = 0$. They depend generally on the shape of the density—for example, if the density is skewed, then managers place fewer scenarios in the thin tail.

For general μ and σ, optimal scenarios then follow by an affine transformation: when the location of the random variable is shifted by the mean μ, and the variable is scaled by volatility σ, optimally covering the outcome space applies the same operations to all scenarios. For example, double volatility calls for doubling all distances of scenarios from the mean. This means in particular that span—the difference between the best and worst case scenarios—does not depend on μ and must be proportional to σ.

The proposition implies that, under certain conditions, span delivers an index of subjective volatility/uncertainty. Its level depends on features of the subjective distribution, captured by $(\hat{s}_n - \hat{s}_1)$, that we cannot infer from our survey question. However, movements in subjective volatility over time will be picked up by movements in span. We thus obtain a simple recipe for using statistics of span to, say, calibrate a model with stochastic subjective volatility: relative movements in subjective volatility should be consistent with measured movements in span.

2.4 Properties of subjective uncertainty

In this section, we present stylized facts on span and forecast errors. Detailed tables of summary statistics are provided in Appendix F, here we discuss selected results.

Sales growth is hard to predict. Realized firm sales growth has a standard deviation of 14.7 percentage points (pp) and an interquartile (IQ) range from −5% to 10%. Relative to this variation, the distribution of forecasts is compressed, with an IQ range from zero to 5%. The variance of forecasts is about half that of the realizations. Forecasts display little bias on average: The average forecast is essentially the same as the average realization. For an average firm, the standard deviation of forecast errors is 10.2 pp, similar in magnitude to the standard deviation of its sales growth of 11.4 pp. Together, these moments indicate that predicting sales growth is difficult: unpredictable variation is close to total variation.

One might suspect that firms provide forecasts in a mechanical way by simply using past growth or some constant baseline growth rate. In our data, both hypotheses are false. Indeed, the difference between a firm's forecast and its last realization of growth has a standard deviation of 17.2 pp, larger than that of the forecast itself at 14 pp. At the same time, the difference between a firm's forecast and its firm level mean growth rate has a standard deviation of 10.8 pp. In other words, both simple forecasting rules generate growth predictions that deviate substantially from firms' actual forecasts. We conclude that firms' forecasts are nontrivial functions of past growth.

Best and worst case scenarios and the magnitude of subjective uncertainty. Firms' best and worst case scenarios bracket forecasts almost symmetrically. The average worst and best case scenarios are −4.8% and 7.4%, respectively. The midpoint between the scenarios is 1.3% and hence less than one percentage point below the average forecast of 2.2%. Both scenarios have slightly higher standard deviations and wider IQ ranges than forecasts. A key difference between the variables is that the distribution of the lower (upper) bound is negatively (positively) skewed.

Our measure of subjective uncertainty is similar in magnitude to firm-level unconditional volatility. Indeed, the mean span for the average firm is 12.3 pp, while its time-series standard deviation of growth rates is 11.4 pp. Since growth is hard to predict, the span reported by the average firm is also similar in magnitude to the typical absolute forecast error experienced by a firm, 9.4 pp.

Subjective uncertainty varies in the cross section. To assess variation of subjective uncertainty in the cross section, we compute the average span for each firm. The cross-sectional standard deviation of average span is 7.4 pp. It is similar in magnitude to the cross-sectional standard deviation of the average absolute forecast error of 9.6 pp. Firms thus differ substantially in both the size of the typical shock they experience and in the way their planning deals with perceived uncertainty. Both variables are positively, if imperfectly correlated in the cross section of firms, with a coefficient of 0.43. Firms that make larger forecast errors on average thus tend to perceive more uncertainty on average.

Subjective uncertainty varies in the time series at the firm level. Our data also show substantial time variation in subjective uncertainty at the firm level. The time-series standard deviation of span for the average firm is 5.9 pp and hence more than half of the standard deviation of span in the pooled sample. Time-series variation in subjective uncertainty is also substantial compared to other changes in firms' beliefs. For example, the cross-sectional mean of firms' time-series standard deviation of forecasts is 7.4 pp (and thus only 1.5 pp higher than the aforementioned number for span), and numbers for best and worst scenarios are only slightly higher (in both cases approximately 8.1 pp).

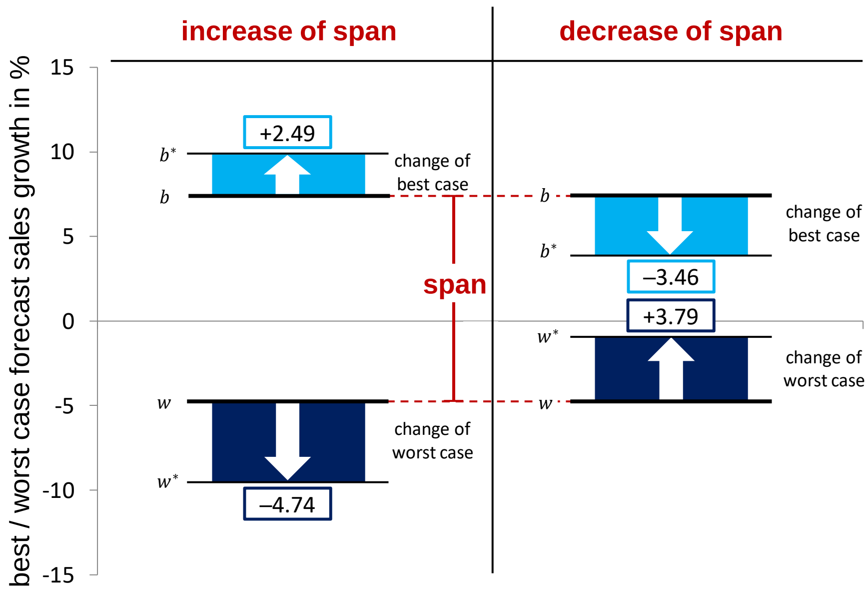
What do *changes* in uncertainty look like? On average, they consist of moves in both the best and worst case scenarios. In particular, for all instances where a firm increases its span from one quarter to the next, the mean change in the worst case scenario is -4.7 pp whereas the mean change in the best case scenario is $+2.6$ pp, see Figure 2. In other words, the average increase in uncertainty thus consists of an outward expansion of span that is slightly asymmetric. The average decrease in span is a symmetric downward compression: Conditional on a decrease in span, the worst case increases by 3.9 pp and the best case decreases by 3.4 pp.

Finally, the variation of subjective uncertainty in both the time series and the cross section is overwhelmingly firm-specific. Indeed, regressions of span on fixed effects achieve an R-squared of 0.006 for time fixed effects, .030 for time and industry fixed effects 0.030, and .084 for time-industry fixed effects.[15] This fact does not imply that we cannot uncover patterns in the variation of span, as we will see below. It simply means that the cross-sectional patterns are not driven by industry, but rather by differences in firm perceptions within industries.

[15]The variation in firms' forecast errors and absolute forecast errors is also overwhelmingly firm-specific. For forecast errors, a regression on time fixed effects yields an R-squared of 0.010, a regression on time and industry effects an R-squared of 0.023, and a regression on time-industry fixed effects an R-squared of 0.12. For absolute forecast errors, a regression on time fixed effects yields an R-squared of 0.014, a regression on time and industry effects an R-squared of 0.033, and a regression on time-industry fixed effects an R-squared of 0.12.

Similarly, time-series patterns are largely driven by individual firm experiences as opposed to, say, the state of the business cycle.

Figure 2: Changes in subjective uncertainty



Notes: The figure illustrates how, on average in the cross section of firm-quarter observations, changes in the best and worst case scenarios generate increases (left part of the figure) and decreases (right part of the figure) of subjective uncertainty (span). The plot shows the pooled averages of span as well as the pooled averages of the best and worst case sales growth rates, which are denoted by b and w, respectively. b^* and w^* are the best and worst case scenarios after the average changes of span. The figure is based on the 2,762 firm-quarter observations of *baseline sample span*.

3 Uncertainty and change

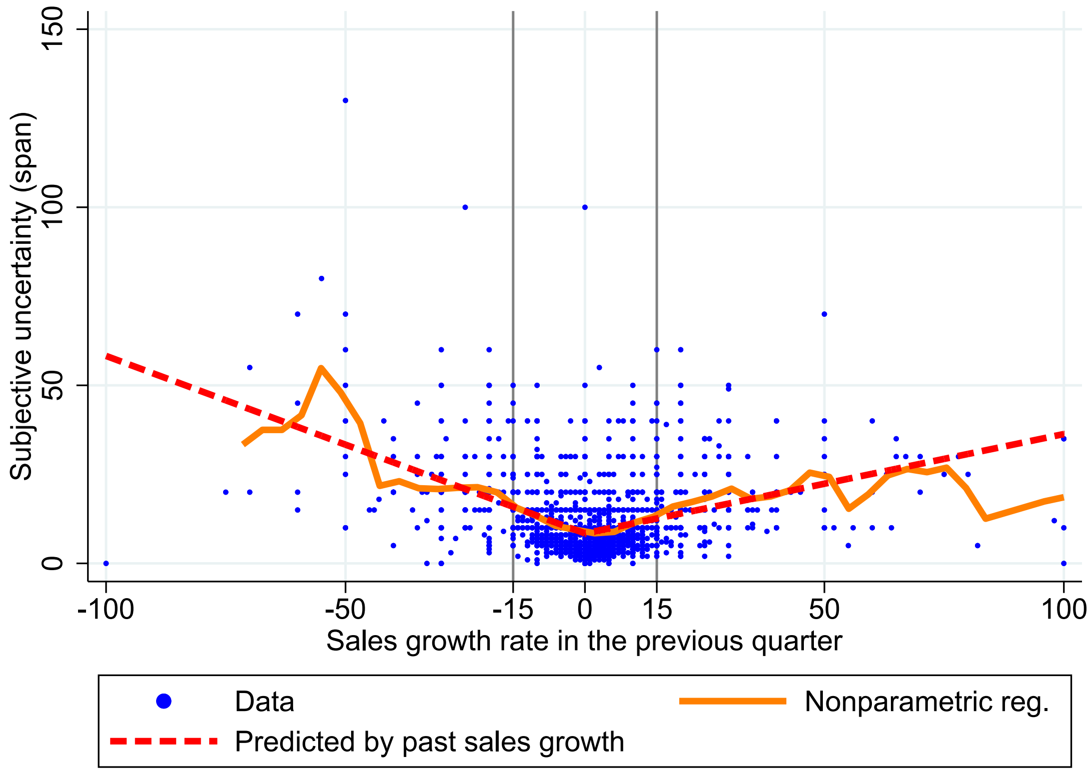
How does firms' subjective uncertainty relate to their experience? In this section, we first present our key stylized fact on subjective uncertainty and change: The relationship between subjective uncertainty and past sales growth looks like an asymmetric V. We then lay out a simple organizing framework that guides our subsequent analysis of how the fact reflects both cross-sectional and time-series variation.

3.1 Uncertainty and past growth: An asymmetric V

Figure 3 displays a scatter plot of the survey responses, with span at the beginning of a quarter measured along the vertical axis, and quarter-to-quarter sales growth realized in the quarter before along the horizontal axis. Vertical gray lines indicate the interdecile range which reaches from -15% to $+15\%$ as reported in Table 13 in Appendix F.

Firms that have experienced larger changes are more uncertain. In particular, the relationship between subjective uncertainty and past sales growth looks like the letter V with a minimum near zero. This is illustrated in the figure by two lines: The solid line is a non-

Figure 3: Uncertainty and past sales growth



Notes: Every dot represents a firm-quarter observation. The solid organge line is the prediction from a kernel-weighted local polynomial regression of degree zero with an Epanechnikov kernel where the bandwidth was selected based on the rule of thumb suggested by Fan and Gijbels (1996). The dashed red line depicts the predicted values from a piecewise linear regression of subjective uncertainty on past sales growth, with a break at zero. The thin vertical lines mark the interdecile range that extends from −15% to 15%, see Table 13 in Appendix F.

parametric regression line, while the dashed line is from a simple piecewise linear regression with a breakpoint at zero.[16] The two lines are very similar, and they virtually coincide in the relevant range where most observations are located.

Firms perceive higher uncertainty after negative change than after positive change. Indeed, the slope of the left-hand branch of the letter V is about twice as large in absolute value as the slope of the right-hand branch. After a one percentage point lower negative sales growth, next quarter's span is 50 basis points wider. By contrast, after one percentage point higher positive sales growth, span is wider by slightly more than 25 basis points. The regression coefficients are reported in column (2) of Table 4, discussed further below.

The V-shaped regression line relates uncertainty to change; it stands in contrast to the simple linear (often negative) relationship between uncertainty and growth emphasized in

[16]We have compared the in-sample fit of a piecewise linear regression model with a breakpoint at zero with that of a quadratic model. Both the Akaike and Bayesian information criteria favor the piecewise linear model.

the literature. At the same time, asymmetry implies that uncertainty and growth are in fact negatively correlated. Indeed, a linear regression returns a small but significantly negative coefficient of -.06, shown below in column (1) of Table 4. However, ignoring the V-shape drastically lowers the explanatory power of past sales growth rates for uncertainty from an R-squared of 0.19 for a piecewise linear regression to an R-squared of just below 0.01 for the simple linear regression. In other words, a simple linear model between uncertainty and sales growth appears to be misspecified.

3.2 Uncertainty and change: An organizing framework

Our organizing framework relates a firm's subjective uncertainty to the distribution of growth measured by an econometrician. We use it in later sections to guide our detailed discussion of uncertainty and change in both the cross section and the time series. To be precise about timing, it is helpful to measure time in quarters and define date t as the beginning of a quarter when the survey is in the field. We further write g^i_{t+1} for the growth rate of firm i's sales over the quarter between dates t and $t+1$ relative to sales that occurred over the previous quarter between dates $t-1$ and t. In other words, g^i_{t+1} is the growth rate that firm i forms beliefs about when it answers our survey questions at date t. Firm i's information set at that point in time includes its last observed growth rate g^i_t. It may also include other signals that represent news arrived up to t, which we collect in a vector z^i_t. We then use the vector s^i_t to represent all information from past growth rates or other signals that is relevant for forecasting the future dynamics of growth.

We represent firm i's belief about its sales growth by the state space system:

$$\begin{aligned} g^i_{t+1} &= f\left(s^i_t, x^i\right) + \sigma\left(s^i_t, x^i\right) \varepsilon^i_{t+1} && (1) \\ s^i_t &= S\left(s^i_{t-1}, g^i_t, z^i_t; x^i\right), && (2) \end{aligned}$$

where x^i is a vector of firm characteristics, which we think of as fixed in the medium run, and ε^i_{t+1} is an error that has mean zero and variance one under the firms' subjective belief. The observation equation allows firm i's forecast $f(s^i_t, x^i)$ to depend on the state as well as its fixed characteristics. The state is updated every period to incorporate new information in g^i_t and z^i_t according to the function S. When firm i answers our survey questions at t, it provides its forecast $f(s^i_t, x^i)$ as well as best and worst case scenarios. We also observe the subsequent realization g^i_{t+1} and hence the firm's subjective forecast error. We further identify span, the difference between firm i's best and worst case scenarios, with firm i's subjective conditional volatility $\sigma(s^i_t, x^i)$. The model of scenario analysis in Section 2.3 justifies this step for a large class of distributions.

Examples. The state space system (1)-(2) nests many models used to describe firms' subjective uncertainty in economic models. As a simple example, consider the case of *iid* growth together with an orthogonal uncertainty shock:

$$
\begin{aligned}
g_{t+1}^i &= f + \sigma_t^i \varepsilon_{t+1}^i && (3) \\
\sigma_t^i &= S\left(\sigma_{t-1}^i, z_t^i\right). && (4)
\end{aligned}
$$

Here, the only relevant state is stochastic volatility σ_t^i. Rational expectations models with uncertainty shocks often assume that σ_t^i is correlated across firms and high in recessions, which helps generate the observed heightened dispersion of firm growth rates in bad times.

The system (1)-(2) also nests many popular learning rules. Examples include Bayesian models where firms track some latent state such as a regime, or constant gains learning where firms recursively estimate parameters of the one-step-ahead predictive distribution while down-weighting past observations. The common denominator of all these setups is that the state vector contains statistics of the empirical distribution that are relevant for predicting the future dynamics of growth. A natural property in many settings is that high growth g_t^i increases the forecast f and that a large absolute value of the forecast error increases subjective uncertainty σ.

Comparing beliefs and the true data generating process. We would like to distinguish firms' subjective uncertainty from actual volatility, as reflected in the size of innovations measured by an econometrician. We thus consider a change of measure from the firm's belief to the "econometrician's belief," that is, the probability measure that characterizes the true data generating process. We assume that under the econometrician's belief the distribution of growth rates has the alternative representation:

$$
\begin{aligned}
g_{t+1}^i &= f\left(s_t^i, x^i\right) + b\left(s_t^i, x^i\right) + \hat{\sigma}\left(s_t^i, x^i\right)\hat{\varepsilon}_{t+1}^i && (5) \\
s_t^i &= \hat{S}\left(s_{t-1}^i, g_t^i, z_t^i\right), && (6)
\end{aligned}
$$

where again the error has mean zero and variance one, now under the econometrician's belief.

The new observation equation allows for two key differences between firms' belief and the true data generating process. First, firms might have biased forecasts, represented by the function b. Second, the size of the typical innovation $\hat{\sigma}$ might be different from firms' subjective uncertainty captured by σ. Both differences may vary either in the cross section with firms' fixed characteristics x^i or over time with the information set captured by s_t^i. In the special case of rational expectations, there is no bias ($b = 0$) and subjective uncertainty mirrors actual volatility, that is, $\sigma = \hat{\sigma}$.

4 Uncertainty and change in the cross section

In this section, we ask what type of firms perceive more subjective uncertainty *on average*. In other words, we now relate average firm-level subjective uncertainty to measures of change over the medium term in a firm's environment. We compute, for each firm, its average span, that is, the time-series mean of all observations of span for the firm. We then regress average span on a number of firm-level characteristics. In terms of the framework of Section 3.2, we thus characterize the dependence of subjective uncertainty σ on fixed characteristics x^i, assuming that time-averaging removes the effects of information s_t^i. We also compare the cross-sectional properties of average span with those of firms' average absolute forecast errors. From this analysis, our second key result emerges: There is substantial but different heterogeneity in subjective uncertainty and absolute forecast errors across firm types and firm environments.

4.1 Change in firms' environment

We define two variables that measure the medium-term dynamics in a firm's environment, based on its realized sales growth rates (that is, answers to question 1 of our survey module). First, we refer to a firm's sample average sales growth as its *trend*. Second, the *turbulence* experienced by a firm is defined as the sample standard deviation of its sales growth rates. We emphasize that turbulence differs from span for two reasons: First, it is based on realized growth rates. Second, it is an unconditional volatility measure over three years, whereas span measures conditional uncertainty one quarter ahead.

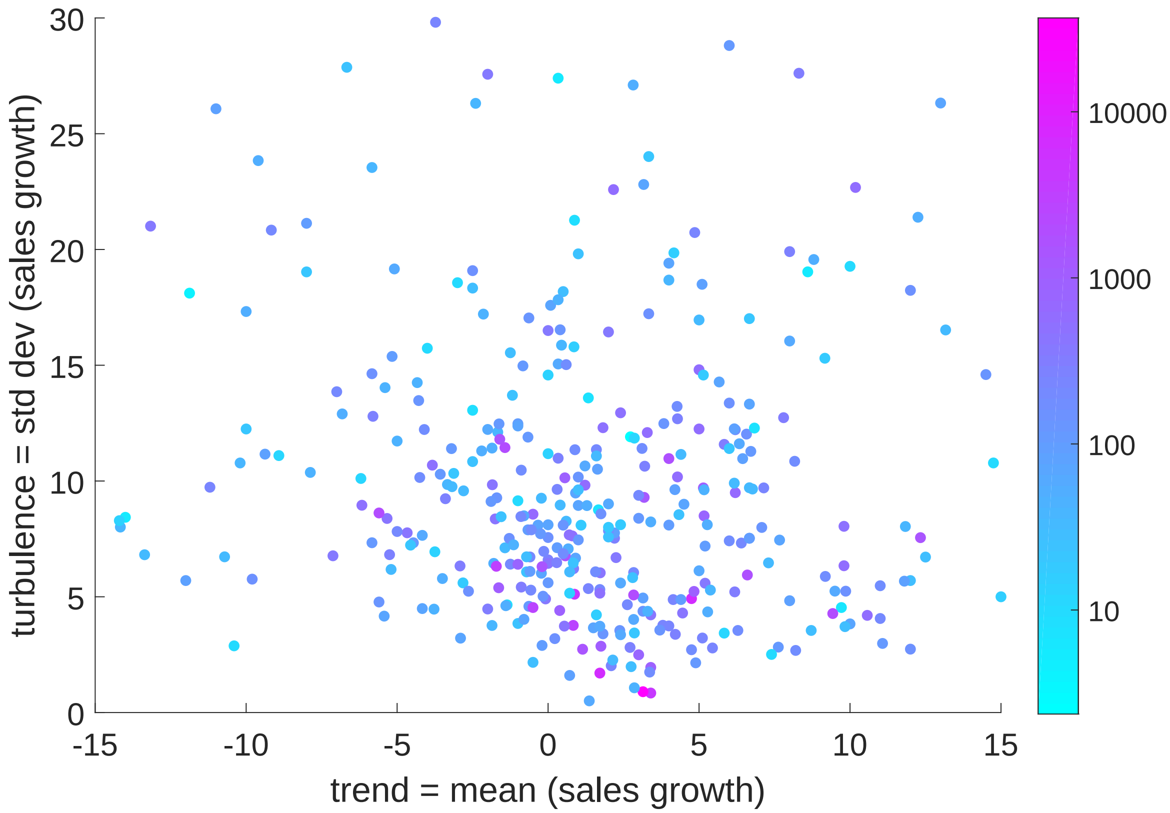
To tractably account for potentially nonlinear effects of these firm characteristics on average span, we code the firm characteristics as dummies. In particular, we use turbulence dummies that indicate quartiles of the distribution of firm-level standard deviations of realized sales growth rates with the lowest quartile as the baseline. We proceed similarly for trend. However, since the middle two quartiles for trend turn out to be very similar, we introduce dummies only for a low trend (bottom 25%) as well as a high trend (top 25%), so that for trend the middle group is the baseline.

Finally, we divide firms into four size categories, with size measured as average employment over our sample. Here we follow the German Statistical Office in their definition of tiny, small, medium-sized, and large firms; lower bounds for the latter three groups are at 10, 50, and 250 employees, respectively. We work with three dummies, with tiny firms as the baseline.

Figure 4 provides a scatter plot of trend and turbulence, respectively. Every dot represents a firm, and the color of the dot indicates firm size, as measured by the number of employees. Size increases from light blue to pink according to the color bar provided on the right-hand side of the figure.

The main takeaway from Figure 4 is that, while trend and turbulence vary substantially, they are not particularly correlated. Firms that grow or shrink along strong trends need not typically experience volatile growth rates and vice versa: The correlation between a firm's average sales growth rate and its standard deviation of those sales growth rates is at a statistically insignificant -0.046. Moreover, the correlation of either environment variable with size is also rather weak. While the very largest firms (identified by bright pink dots) do tend to cluster where turbulence is low (correlation is -0.107), we observe firms of all sizes spread out over the plane. The correlation between size and the average sales growth rates is indeed a statistical zero.

Figure 4: Scatter plot of trend and turbulence with firm size



Notes: Every dot represents a firm identified by its trend (time-series average realized sales growth rate by firm) and turbulence (time-series standard deviation of realized sales growth rates by firm). Color indicates number of employees according to the color bar on the right-hand side.

4.2 Subjective uncertainty, size, trend and turbulence

Table 2 presents regression results on the cross-sectional nexus between uncertainty and firm characteristics. The first three columns ask how much variation in span can be explained by each fixed characteristic—size, trend and turbulence—separately. All three characteristics show a statistically and economically strong association with span. Column (1) says that larger firms perceive less uncertainty. Average span in the entire population of firms is about 12 pp, and it falls monotonically from 18 pp for very small firms (the omitted category) to 10 pp for large firms.

Columns (2) and (3) show that cross-sectional variation in trend and turbulence—each by itself—is sufficient to induce a V-shaped relationship between growth and uncertainty, as observed in the scatter plot in Figure 3. On the one hand, trend and span are directly related by an asymmetric V: Rapidly shrinking or growing firms report higher average spans than firms with normal growth, by 6 and 2 pp, respectively. On the other hand, more turbulent firms also exhibit monotonically higher spans. Since more turbulent firms' growth rate realizations fall more into the tails of realized sales growth rates, this effect also generates a V-pattern.

Table 2: Regressions of time-series averages of subjective uncertainty, of absolute forecast errors, and of forecast errors by firm on firm characteristics

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Dependent variable:	avg. span	avg. span	avg. span	avg. span	avg. span	avg. abs. FE	avg. FE
Dummy small firms	-3.267*			-1.610	-1.936	1.217	4.147
	(1.923)			(1.655)	(1.729)	(2.255)	(2.800)
Dummy medium-sized firms	-6.402***			-3.711**	-4.308***	-0.165	1.557
	(1.794)			(1.550)	(1.623)	(2.067)	(2.508)
Dummy large firms	-8.834***			-5.051***	-5.705***	-0.568	3.036
	(1.827)			(1.603)	(1.716)	(2.056)	(2.486)
Dummy ’bad’ sales growth trend		5.940***		3.233***	3.209***	2.663***	-5.322***
		(0.936)		(0.821)	(0.850)	(0.934)	(1.221)
Dummy ’good’ sales growth trend		2.177***		0.444	0.190	2.489**	5.358***
		(0.800)		(0.730)	(0.739)	(1.126)	(1.364)
Dummy medium low turbulence			2.287***	1.731***	1.751***	2.816***	-0.0552
			(0.623)	(0.613)	(0.664)	(0.515)	(0.733)
Dummy medium high turbulence			6.028***	5.052***	4.985***	5.259***	0.0525
			(0.725)	(0.701)	(0.723)	(0.561)	(0.891)
Dummy high turbulence			11.28***	9.625***	9.216***	13.33***	0.0624
			(0.892)	(0.865)	(0.898)	(1.393)	(1.640)
Constant	18.16***	10.32***	7.456***	10.61***	10.96***	2.734	-2.741
	(1.731)	(0.456)	(0.366)	(1.567)	(1.790)	(2.082)	(2.498)
Industry dummies					YES		
No. of observations	400	400	400	400	400	389	389
No. of firms	400	400	400	400	400	389	389
No. of parameters (excl. intercept)	3	2	3	8	21	8	8
R-squared	0.10	0.11	0.34	0.41	0.43	0.35	0.14

Notes: avg. span denotes the time-series average of firm-level span, avg. abs. FE denotes the time-series average of the firm-level absolute forecast error, and avg. FE denotes the time-series average of the firm-level forecast error. Results from OLS regressions. Industry dummies are defined in Appendix D. Robust standard errors in parentheses; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Each of the three firm characteristics has independent effects on the average subjective uncertainty of firms. This is established in column (4) where we consider all three in the same regression. The positive turbulence gradient is qualitatively and quantitatively unchanged compared to the results in column (3). For trend, the interaction with other characteristics is more subtle. In particular, once size and turbulence are controlled for, growing firms no longer perceive higher uncertainty. At the same time, the negative branch of the V remains large and statistically significant.

While trend and turbulence are correlated with size, controlling for them does not remove an independent role for size in explaining subjective uncertainty. Indeed, comparing columns (1) and (4), the negative size gradient is quantitatively reduced, but remains in place qualitatively. Column (5) further shows that our three firm characteristics are not simply reflective of industry characteristics: Including industry dummies neither changes significantly the R-squared compared to column (4) nor the coefficient estimates.

4.3 Subjective uncertainty, forecast errors and bias in the cross section

How does firms' perceived uncertainty relate to the size of shocks they experience? Column (6) of Table 2 reports a regression of firms' average absolute value of its subjective forecast error, a measure of the size of shocks experienced by the firm. Along all three cross-sectional dimensions we consider, subjective uncertainty is significantly different from the size of shocks experienced by the typical firm. First there is no independent effect of size, once we control for trend and turbulence. It is true that, unconditionally, larger firms experience smaller shocks (see the first column of Table 37 in Appendix G, where we document further cross-sectional results for (absolute) forecast errors and unconditional volatility). However, this relationship is entirely explained by their trend and turbulence. We conclude that the additional effect of size on span is a subjective phenomenon: Large firms' perceive lower uncertainty even if they face the same size of shocks as smaller firms.[17]

A second special feature of subjective uncertainty is its asymmetric dependence on trend. For the same size of shocks, shrinking firms perceive higher subjective uncertainty; growing firms do not. At the same time, both growing and shrinking firms experience larger shocks, and the effect is roughly symmetric. Summarizing the results for both size and trend, we have that successful firms—either growing or large—report lower uncertainty when faced with similar-sized shocks. This fact is consistent with mechanisms that make uncertainty matter more to decision makers in bad times, so their planning considers a wider span of scenarios. In other words, successful firms exhibit less subjective uncertainty.[18]

As discussed in Section 3.2, forecast errors experienced by firms may in part reflect system-

[17]Columns 7 to 10 of Table 37 in Appendix G shows that a similar finding holds for the unconditional sample volatility of realized growth rates: On its own, it is negatively correlated with firm size, but the relation vanishes after controlling for the trend and the turbulence dummies. Importantly, this means that both unconditional and conditional realized volatilities have a different size gradient than subjective (conditional) uncertainty.

[18]While subjective uncertainty and the size of shocks decouple for successful firms, they are not unrelated in the cross section. In particular, more turbulent firms—those with higher sample volatility—also perceive higher uncertainty and generate higher absolute forecast errors, although the gradient is steeper for the size of shocks, which means that highly turbulent firms appear too confident relative to their shock environment.

atic bias in firms' forecasts. Column (7) of Table 2 shows a regression of the mean forecast error on characteristics. For the size and turbulence categories, the coefficients on the dummies are not statistically significant (this is true for the joint regression as presented here, or separately for each of the three firm characteristics, as columns 4 to 6 of Table 37 in Appendix G show). Consistent with this result, group means all lie around zero when firms are sorted into size or turbulence categories (see Appendix F, Tables 15 to 22 and Tables 29 to 36).

At the same time, there is evidence that firms on trends make biased forecasts. In particular, growing firms make large positive forecast errors, defined above as realized growth minus forecast. In other words, growing firms are regularly positively surprised; their forecasts are biased towards zero. Analogously, shrinking firms make large negative forecast errors: Again the forecast is biased towards zero—firms do not sufficiently anticipate the trend they are on.

4.4 Heteroskedasticity and firm characteristics

So far in this section, we have characterized firm-level average subjective uncertainty. We now ask which firms experience greater *time variation* in subjective uncertainty. As a simple measure of firm-level subjective heteroskedasticity, we compute, for each firm, the sample standard deviation of span over all observations for that firm. We then study the cross section of these subjective heteroskedasticity measures by regressing them on the same firm-level characteristics we have used for average span.

Time variation in subjective uncertainty is larger for smaller firms, firms on positive or negative trends, firms in more turbulent environments. These facts are shown in Columns (1)-(3) of Table 3 respectively. Column (4) shows the individual effects displayed in columns (1) to (3) largely survive their joint inclusion in the regression, and that the cross-sectional structure of subjective heteroskedasticity is similar to that of average subjective uncertainty.

Turbulence, thus, generates not only higher, but also *more variable* subjective uncertainty. The same holds for smaller firms and firms on bad or good trends. In other words, planning at firms that live in a more volatile environment, that are smaller, and that are on unusual trends not only uses scenarios that are further apart but also varies those scenarios more over time.

Table 3: Regressions of time-series standard deviations of subjective uncertainty by firm on firm characteristics

Dependent variable:	(1) std. span	(2) std. span	(3) std. span	(4) std. span
Dummy small firms	-3.053**			-2.084*
	(1.485)			(1.254)
Dummy medium-sized firms	-4.252***			-2.655**
	(1.470)			(1.292)
Dummy large firms	-5.094***			-2.798**
	(1.481)			(1.294)
Dummy 'bad' sales growth trend		3.603***		2.164***
		(0.802)		(0.684)
Dummy 'good' sales growth trend		1.672***		0.771*
		(0.475)		(0.450)
Dummy medium low turbulence			1.254***	1.000**
			(0.384)	(0.387)
Dummy medium high turbulence			2.907***	2.369***
			(0.321)	(0.353)
Dummy high turbulence			6.364***	5.463***
			(0.797)	(0.717)
Constant	9.767***	4.543***	3.237***	5.337***
	(1.415)	(0.208)	(0.184)	(1.251)
No. of observations	397	397	397	397
No. of firms	397	397	397	397
No. of parameters (excl. intercept)	3	2	3	8
R-squared	0.052	0.086	0.22	0.27

Notes: std. span denotes the time-series standard deviation of firm-level span. Results from OLS regressions. Standard errors in parentheses, clustered by firm; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

5 Uncertainty and change over time

We have seen in the previous section that the V-shaped relationship between growth and uncertainty in Figure 3 in part reflects fixed differences between firms. We now turn to time-series variation: We ask how much of a V remains once we control for fixed characteristics. In terms of the organizing framework of Section 3.2, we ask whether variation of span σ with firms' information s_t^i also contributes to the V-shape, via the correlation of s_t^i with past growth. From this analysis follows our third key finding that the V-shaped nexus between subjective uncertainty and past sales growth is the result of both time-series and cross-sectional forces.

Formally, all our regression specifications take the basic form:

$$span_t^i = \beta^- g_t^{i,-} + \beta^+ g_t^{i,+} + \gamma' x^i + \epsilon_t^i, \tag{7}$$

where $g_t^{i,-} = g_t^i \ I(g_t^i{<}0)$, $g_t^{i,+} = g_t^i \ I(g_t^i \geq 0)$, $I(\cdot)$ is the indicator function, and x^i is a vector of fixed firm characteristics that do not depend on time.

We include the three characteristics studied in the previous section: trend, turbulence, and size. Trend and turbulence are again coded as time-invariant dummies. As the unit of observation is now a firm-quarter pair, we measure the size of the firm as the number of employees at the end of the previous calendar year. We then form three size dummies: Small firms have 10-50 employees, medium-sized firms 51-250 employees and large firms more than 250 employees. The baseline "tiny" firm has fewer than 10 employees.[19]

5.1 Time variation in subjective uncertainty and growth

Table 4 reports the regression results. As a benchmark, we start in columns (1) and (2) with a simple linear regression and a piecewise linear regression with a break at zero, respectively. The two columns provide formal counterparts to the scatter plot in Figure 3. The next four columns augment the piecewise linear specification with dummies for fixed characteristics, first adding size, trend and turbulence separately, and then in column (6) adding all characteristics together.

The main result from Table 4 is that a strongly significant asymmetric V remains even if we control for fixed characteristics. Indeed, the coefficients on both negative and positive past sales growth are statistically significant and quantitatively relevant in all specifications. Column (6) says that, holding fixed all characteristics, after a one percentage point lower negative sales growth rate, next quarter's span is 31 basis points wider. Similarly, a one percentage point higher positive sales growth rate is followed by a 18 basis points wider span. This translates into a 4.6 (2.6) pp increase in span for a one-standard-deviation decrease (increase) in previous-quarter sales growth rates. Responses to past growth thus account for a considerable part of time variation in subjective uncertainty.

[19] While size, therefore, does vary over time, change is so slow that the size dummies are essentially time-invariant. We observe only 56 jumps from one size category to another in our sample.

Table 4: Regressions of subjective uncertainty on past sales growth and firm characteristics

Dependent variable: span between best and worst case sales growth rate for quarter t	(1) POLS	(2) POLS	(3) POLS	(4) POLS	(5) POLS	(6) POLS	(7) FE	(8) POLS
Sales growth rate in quarter $t-1$	-0.0598**							
	(0.0259)							
Negative sales growth rate in quarter $t-1$		-0.498***	-0.470***	-0.436***	-0.351***	-0.306***	-0.272***	-0.304***
		(0.0612)	(0.0602)	(0.0626)	(0.0661)	(0.0675)	(0.0786)	(0.0686)
Positive sales growth rate in quarter $t-1$		0.279***	0.266***	0.280***	0.166***	0.180***	0.159***	0.173***
		(0.0329)	(0.0317)	(0.0327)	(0.0335)	(0.0314)	(0.0319)	(0.0303)
Dummy small firms			-4.480*			-3.959*		-3.508*
			(2.560)			(2.178)		(1.897)
Dummy medium-sized firms			-6.677***			-5.452**		-5.157***
			(2.516)			(2.141)		(1.972)
Dummy large firms			-7.858***			-6.295***		-5.970***
			(2.570)			(2.170)		(2.014)
Dummy 'bad' sales growth trend				3.711***		2.248***		2.300***
				(0.951)		(0.856)		(0.858)
Dummy 'good' sales growth trend				0.410		-0.434		-0.618
				(0.658)		(0.645)		(0.667)
Dummy medium low turbulence					1.699***	1.388**		1.340**
					(0.578)	(0.591)		(0.649)
Dummy medium high turbulence					5.157***	4.560***		4.572***
					(0.752)	(0.764)		(0.772)
Dummy high turbulence					7.702***	6.748***		6.525***
					(1.035)	(0.969)		(0.979)
Intercept	12.22***	8.428***	14.76***	7.695***	6.206***	11.37***	10.06***	9.774***
	(0.392)	(0.435)	(2.532)	(0.435)	(0.425)	(2.154)	(0.480)	(3.014)
Time-industry dummies								YES
No. of observations	2,762	2,762	2,762	2,762	2,762	2,762	2,762	2,762
No. of firms	400	400	400	400	400	400	400	400
No. of parameters (excl. intercept)	1	2	5	4	5	10	401	199
R-squared	0.0079	0.19	0.22	0.21	0.26	0.29	0.57	0.34

Notes: The regressions are based on the sample of firms with at least five answers to question 1. In addition, we require an answer to both questions 2.a for the firm's span, leading us to the *baseline sample span* with 2,762 observations (see Table 11 in Appendix C). Standard errors in parentheses, clustered by firm. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. "POLS" stands for "pooled ordinary least squares regression"; "FE" stands for a fixed-effect regression. See Appendix D for the definition of industries.

The impact of firm characteristics is also significant. First, introducing firm characteristics dummies improves the fit of the regression: For example, the R-squared improves from 0.19 in column (2) to 0.29 in column (6). Coefficients on the dummies reproduce the cross-sectional effects discussed in the previous section. For example, firms with more than 250 employees are more than 6 pp less uncertain on average than tiny firms. Firms that experience more than median turbulence are at least 4.5 pp more uncertain than those with low turbulence. The impact of trend is asymmetric: Firms on a bad trend are more than 2 pp more uncertain than those on a normal trend, whereas a good trend has no significant effect on span.

It is natural to conjecture that fixed characteristics other than size, trend and turbulence matter for subjective uncertainty. We thus re-estimate the regression in column (7) with firm fixed effects. As expected, we find a large increase in R^2. Remarkably, however, there is virtually no change in the coefficients on past growth. We can thus conclude that size, trend and turbulence dummies exhaustively control for the impact of firm characteristics on the uncertainty-growth relationship.

In column (8), we include time-industry dummies. This neither alters our coefficient estimates nor markedly improves the fit of the regression, which is consistent with variation in subjective uncertainty being largely firm-specific. We conclude that our results are not driven by industry-composition effects, industry-specific or aggregate trends and cycles.

We finally note that a comparison between column (1) and column (2) shows that the data clearly prefer a piecewise linear specification, a V-shape, to model the uncertainty-growth nexus. A linear specification as in column (1), the traditional focus of the literature, finds the usual negative correlation between growth and uncertainty but the fit of this regression is small compared to the V-shaped regression in column (2).

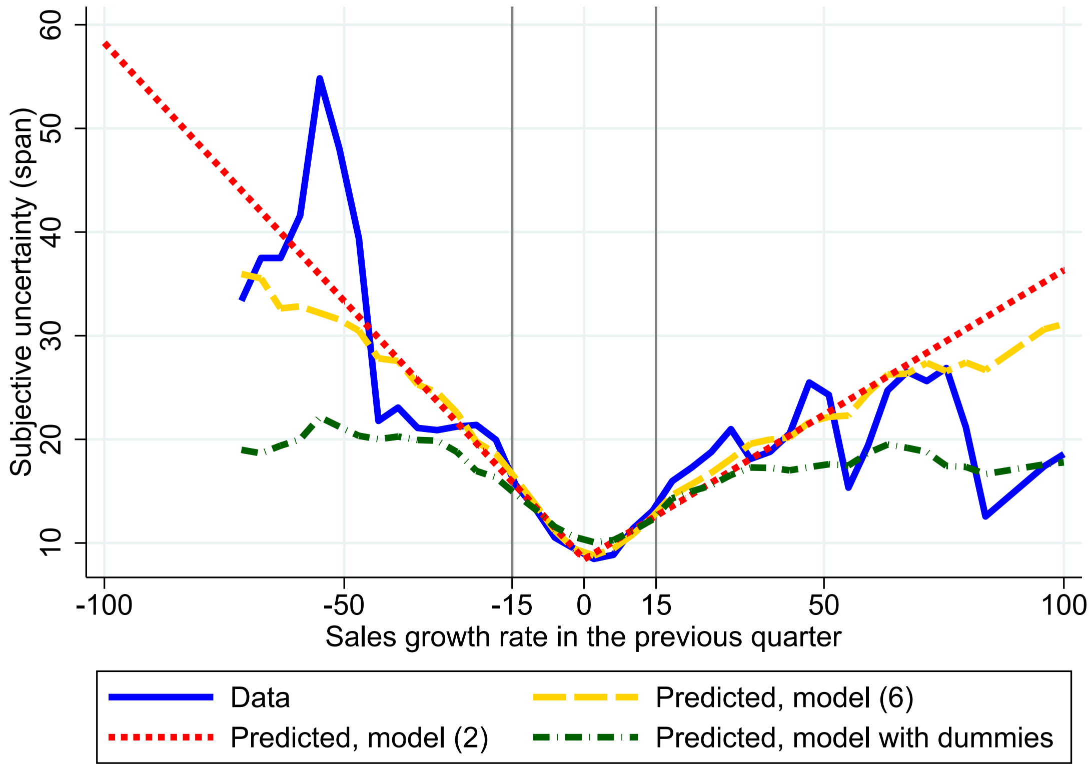
5.2 Comparing cross-sectional and time-series variation in subjective uncertainty

The coefficients on positive and negative growth in column (6) in Table 4 effectively isolate large and asymmetric *time-series* responses of firms to past growth. Interestingly, the asymmetric V-shaped response induced by time-series responses is quite similar to that induced purely by cross-sectional heterogeneity. To see this, consider Figure 5, where we show the nonparametric regression line from Figure 3 along with several regression lines motivated by the findings in Table 4, in order to compare the two forces.

As a benchmark, the solid line is the nonparametric regression line fitted to the data, and the dotted line is the fitted line from the regression model in column (2) of Table 4, both already shown in Figure 3. The dashed and the dash-dotted nonparametric regression lines are fitted not to the data, but to clouds of predicted values from two parametric regressions.[20] Specifically, the dashed line is fitted to the predicted values from the regression model in column (6) of Table 4. The dash-dotted line is fitted to the predicted values from a model with

[20]These predicted values form clouds rather than (piecewise) straight lines because growth is not the only regressor; there are also the fixed firm characteristics. For example, for two firms that experienced the same sales growth rate in the previous quarter, the model in column (6) of Table 4 will predict different spans depending on the firms' size, trend, and turbulence.

Figure 5: Cross-sectional and time-series relationships between uncertainty and past sales growth



Notes: Besides a linear fitted line with break at zero that corresponds to column (2) of Table 4 (dotted red line), the chart presents three nonparametric regression lines. Respectively, the nonparametric regression lines are based on the full sample (solid blue line), the cloud of predicted values of column (6) of Table 4 (dashed yellow line), and the cloud of predicted values from a model with size, trend, and turbulence dummies as regressors (dash-dotted green line). The nonparametric regressions are the predictions from kernel-weighted local polynomial regressions of degree zero with an Epanechnikov kernel where the bandwidth was selected based on the rule of thumb suggested by Fan and Gijbels (1996). The thin vertical lines mark the interdecile range that extends from −15% to 15%, see Table 13 in Appendix F.

only the three classes of dummies, thus reflecting only cross-sectional variation. We use the nonparametric regressions simply as a convenient device to make the predictive essence of the various components of the regression in column (6) of Table 4 visible.

The main takeaway from Figure 5 is that all lines lie effectively on top of each other, especially within the interdecile range. In other words, time-series and cross-sectional variation induce the same V-shape, albeit through very different mechanisms. For the time-series response, the V follows directly from the difference in coefficients on positive and negative growth. For the cross section, the effect is more subtle and comes from the comovement of span with turbulence and trend growth, documented in Table 2: Firms with higher span also see higher absolute values of their growth rates (due to differences in trend or turbulence).[21]

[21] In Appendix H, we show that the V-shaped relationship between sales growth and subjective uncertainty holds separately, and in a quantitatively similar manner, for all firm-level subgroups: the four firm size groups, the four turbulence groups, and the three growth trend groups.

5.3 Subjective uncertainty and volatility in the time series

Section 4.4 showed that subjective uncertainty and conditional volatility vary differently in the cross section of firms. How do subjective and conditional volatility compare in the within-firm time-series dimension? Table 5 compares our baseline regression of span on past growth and fixed characteristics—column (1) here reproduces column (6) of Table 4—to an analogous regression for the absolute value of the firm's forecast error, shown in column (2).

Controlling for fixed characteristics, a firm that observes one percent worse negative growth in the previous quarter not only increases its span by 31 basis points, but also experiences, on average, a forecast error that is 34 basis points higher in absolute value. By contrast, one percent higher positive growth increases span by 18 basis points and the absolute value of the forecast error by 12 basis points. The asymmetric V that emerges in the time series of firms' uncertainty is thus also present in firms' experience of shocks. Columns (3) and (4) of Table 5 show that this asymmetric V persists for both subjective uncertainty and the shock size, even controlling for firms' sales growth expectations, that is, a forward-looking first moment.

The differences between subjective uncertainty and conditional volatility observed in the cross section appear to be largely orthogonal to the time-series nexuses of uncertainty and past growth, which are similar. Indeed, the regression coefficients in columns (1) and (2) of Table 5 display the same patterns as columns (4) and (6) of Table 2: Large firms and very turbulent firms are less uncertain than one might expect given the size of the shocks they face, and the asymmetric relationship between trend growth and uncertainty is a subjective phenomenon.

Finally, we show that subjective uncertainty and conditional volatility in fact move together as the regressors change: They jointly increase after high or low growth in the time series. This result does not follow yet from the regressions in columns (1)-(4) of Table 5 alone. The latter only relate span and the absolute forecast error separately to the regressors. It is then possible, for example, that sales growth consists of two orthogonal components that each co-move with only one of the uncertainty measures. A firm might thus experience some episodes with large negative growth and high span, and other episodes with large negative growth followed by high absolute forecast errors, without any connection between the two.

To rule out this case, columns (5) and (6) of Table 5 show versions of columns (1) and (2) for subjective uncertainty and conditional volatility (absolute forecast errors), respectively, but with the (contemporaneous) other uncertainty measure added to the regression. If in fact span were correlated with one component of sales growth and the absolute forecast error with another, then the absolute forecast error should be conditionally correlated with span controlling for sales growth (as well as our other regressors). In effect, including the absolute forecast error purifies sales growth into the component that is related to span only. We find, however, that the estimated coefficients on all regressors in columns (5) and (6) are essentially the same as in columns (1) and (2), respectively, while the other uncertainty measure does not matter significantly. We conclude that our results reflect comovement of subjective uncertainty, conditional volatility and the regressors, as opposed to the composition of separate forces as in the example above.

Table 5: Regressions of subjective uncertainty and the absolute forecast error on past sales growth, firm characteristics, and additional controls

	(1)	(2)	(3)	(4)	(5)	(6)
Dependent variable:	span	firms' abs(FE)	span	firms' abs(FE)	span	firms' abs(FE)
Negative sales growth rate in quarter $t-1$	-0.306***	-0.337***	-0.283***	-0.345***	-0.364***	-0.298***
	(0.0675)	(0.0689)	(0.0691)	(0.0679)	(0.0772)	(0.0775)
Positive sales growth rate in quarter $t-1$	0.180***	0.123**	0.172***	0.126**	0.221***	0.105**
	(0.0314)	(0.0529)	(0.0318)	(0.0536)	(0.0351)	(0.0494)
Dummy small firms	-3.959*	-0.632	-4.251*	-0.493	-3.418	-1.142
	(2.178)	(1.545)	(2.175)	(1.544)	(2.310)	(1.496)
Dummy medium-sized firms	-5.452**	-1.672	-5.745***	-1.553	-5.604**	-1.516
	(2.141)	(1.460)	(2.149)	(1.465)	(2.260)	(1.422)
Dummy large firms	-6.295***	-1.810	-6.571***	-1.691	-6.170***	-1.598
	(2.170)	(1.598)	(2.174)	(1.581)	(2.332)	(1.647)
Dummy 'bad' sales growth trend	2.248***	1.340*	2.380***	1.218	1.528	1.297*
	(0.856)	(0.811)	(0.858)	(0.865)	(1.004)	(0.785)
Dummy 'good' sales growth trend	-0.434	1.417*	-0.560	1.558*	-1.044	1.171
	(0.645)	(0.826)	(0.643)	(0.864)	(0.710)	(0.831)
Dummy medium low turbulence	1.388**	1.802***	1.514**	1.770***	0.706	1.797***
	(0.591)	(0.356)	(0.596)	(0.354)	(0.643)	(0.355)
Dummy medium high turbulence	4.560***	4.160***	4.868***	4.151***	4.279***	3.706***
	(0.764)	(0.472)	(0.770)	(0.473)	(0.971)	(0.557)
Dummy high turbulence	6.748***	9.161***	6.899***	9.153***	5.655***	8.184***
	(0.969)	(0.953)	(0.978)	(0.954)	(1.261)	(0.959)
Forecast sales growth rate for quarter t			0.0510	-0.0357		
			(0.0321)	(0.0458)		
Absolute forecast error in quarter t					0.0788	
					(0.0499)	
span						0.102
						(0.0667)
Constant	11.37***	3.782**	11.56***	3.702**	11.06***	3.135*
	(2.154)	(1.484)	(2.173)	(1.478)	(2.377)	(1.616)
No. of observations	2,762	1,664	2,710	1,664	1,621	1,621
No. of firms	400	389	399	389	381	381
No. of parameters (excl. intercept)	10	10	11	11	11	11
R-squared	0.29	0.25	0.29	0.25	0.32	0.25

Notes: Span is our measure of subjective uncertainty and firms' abs(FE) denotes firms' absolute forecast error. All equations are estimated by pooled OLS. Standard errors in parentheses, clustered by firm; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. The forecast sales growth rate for quarter t is the answer to question 2.b of the survey (see Section 2.2). The regressions in columns (1) and (3) are based on the *baseline sample span* with 2,762 observations; for column (3) we additionally need the aforementioned contemporaneous forecast. Columns (2) and (4) are based on the *baseline sample forecast* with forecast errors, leading to 1,664 observations (see Table 11 in Appendix C). Finally, columns (5) and (6) start from the same *baseline sample forecast* with forecast errors, and, in addition, we require a contemporaneous span observation, leading to 1,621 observations.

6 The dynamics of subjective uncertainty

In this section, we further explore the dynamics of subjective uncertainty. Our approach is motivated by two properties of many common learning rules. First, changes in uncertainty tend to propagate over time. Second, we would expect higher absolute forecast errors to increase uncertainty. In principle, this property alone could induce a V-shaped relationship between uncertainty and growth, because large absolute growth rates tend to go hand in hand with large absolute forecast errors, which in turn raises uncertainty in the subsequent quarter. Our fourth key finding, however, is that predictable change in past growth dominates past growth surprises in explaining the dynamics of subjective uncertainty.

Table 6 compares the effects of predictable and unpredictable variation in past sales growth on subjective uncertainty. For comparison, column (1) replicates our baseline result from column (6) of Table 4 on the somewhat smaller sample for which we observe firm forecast errors—this is the sample used throughout this section. The V-shape of subjective uncertainty in previous-quarter sales growth is again present. In column (2), we replace sales growth with previous-quarter forecast errors in sales growth: We find again a V-shape with somewhat smaller coefficients. This result is consistent with learning rules that increase uncertainty when a surprise occurs. We note that the R-squared of this regression is slightly lower than

Table 6: Regressions of subjective uncertainty on past sales growth and past forecast errors, with dynamic models

Dependent variable: span for quarter t	(1)	(2)	(3)	(4)	(5)	(6)
Subjective uncertainty in quarter $t-1$				0.273***	0.271***	0.276***
				(0.0706)	(0.0748)	(0.0695)
Negative sales growth rate in quarter $t-1$	-0.358***		-0.314***	-0.322***		-0.333***
	(0.0755)		(0.0825)	(0.0754)		(0.0910)
Positive sales growth rate in quarter $t-1$	0.148***		0.0990**	0.138***		0.0936*
	(0.0394)		(0.0450)	(0.0381)		(0.0478)
Negative forecast error in quarter $t-1$		-0.232***	-0.0599		-0.166***	0.0167
		(0.0573)	(0.0567)		(0.0586)	(0.0607)
Positive forecast error in quarter $t-1$		0.130***	0.0747		0.116***	0.0641
		(0.0392)	(0.0488)		(0.0363)	(0.0498)
Size, trend, and turbulence dummies	YES	YES	YES	YES	YES	YES
Constant	11.70***	11.61***	11.48***	8.587***	8.719***	8.465***
	(2.562)	(2.610)	(2.544)	(2.055)	(2.039)	(2.008)
No. of observations	1,520	1,520	1,520	1,489	1,489	1,489
No. of firms	373	373	373	367	367	367
No. of parameters (excl. intercept)	10	10	12	11	11	13
R-squared	0.35	0.32	0.35	0.40	0.38	0.41

Notes: Results from pooled OLS regressions. They are based on the sample of firms with at least five answers to question 1. In addition, we require an answer to both questions 2.a for the firm's span, leading us to the *baseline sample span* with 2,762 observations. We further need the lag of the forecast error for columns (2) and (3), leading to 1,520 observations. For reasons of comparability we estimate the regression in column (1) on that same sample. In columns (4) to (6), we additionally require the lag of span leading to 1,489 observations. See also Table 11 in Appendix C. Standard errors in parentheses, clustered by firm. * p < 0.10, ** p < 0.05, *** p < 0.01.

that in column (1): Past sales growth has a marginally higher explanatory power than past growth surprises.

Column (3) contains the main result of this section: Predictable change in past growth drives out surprises. The specification shown in Column (3) includes both sales growth and the forecast error, allowing for asymmetry for both variables. Clearly, the asymmetric V in sales growth wins the horse race between change and unanticipated change, coefficients on forecasts errors lose statistical significance and no longer reflect an asymmetric V. At the same time, the R-squared in column (3) does not improve relative to column (1), that is, making a distinction between predictable and unpredictable change does not add explanatory power.

Columns (4) to (6) repeat the same steps, but this time with lagged span included in the regression, in order to quantify propagation of subjective uncertainty. We first note that subjective uncertainty displays a mild persistence because it depends on its own lag in all three specifications. With respect to the relevance of sales growth versus forecast error, the result is the same: In a horse race between these two regressors to determine subjective uncertainty, it is sales growth that enters with an asymmetric V, whereas the data do not ask for the forecast error over and above sales growth.

Why does past growth "drive out" the past forecast error in these regressions? It is helpful to think about managers for whom past growth rates and past forecast errors were very different. They can be in one of two situations. One is that the forecast error was small in absolute value, while growth was far away from its mean—that is, a predictable unusual growth event. The other is that the forecast error was large (again in absolute value), even though growth was not unusual: The manager expected an unusual event to occur, but that event did not actually materialize. In other words, the manager's last forecast was an "unforced error". The driving out result could thus be due to either (i) managers reporting higher uncertainty after a predictable unusual growth event, or (ii) managers not reporting higher uncertainty after an unforced error.

We now show that both possibilities (i) and (ii) contribute to the result that growth drives out forecast errors. We compare group averages of span in a two-by-two table of high versus low absolute growth rates and high versus low absolute forecast errors, relative to the respective means. To control for firm characteristics, we first partial out the size, trend and turbulence dummies from span, absolute growth rates and absolute forecast errors leaving the conditional linear relationship between the latter three variables unchanged.[22] Since our partialling-out regressions include an intercept, the adjusted variables have mean zero.

Table 7 shows how span differs across four groups of firms: We split the adjusted absolute growth rates and the adjusted absolute forecast errors into observations above and below zero (their mean), thereby defining the four quadrants shown in the upper left panel. In particular, managers in situations (i), predictable unusual growth event, and (ii), unforced error, are located in the lower left and upper right cells, respectively. We then compute the average span

[22]Technically, we invoke the Frisch-Waugh theorem which says that there are two equivalent ways to control for some variables z (here: the dummies) in an OLS regression of y (here: span) on x (here: past sales growth and past forecast error). Either regress y on x and z and take the coefficient of x. Alternatively, first regress y on z and x on z and then regress the residuals of these two regressions on each other.

Table 7: Sales growth versus forecast errors as predictors of subjective uncertainty (two-by-two tables)

	Full sample		*Only neg. growth and neg. FE*	
	low abs. FE	high abs. FE	low abs. FE	high abs. FE
After partialling out size, trend, and turbulence dummies				
low abs. growth	-1.82^{***} [obs: 665]	-0.19 [obs: 188])	-2.04^{***} [obs: 127]	-0.02 [obs: 68]
high abs. growth	0.64 [obs: 257]	2.63^{***} [obs: 410]	2.01^{*} [obs: 62]	3.29^{***} [obs: 170]
After partialling out size, trend, and turbulence dummies, and lagged span				
low abs. growth	-1.38^{***} [obs: 637]	-0.82 [obs: 188])	-1.71^{***} [obs: 125]	-0.20 [obs: 68]
high abs. growth	0.52 [obs: 242]	2.14^{***} [obs: 422]	1.51 [obs: 61]	3.01^{***} [obs: 166]

Notes: The cells show group-specific means of adjusted span and, in brackets below, the number of observations per cell. The adjustment in the two upper panels is based on a regression of span on size, trend, and turbulence dummies (1,520 observations). The groups in the four cells are, respectively, defined by the mean values of the residuals of the absolute sales growth and the absolute forecast error regressions on the same variables as span. For the left panels, we use the whole sample, for the right panels a subsample with only negative sales growth rates and negative sales growth forecast errors. The lower panels mirror the upper panels, but, additionally, control for lagged span in the adjustment regressions (1,489 observations). The upper/lower panels refer, respectively, to the samples used in columns (1) to (3) and (4) to (6) of Table 6. * p < 0.10, ** p < 0.05, *** p < 0.01.

for each quadrant. For example, the upper left value of -1.82 means that a firm which, after controlling for firm characteristics, experiences a below-average absolute growth rate and a below-average absolute forecast error, reports a 1.82 pp smaller span than the average firm. In addition, we report the number of observations that fall in each quadrant (in square brackets below average span).

The first takeaway from the upper left panel of Table 7 is that uncertainty, as measured by span, is relatively low (high) if both absolute growth and absolute forecast errors are relatively low (high). In other words, a firm is relatively certain if it experiences a small absolute growth rate near its expectations and it is relatively uncertain if it experiences a large absolute growth rate far away from what it expected. Most observations (665+410=1075 of 1520 and thus 71%) fall in these two cells reflecting that sales growth is difficult to predict, so that low (high) absolute forecast errors and low (high) absolute growth go often hand in hand.

We are particularly interested in the upper right and lower left cells. In the upper right cell, a small absolute growth rate that comes as a large surprise does not alter span: Managers who incorrectly expected something "big" to happen do not experience higher uncertainty even though the size of their forecast error is large. They tend not to update their subjective uncertainty after "unforced errors", because the signal they receive tells them that they are in calm territory. By contrast, the lower left cell tells us that a large absolute growth rate increases uncertainty to a noticeable, if not statistically significant amount even if it comes more or less expectedly. Altogether, it thus appears to be the signal conveyed by the absolute growth rate which shapes uncertainty and not the expectational error.

Since responses of span to growth are generally asymmetric, we next investigate to what extent the span response to predictable growth changes and unforced errors is different when the manager's experience is good or bad. In particular, we ask whether a manager who has gone through a bad experience will be more uncertain compared to after a good experience. The upper right panel of Table 7 reports average span using only those observations that exhibit both negative growth and negative forecast errors. By comparing the upper panels, we can thus assess the asymmetry of the relationships between span and its drivers.

The result is that firms that experience a predictable bad growth spell are particularly important. Coefficient differences between the upper left and upper right panels are moderate in all but one cell: In the lower left, average span is much larger than in the full sample, that is, an (almost expected) large negative growth rate leads to a strong increase in span by 2 pp. In other words, firms in a gloomy situation—expecting an unusually bad outcome and experiencing an even slightly worse realization—drive the lower left cell even for the full sample.[23] This means, that it is firms in a gloom situation that make sales growth dominate sales forecast errors in the determination of firms' subjective uncertainty. The lower panels of Table 7 show that these results are robust to including lagged span in the set of controls.

To summarize, we find that subjective forecast errors are driven by past sales growth rather than past forecast errors. While these two regressors are correlated, suggesting that sales forecasts, as a rule, are difficult, there is a sizeable number of observations exhibiting small (large) absolute sales growth combined with large (small) absolute forecast errors. In these cases, sales growth is the better predictor of span, particularly for firms in a gloomy situation.

What is the economic mechanism that might make firms more uncertain especially after a negative previous-quarter sales growth rather than a negative sales growth surprise? A possible interpretation is that large negative sales growth could indicate a large loss in the customer base of a firm. Whether this loss was predicted or not, in an environment where building up customer relationships is costly and the success of it uncertain, affected firms do not know whether and which new customers can be found in the months going forward, making them more uncertain with respect to future sales growth.

7 Dynamics: Subjective uncertainty vs conditional volatility

In this final section, we now compare the dynamics of subjective uncertainty perceived by firms with the dynamics of conditional volatility experienced by firms. We have already seen in Section 5.3 that the projection of the absolute size of forecast errors on past growth—controlling for fixed firm characteristics—yields coefficients that are quite similar to the coefficients of our baseline span regression. We now ask to what extent firms' updating of subjective uncertainty studied in Section 6 resembles the dynamic behavior of the conditional volatility of unpredictable shocks. In terms of the framework of Section 3.2, we now ask how similar the *dynamics* of σ and $\hat{\sigma}$ are. Our final key finding is that the dynamics of subjective uncertainty and "objective" conditional volatility are very similar. They include mild but statistically sig-

[23]In fact, the average adjusted span in the lower left cell of the complementary group of firms that do not exhibit negative growth and negative forecast error is 0.2, thus almost indistinguishable to the average of zero.

nificant persistence, irrelevance of lagged forecast errors, and predictive importance of lagged absolute sales growth, especially if it is negative. Managers understand the changing uncertainty environment their firms operate in and update their subjective uncertainty accordingly.

To arrive at this result, we take two preliminary steps, the details of which are documented in Appendix I. As a first preliminary step, we distinguish between the two sources of firm forecast errors discussed in Section 3.2: bias and conditional volatility. We therefore "clean" subjective forecast errors by removing the firm-specific forecast bias. To do so, we estimate a LASSO regression to select among the many possible predictors for this bias—we consider our three firm characteristics and their interactions—and then subtract the estimated bias from firms' subjective forecast errors. This approach based on expectational survey data captures the information of actual decision makers irrespective of the quality of their forecasts. In addition, we construct an alternative measure of unbiased forecast errors that an econometrician would compute using a statistical model with only firm-level sales growth data and no expectational data available. This approach based on outcome data only is a conventional econometric forecasting exercise although it may fail to fully capture the information used by decision makers. We will show below that both approaches lead to very similar results, suggesting that the details of the information structure are not key to our results.

As a second preliminary step, we then estimate dynamic models of conditional volatility for both the cleaned subjective forecast errors and for the statistical forecast errors to provide a counterpart to the previous regressions of span on lagged span, past growth, past forecast errors as well as fixed firm characteristics. We choose the conditional standard deviation of forecast errors as our measure of "objective" uncertainty and model it in a power GARCH framework. We select and estimate a power GARCH specification that optimally describes the data as indicated by information criteria. Our choice of explanatory variables turns out to mirror our analysis of subjective uncertainty in Section 6: We include past sales growth rates, past forecast errors, and fixed firm characteristics in the power GARCH equation.

We start our comparison by reporting descriptive summary statistics for all three measures, span and the two measures of conditional volatility, in Table 8. A first result is that the distributions of the predicted conditional standard deviations of the firms' subjective and the statistical forecast errors are remarkably similar. In fact, the sample correlation of the two measures is 0.97. Moreover, the distribution of subjective uncertainty as measured by span is also similar to the two distributions for conditional volatility, with a slightly higher mean and dispersion. The sample correlations between subjective uncertainty and the conditional volatility based on firms' subjective forecast errors is 0.53, and with the one based on statistical forecast errors it is 0.51.

In a final step, we compare the dynamics of subjective uncertainty to the dynamics of conditional volatility. Since the volatility models link the conditional standard deviation to past sales growth and the dummies via an exponential function that ensures nonnegativity, we base our comparison on average partial effects. We report them in columns (2) and (3) of Table 9, while column (1) replicates the coefficient estimates of the dynamic linear model for span reported in column (6) of Table 6 with the only difference that we replace the two insignificant regressors "positive past forecast error" and "negative past forecast error" by the

Table 8: Summary statistics for measures of subjective uncertainty and predicted conditional volatility

Variable	#obs	Mean	Std. Dev.	P10	P25	P50	P75	P90
Span between worst and best case forecast	932	12.1	9.8	4	5	10	15	25
Predicted conditional volatility of firms' subjective forecast errors	932	9.7	7.2	3.5	4.8	7.5	12.3	18.3
Predicted conditional volatility of statistical forecast errors	932	9.7	7.3	3.4	4.2	7.2	12.2	19.8

Notes: The number of usable observations shrinks from 949 to 932 here because 17 quarter-firm observations we used to construct forecast errors have either a missing upper or lower interval bound, or both, in the data, and thus we cannot compute a span for these observations (see Appendix C). The models used to calculate the predicted conditional standard deviations (volatilities) of subjective and statistical forecast errors, can be found in columns (1) and (3), respectively, of Table 40 in Appendix I. P10 to P90 denotes the corresponding percentiles of the distribution.

single regressor "absolute forecast error" to conform with the symmetric specification of the volatility models.[24]

The results indicate that the dynamics of subjective uncertainty and "objective" conditional volatility are remarkably similar. There is mild but statistically significant persistence; lagged forecast errors are largely irrelevant; and lagged absolute sales growth has an asymmetric effect with large negative realizations being roughly twice as important than large positive realizations. Hence, when forming uncertainty beliefs, firms appear to have a realistic impression of the uncertainty dynamics that characterize the underlying data.

By contrast, subjective uncertainty and conditional volatility differ in several cross-sectional dimensions, similar to our findings in Sections 4 and 5. We find again that successful firms—defined as large or fast-growing—plan with narrower spans than their conditional volatility environment relative to those of less successful firms would suggest. We also see again that highly turbulent firms appear too confident relative to their conditional volatility environment.

To summarize, a typical firm's updating of subjective uncertainty over time closely resembles the dynamics of conditional volatility, rendering managers' grasp on the (short-term) dynamics of their uncertainty environment remarkably good. By contrast, in the cross section, successful firms and firms operating in a highly turbulent environment underestimate their uncertainty.

[24]Our results are robust to two modelling decisions, one with respect to the sample choice, another one with respect to the symmetric specification regarding the absolute forecast error in the power GARCH equation. As to the first decision, the regression for span, as elsewhere in the paper, is based on the *baseline sample span*, whereas the power GARCH estimations are based on the *baseline sample forecast* as explained in Table 11 in Appendix C. Had we estimated the regression in column (1) on the appropriate restriction of the *baseline sample forecast*, the results would be essentially the same. As to the second decision about symmetry, we note that our information criteria do not provide us with clear guidance but the asymmetry parameter turns out to be statistically insignificant and the other estimated power GARCH coefficients are nearly identical and thus independent of this symmetry/asymmetry choice (see Table 40 in Appendix I).

Table 9: Comparison of subjective uncertainty and predicted conditional volatility

Dependent variable:	(1) Subjective uncertainty	(2) Conditional volatility of firms' subjective forecast errors	(3) Conditional volatility of statistical forecast errors
Uncertainty/volatility in $t-1$	0.270***	0.235***	0.236***
	(0.0699)	(0.0874)	(0.0921)
Absolute forecast error in $t-1$	0.0373	0.085*	0.008
	(0.0383)	(0.051)	(0.068)
Negative sales growth in $t-1$	-0.285***	-0.223***	-0.220***
	(0.0768)	(0.073)	(0.064)
Positive sales growth in $t-1$	0.112***	0.102**	0.060
	(0.0422)	(0.043)	(0.044)
Dummy small firms	-3.507*	-1.227	-1.316
	(2.066)	(1.068)	(0.811)
Dummy medium-sized firms	-3.920*	-1.891	-1.656**
	(2.056)	(0.947)	(0.709)
Dummy large firms	-4.562**	-2.073**	-1.550*
	(2.022)	(1.006)	(0.790)
Dummy 'bad' sales growth trend	2.219***	1.498**	-0.234
	(0.780)	(0.706)	(0.540)
Dummy 'good' sales growth trend	-0.432	1.542**	0.565
	(0.530)	(0.756)	(0.727)
Dummy medium low turbulence	0.660	2.216***	2.294***
	(0.523)	(0.459)	(0.361)
Dummy medium high turbulence	3.714***	4.094***	4.714***
	(0.773)	(0.634)	(0.525)
Dummy high turbulence	4.528***	9.462***	12.057***
	(0.883)	(1.314)	(1.552)
No. of observations	1,489	949	949

Notes: In the first column, pooled OLS regression coefficients are displayed. Note that linear regression coefficients are the same as average partial effects. The second and third column show average partial effects. The regression in column (1) is based on *baseline sample span* with 2,762 observations, where, in addition, we need observations on the lag of the forecast error and the lag of span, leading to 1,489 observations. The average partial effects in column (2) and (3) are based on *baseline sample forecast* with 2,778 observations, where we need, in addition, observations on the forecast error and its lag, leading to 949 observations (see also Table 11 in Appendix C). They are from the power GARCH models shown in columns (1) and (3), respectively, of Table 40 in Appendix I. All standard errors below the coefficients and the average partial effects are clustered by firm. * p < 0.10, ** p < 0.05, *** p < 0.01.

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Appendix A Representativeness of the sample

In this appendix, we investigate whether participation in our uncertainty module is driven by selection, conditional on participation in the main manufacturing survey. We base our analysis on all 34,684 complete firm-quarter responses available in the main survey for the months the uncertainty module was executed. We then ask whether firm size, time dummies, industry dummies, and interacted time-industry dummies are able to predict participation in the uncertainty module. To this end, we run a probit regression of a participation dummy that is 1 for the 5,564 observations of the uncertainty module and zero otherwise, on these predictors and report the estimated coefficients in column (1) of Table 10. We find that there is no statistically significant selection with respect to quarter/survey wave and industry suggesting that the uncertainty sample does not misrepresent specific quarters or industries. While firm size turns out to be significantly negative indicating that large firms are slightly underrepresented in the uncertainty module compared to the main manufacturing survey, the pseudo R-squared of 0.016 shows that this selection is quantitatively irrelevant. This is also reflected by an ROC curve which differs only slightly from the diagonal that indicates no discriminatory power, see the left panel of Figure 6.[25]

We repeat the analysis starting from the subset of 23,486 complete firm-quarter responses available from the online part of the main survey. We thus account for the fact that some firms reply to the main survey by fax and thus, essentially mechanically, do not participate in the uncertainty module, which is solely implemented online. The results of an analogous probit regression are reported in column (2) of Table 10. Again, the very low pseudo R-squared suggests that selectivity is not a relevant issue for the uncertainty module. This conclusion is supported by a largely unaltered ROC curve near the non-discriminatory diagonal, see the right panel of Figure 6.

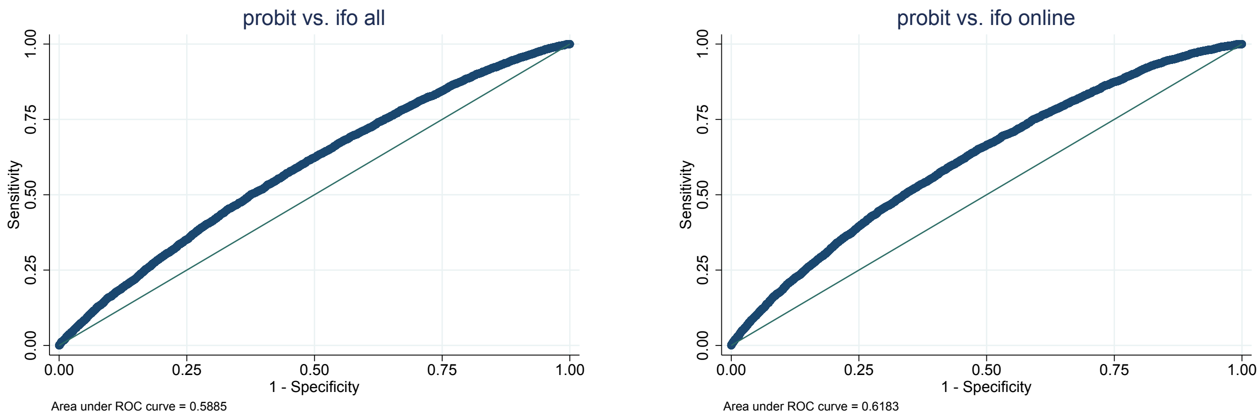
[25]The receiver operating characteristic (ROC) visualizes the discriminatory power of a binary classifier as follows: By varying the classification threshold—here: the probability above which an observation is predicted to participate in the uncertainty module—the classifier can produce any true positive rate (type II error). The ROC curve plots the true positive rate so obtained against its corresponding false positive rate (type I error). In the case of no discriminatory power, true and false positive rates are always the same, the ROC curve equals the diagonal, and the area under the ROC curve (AUC) is 0.5. A good classifier has a ROC curve well above the diagonal and an AUC that is near the maximum of 1.0.

Table 10: Probit regression of a dummy indicating uncertainty module participation

	(1) probit vs. ifo all	(2) probit vs. ifo online
Log of number of employees	-0.0753***	-0.134***
	(0.0127)	(0.0139)
Dummy survey wave 1	-0.0441	0.0707
	(0.221)	(0.242)
Dummy survey wave 2	0.0647	0.235
	(0.260)	(0.289)
Dummy survey wave 3	0.248	0.452*
	(0.232)	(0.258)
Dummy survey wave 4	0.276	0.439*
	(0.217)	(0.240)
Dummy survey wave 5	0.126	0.253
	(0.204)	(0.225)
Dummy survey wave 6	0.317	0.490**
	(0.195)	(0.211)
Dummy survey wave 7	0.224	0.387*
	(0.206)	(0.225)
Dummy survey wave 8	-0.244	-0.150
	(0.247)	(0.268)
Dummy survey wave 9	0.110	0.228
	(0.235)	(0.255)
Dummy survey wave 10	0.224	0.361*
	(0.186)	(0.201)
Dummy survey wave 11	-0.0333	0.0732
	(0.179)	(0.196)
Dummy survey wave 12	0.154	0.275
	(0.182)	(0.202)
Dummy survey wave 13	0.321	0.334
	(0.218)	(0.232)
Dummy industry 1	0.130	0.415
	(0.245)	(0.262)
Dummy industry 2	-0.301	-0.153
	(0.271)	(0.289)
Dummy industry 3	-0.0686	0.154
	(0.229)	(0.242)
Dummy industry 4	-0.0697	0.0116
	(0.250)	(0.264)
Dummy industry 5	0.136	0.335
	(0.237)	(0.250)
Dummy industry 6	-0.129	0.000307
	(0.240)	(0.253)
Dummy industry 7	-0.0828	0.185
	(0.250)	(0.267)
Dummy industry 8	0.109	0.379
	(0.257)	(0.275)
Dummy industry 9	-0.360	-0.184
	(0.232)	(0.245)
Dummy industry 10	0.120	0.282
	(0.259)	(0.276)
Dummy industry 11	-0.0901	0.0704
	(0.240)	(0.254)
Dummy industry 12	-0.0935	0.0302
	(0.222)	(0.232)
Dummy industry 13	-0.240	-0.0867
	(0.283)	(0.301)
Constant	-0.605***	-0.256
	(0.214)	(0.224)
Additional time-industry dummies	YES	YES
No. of observations	34,684	23,486
No. of firms	3,428	2,416
No. of parameters (excl. intercept)	196	196
Pseudo R-squared	0.016	0.030

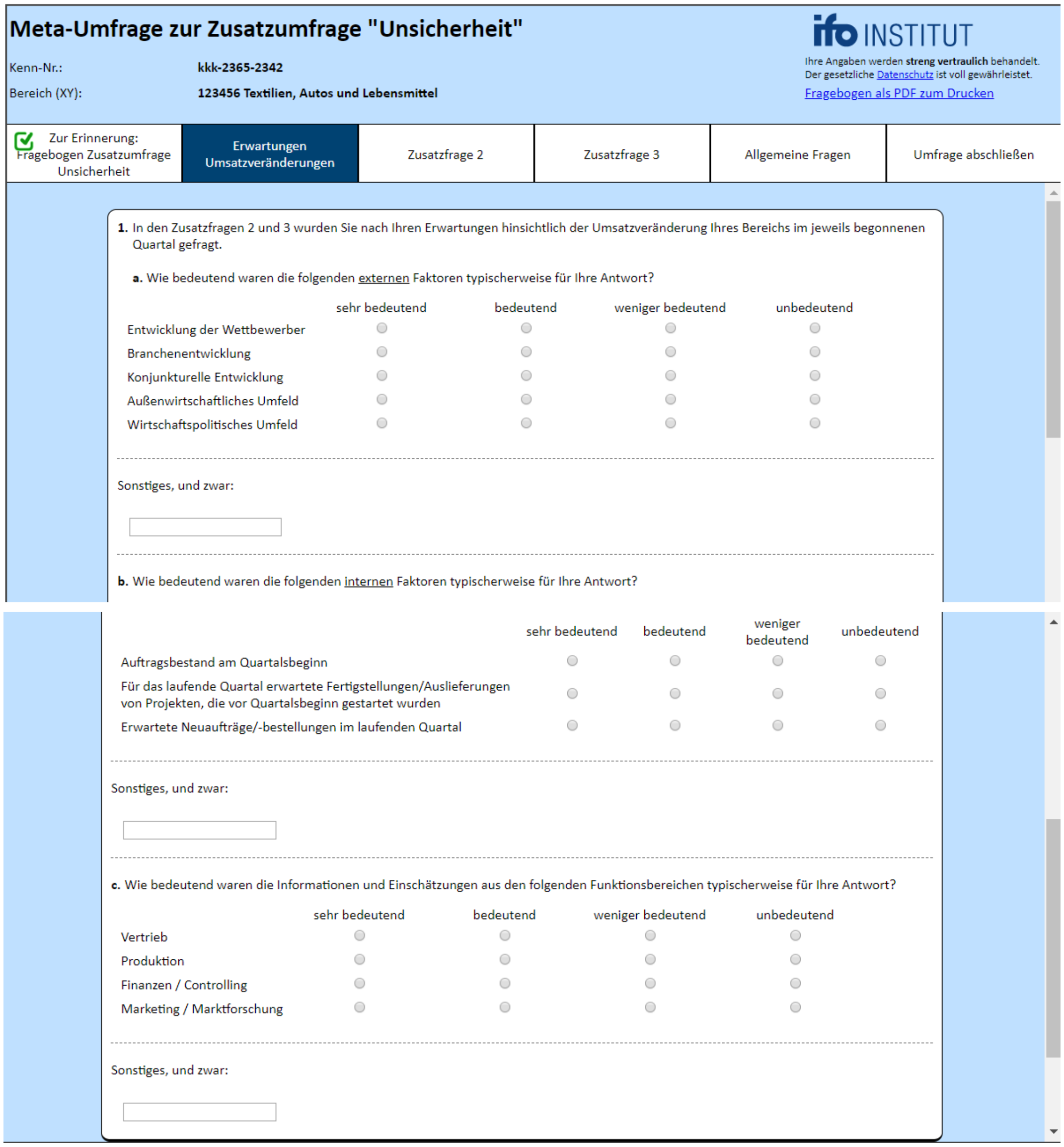
Notes: Standard errors in parentheses, clustered by firm; * p < 0.10, ** p < 0.05, *** p < 0.01. For the definition of industries see Appendix D.

Figure 6: ROC curves for probit estimations



Notes: The two plots depict ROC curves that correspond to the probit estimations in columns 1 and 2 of Table 10, respectively.

Appendix B Questionnaire for the one-time special survey from fall 2018



Meta-Umfrage zur Zusatzumfrage "Unsicherheit"

Kenn-Nr.: **kkk-2365-2342**

Bereich (XY): **123456 Textilien, Autos und Lebensmittel**

ifo INSTITUT

Ihre Angaben werden **streng vertraulich** behandelt.
Der gesetzliche Datenschutz ist voll gewährleistet.
Fragebogen als PDF zum Drucken

Zur Erinnerung: Fragebogen Zusatzumfrage Unsicherheit	Erwartungen Umsatzveränderungen	Zusatzfrage 2	Zusatzfrage 3	Allgemeine Fragen	Umfrage abschließen

1. In den Zusatzfragen 2 und 3 wurden Sie nach Ihren Erwartungen hinsichtlich der Umsatzveränderung Ihres Bereichs im jeweils begonnenen Quartal gefragt.

a. Wie bedeutend waren die folgenden externen Faktoren typischerweise für Ihre Antwort?

	sehr bedeutend	bedeutend	weniger bedeutend	unbedeutend
Entwicklung der Wettbewerber				
Branchenentwicklung				
Konjunkturelle Entwicklung				
Außenwirtschaftliches Umfeld				
Wirtschaftspolitisches Umfeld				

Sonstiges, und zwar:

b. Wie bedeutend waren die folgenden internen Faktoren typischerweise für Ihre Antwort?

	sehr bedeutend	bedeutend	weniger bedeutend	unbedeutend
Auftragsbestand am Quartalsbeginn				
Für das laufende Quartal erwartete Fertigstellungen/Auslieferungen von Projekten, die vor Quartalsbeginn gestartet wurden				
Erwartete Neuaufträge/-bestellungen im laufenden Quartal				

Sonstiges, und zwar:

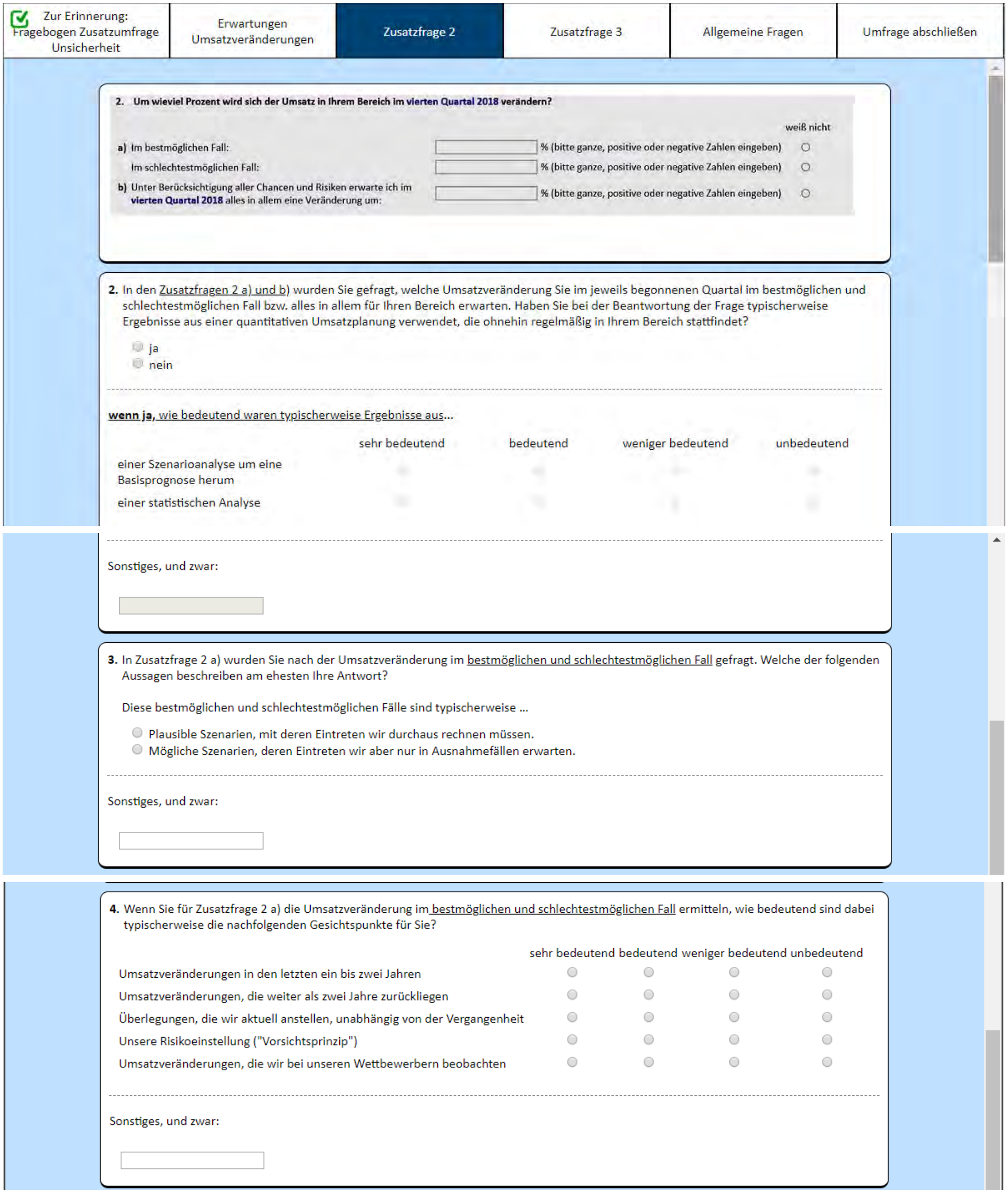
c. Wie bedeutend waren die Informationen und Einschätzungen aus den folgenden Funktionsbereichen typischerweise für Ihre Antwort?

	sehr bedeutend	bedeutend	weniger bedeutend	unbedeutend
Vertrieb				
Produktion				
Finanzen / Controlling				
Marketing / Marktforschung				

Sonstiges, und zwar:

Figure 7: Original survey questionnaire in German

Notes: Original questionnaire from ifo’s one-time online special survey on its “uncertainty module” in German, from fall 2018


Zur Erinnerung: Fragebogen Zusatzumfrage Unsicherheit | Erwartungen Umsatzveränderungen | Zusatzfrage 2 | Zusatzfrage 3 | Allgemeine Fragen | Umfrage abschließen

2. Um wieviel Prozent wird sich der Umsatz in Ihrem Bereich im vierten Quartal 2018 verändern?

		weiß nicht
a) Im bestmöglichen Fall:	% (bitte ganze, positive oder negative Zahlen eingeben)	○
Im schlechtestmöglichen Fall:	% (bitte ganze, positive oder negative Zahlen eingeben)	○
b) Unter Berücksichtigung aller Chancen und Risiken erwarte ich im **vierten Quartal 2018** alles in allem eine Veränderung um:	% (bitte ganze, positive oder negative Zahlen eingeben)	○

2. In den Zusatzfragen 2 a) und b) wurden Sie gefragt, welche Umsatzveränderung Sie im jeweils begonnenen Quartal im bestmöglichen und schlechtestmöglichen Fall bzw. alles in allem für Ihren Bereich erwarten. Haben Sie bei der Beantwortung der Frage typischerweise Ergebnisse aus einer quantitativen Umsatzplanung verwendet, die ohnehin regelmäßig in Ihrem Bereich stattfindet?

○ ja
○ nein

wenn ja, wie bedeutend waren typischerweise Ergebnisse aus...

	sehr bedeutend	bedeutend	weniger bedeutend	unbedeutend
einer Szenarioanalyse um eine Basisprognose herum	○	○	○	○
einer statistischen Analyse	○	○	○	○

Sonstiges, und zwar:

3. In Zusatzfrage 2 a) wurden Sie nach der Umsatzveränderung im bestmöglichen und schlechtestmöglichen Fall gefragt. Welche der folgenden Aussagen beschreiben am ehesten Ihre Antwort?

Diese bestmöglichen und schlechtestmöglichen Fälle sind typischerweise ...

○ Plausible Szenarien, mit deren Eintreten wir durchaus rechnen müssen.
○ Mögliche Szenarien, deren Eintreten wir aber nur in Ausnahmefällen erwarten.

Sonstiges, und zwar:

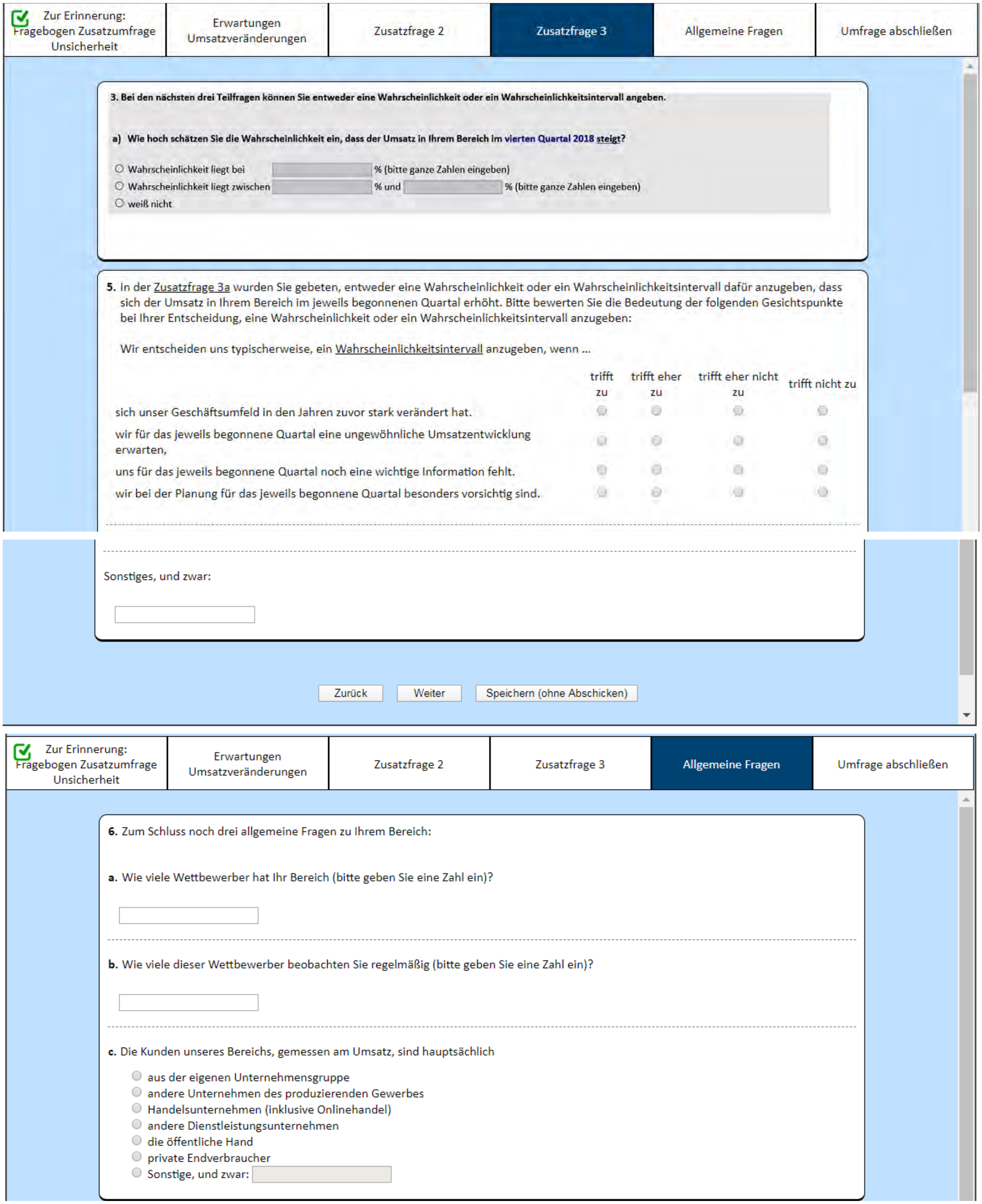
4. Wenn Sie für Zusatzfrage 2 a) die Umsatzveränderung im bestmöglichen und schlechtestmöglichen Fall ermitteln, wie bedeutend sind dabei typischerweise die nachfolgenden Gesichtspunkte für Sie?

	sehr bedeutend	bedeutend	weniger bedeutend	unbedeutend
Umsatzveränderungen in den letzten ein bis zwei Jahren	○	○	○	○
Umsatzveränderungen, die weiter als zwei Jahre zurückliegen	○	○	○	○
Überlegungen, die wir aktuell anstellen, unabhängig von der Vergangenheit	○	○	○	○
Unsere Risikoeinstellung ("Vorsichtsprinzip")	○	○	○	○
Umsatzveränderungen, die wir bei unseren Wettbewerbern beobachten	○	○	○	○

Sonstiges, und zwar:

Figure 8: Original survey questionnaire in German

Notes: Original questionnaire from ifo's one-time online special survey on its "uncertainty module" in German, from fall 2018


Zur Erinnerung: Fragebogen Zusatzumfrage Unsicherheit | Erwartungen Umsatzveränderungen | Zusatzfrage 2 | Zusatzfrage 3 | Allgemeine Fragen | Umfrage abschließen

3. Bei den nächsten drei Teilfragen können Sie entweder eine Wahrscheinlichkeit oder ein Wahrscheinlichkeitsintervall angeben.

a) Wie hoch schätzen Sie die Wahrscheinlichkeit ein, dass der Umsatz in Ihrem Bereich im vierten Quartal 2018 steigt?

○ Wahrscheinlichkeit liegt bei ____ % (bitte ganze Zahlen eingeben)
○ Wahrscheinlichkeit liegt zwischen ____ % und ____ % (bitte ganze Zahlen eingeben)
○ weiß nicht

5. In der Zusatzfrage 3a wurden Sie gebeten, entweder eine Wahrscheinlichkeit oder ein Wahrscheinlichkeitsintervall dafür anzugeben, dass sich der Umsatz in Ihrem Bereich im jeweils begonnenen Quartal erhöht. Bitte bewerten Sie die Bedeutung der folgenden Gesichtspunkte bei Ihrer Entscheidung, eine Wahrscheinlichkeit oder ein Wahrscheinlichkeitsintervall anzugeben:

Wir entscheiden uns typischerweise, ein Wahrscheinlichkeitsintervall anzugeben, wenn ...

	trifft zu	trifft eher zu	trifft eher nicht zu	trifft nicht zu
sich unser Geschäftsumfeld in den Jahren zuvor stark verändert hat.	○	○	○	○
wir für das jeweils begonnene Quartal eine ungewöhnliche Umsatzentwicklung erwarten,	○	○	○	○
uns für das jeweils begonnene Quartal noch eine wichtige Information fehlt.	○	○	○	○
wir bei der Planung für das jeweils begonnene Quartal besonders vorsichtig sind.	○	○	○	○

Sonstiges, und zwar:

Zurück | Weiter | Speichern (ohne Abschicken)

Zur Erinnerung: Fragebogen Zusatzumfrage Unsicherheit | Erwartungen Umsatzveränderungen | Zusatzfrage 2 | Zusatzfrage 3 | Allgemeine Fragen | Umfrage abschließen

6. Zum Schluss noch drei allgemeine Fragen zu Ihrem Bereich:

a. Wie viele Wettbewerber hat Ihr Bereich (bitte geben Sie eine Zahl ein)?

b. Wie viele dieser Wettbewerber beobachten Sie regelmäßig (bitte geben Sie eine Zahl ein)?

c. Die Kunden unseres Bereichs, gemessen am Umsatz, sind hauptsächlich

○ aus der eigenen Unternehmensgruppe
○ andere Unternehmen des produzierenden Gewerbes
○ Handelsunternehmen (inklusive Onlinehandel)
○ andere Dienstleistungsunternehmen
○ die öffentliche Hand
○ private Endverbraucher
○ Sonstige, und zwar: ____

Figure 9: Original survey questionnaire in German

Notes: Original questionnaire from ifo's one-time online special survey on its "uncertainty module" in German, from fall 2018

Appendix C Sample creation

In this appendix, we describe the construction of our baseline sample and also explain the number of observations from specific regressions in this paper. The observation numbers that remain after each step are listed in Table 11. Our starting sample from 14 survey waves consists of 5,564 firm-quarter observations. A firm-quarter observation is in the starting sample if the firm at that point in time answered at least one question of our online survey module and pressed the send button to return it.

Table 11: Sample creation

	firm-quarter obs. in sample	firm-quarter obs. excluded	firms in sample	firms excluded
Original sample	5,564		1,426	
Require response to question 1				
Response to question 1 exists	5,194	370	1,378	48
Text comment				
Wrong reference time excluded	5,095	99	1,368	10
Uncertain data quality excluded	5,067	28	1,367	1
Outliers to question 1				
Outliers in question 1 responses excluded	5,045	22	1,365	2
Number of observations by firm				
At least 5 clean responses to question 1	3,094	1,951	401	964
Outliers and inconsistencies to question 2				
Inconsistent & outlier responses to question 2 excluded	2,945	149	401	0
Require responses to question 2.a				
Baseline sample span: Responses to question 2.a both exist	2,762	183	400	1
Lag of forecast error exists	1,520	1,242	373	27
Lag of forecast error and lag of span exist	1,489	31	367	6
Lag of span exists	1,513	1,249	372	28
Require response to question 2.b				
Baseline sample forecast: Response to question 2.b exists	2,778	167	400	1
Forecast error exists	1,664	1,114	389	11
Lag of forecast error exists	949	715	292	97
Lag of forecast error, and span exist	932	17	289	3

We start by excluding 370 firm-quarter observations that were lacking an answer to question 1, realized sales growth. Then we carefully read the free text comments respondents can give below each of the questions (see Figure 1 in the main text for the questionnaire). We exclude 99 observations for which a comment indicates that the respondent was not able to calculate sales growth rates on a quarterly basis. For example, some firms in some quarters stated that they responded with annual growth rates. Moreover, we drop 28 observations for which the comment raises doubts about the validity and quality of the answer. For example, some firms in some quarters were not able to state realized past growth rates and used estimates instead. Overall, we exclude 497 firm-quarter observations based on missing or low-quality answers to question 1, leaving us with 5,067 firm-quarter observations.

Next, we exclude outliers which we define for question 1 (sales growth rate realizations) as lying outside the interval [-100%, 100%]. 22 firm-quarter observations are thus excluded. We have also experimented with a [-15%, 15%] cutoff, that is the bottom and top deciles of the final baseline sample, and found very similar results. We set the upper bounds quite high because large (two-digit) growth rates typically appear to be deliberate responses as many text comments reveal. Firms give explanations such as "Many projects were moved into this quarter" and "Invoice of a major project." This leaves us with 5,045 firm-quarter observations. After these cleaning steps, we require for the firm-quarter observations of a firm to remain in the sample that it have at least five clean firm-quarter observations on question 1, leaving us with 3,094 firm-quarter of 401 firm observations. It is this sample that we base the calculation of the trend and turbulence dummies on.

Subsequently, we exclude, respectively, outliers and inconsistencies related to question 2. Question 2-outliers were excluded according to the following two criteria:

1. The best case and worst case sales growth rates elicited in question 2.a lie outside the intervals [-100%; 300%] and [-100%; 100%], respectively.

2. The forecast growth rate elicited in question 2.b lies outside the interval [-100%; 100%].

Then we check whether respondents order the numbers in question 2 consistently, that is, as worst case < forecast < best case. We exclude firm-quarter observations with the orderings worst case $\geq$ forecast $\leq$ best case or worst case $\leq$ forecast $\geq$ best case because it is unclear what the respondents had in mind with these answers. However, we keep those firm-quarter observations with the inverse ordering worst case $\geq$ forecast $\geq$ best case and simply swap the worst case and best case numbers; we do this for 76 firm-quarter observations. Most likely inverse orderings were not intended by the respondent and rather a simple clerical error. Altogether, we eliminate 149 firm-quarter observations in this step.

In a final step, we eliminate those 183 firm-quarter observations which do not have answers to question 2.a, the best or worst case scenarios for sales growth, leaving us with our baseline sample of 2,762 of firm-quarter observations for 400 firms (*baseline sample span*). For some other exercises, for which we do not need span observations but the answer to question 2.b, that is, the forecast growth rate, we use a slightly bigger sample of 2,778 firm-quarter observations (*baseline sample forecast*).

Starting from the baseline sample span, for some further exercises, we additionally need a lag of the forecast error, which leaves us with 1,520 firm-quarter observations. For a subsample of 1,489 observations, we also have the lag of span. For another exercise, we again start from the baseline sample span and require a lag of span. This subsample contains 1,513 firm-quarter observations.

Finally, we use the slightly larger baseline sample forecast to analyze forecast errors, which is possible for 1,664 firm-quarter observations. For some exercises, we use consecutive forecast errors. We, therefore, have to eliminate isolated forecast errors that have no forecast error surrounding them. This reduces the sample to 1,329 firm-quarter observations of which 380 are used as lagged "pre-sample" observations so that an effective sample size of 949 firm-quarter observations remains. For 932 of these observations we also observe span.

Appendix D Definition of manufacturing industries

Table 12 presents the definition of the 14 industries we use. They are based on the original 24 two-digit manufacturing industries, which are defined by the WZ08 (WZ stands for the German *Wirtschaftszweig*) code of the German Statistical Office. Since not all these industries have a large number of observations in the survey, we aggregate, for the purposes of this paper, some of them, resulting in 14 manufacturing industries. The column with the number of observations by industry refers to our baseline sample of 2,762 observations.

Table 12: Definition of industries

Industry	Industry WZ08	Industry WZ08 name	No. of obs.
1	10, 11, 12	Food products; Beverages; Tobacco products	184
2	13, 14, 15	Textiles; Wearing apparel; Leather and related products	66
3	16, 17, 31	Wood, products of wood and cork except furniture, articles of straw and plaiting materials; Paper and paper products; Furniture	286
4	18	Printing and reproduction of recorded media	191
5	19, 20, 21	Coke and refined petroleum products; Chemicals and chemical support; Basic pharmaceutical products and pharmaceutical preparations	262
6	22	Rubber and plastic products	228
7	23	Other non-metallic mineral products	133
8	24	Basic metals	120
9	25	Fabricated metal products, except machinery and equipment	324
10	26	Computer, electronic and optical products	102
11	27	Electrical equipment	201
12	28	Machinery and equipment n.e.c.	445
13	29, 30	Motor vehicles, trailers and semi-trailers; Other transport equipment	116
14	32, 33	Other manufacturing; Repair and installation of machinery and equipment	104
All			2,762

Appendix E Proof of proposition in Section 2.3

This appendix provides the proof of the proposition in Section 2.3, restated here:

Proposition. Suppose g can be written as $g = \mu + \sigma\varepsilon$ where ε has a continuous density with mean zero and unit variance: Then the optimal span is a linear function of σ, that is, $\hat{g}_n - \hat{g}_1 = \sigma(\hat{s}_n - \hat{s}_1)$, where $(\hat{s}_n - \hat{s}_1)$ is the optimal span for $\sigma = 1$.

Proof. Since $g = \mu + \sigma\varepsilon$ and ε has density $\mathfrak{g}$, the density of g is $\mathfrak{g}((g-\mu)/\sigma)\sigma^{-1}$. A scenario $\hat{g}_i$ contributes to cost only when g realizes in an interval where $\hat{g}_i$ is the closest scenario. For an interior scenario other than the best or worst case, that interval is bounded by $\frac{1}{2}(\hat{g}_{i-1} + \hat{g}_i)$ and $\frac{1}{2}(\hat{g}_i + \hat{g}_{i+1})$. We can therefore write the firm's expected cost as:

$$\int_{-\infty}^{(\hat{g}_1+\hat{g}_2)/2} (g-\hat{g}_1)^2\, \mathfrak{g}\left(\frac{g-\mu}{\sigma}\right)\sigma^{-1}dg +$$
$$\sum_{i=2}^{n-1}\int_{(\hat{g}_{i-1}+\hat{g}_i)/2}^{(\hat{g}_i+\hat{g}_{i+1})/2} (g-\hat{g}_i)^2\, \mathfrak{g}\left(\frac{g-\mu}{\sigma}\right)\sigma^{-1}dg + \int_{(\hat{g}_{n-1}+\hat{g}_n)/2}^{\infty} (g-\hat{g}_n)^2\, \mathfrak{g}\left(\frac{g-\mu}{\sigma}\right)\sigma^{-1}dg.$$

The first-order conditions with respect to $\hat{g}_1, ..., \hat{g}_n$, and using Leibniz' rule, are:

$$\int_{(\hat{g}_{i-1}+\hat{g}_i)/2}^{(\hat{g}_i+\hat{g}_{i+1})/2} (g-\hat{g}_i)\, \mathfrak{g}\left(\frac{g-\mu}{\sigma}\right)\sigma^{-1}dg = 0; \qquad i = 2, ..., n-1$$
$$\int_{(\hat{g}_{n-1}+\hat{g}_n)/2}^{\infty} (g-\hat{g}_n)\, \mathfrak{g}\left(\frac{g-\mu}{\sigma}\right)\sigma^{-1}dg = \int_{-\infty}^{(\hat{g}_1+\hat{g}_2)/2} (g-\hat{g}_1)\, \mathfrak{g}\left(\frac{g-\mu}{\sigma}\right)\sigma^{-1}dg = 0$$

We note that increasing any given $\hat{g}_i$ increases the upper bound of the interval containing $\hat{g}_i$ as well as the lower bound of the next higher interval. Since the distribution is continuous, these effects cancel, so that the marginal benefit of increasing $\hat{g}_i$ is proportional to expected marginal cost over the interval containing $\hat{g}_i$.

Now consider the change of variable $s = (g-\mu)/\sigma$:

$$\int_{(\hat{g}_{i-1}+\hat{g}_i-2\mu)/2\sigma}^{(\hat{g}_i+\hat{g}_{i+1}-2\mu)/2\sigma} \left(s - \frac{\hat{g}_i-\mu}{\sigma}\right)\mathfrak{g}(s)\,ds = 0; \qquad i = 2, ..., n-1$$
$$\int_{(\hat{g}_{n-1}+\hat{g}_n-2\mu)/2\sigma}^{\infty} \left(s - \frac{\hat{g}_n-\mu}{\sigma}\right)\mathfrak{g}(s)\,ds = \int_{-\infty}^{(\hat{g}_1+\hat{g}_2-2\mu)/2\sigma} \left(s - \frac{\hat{g}_1-\mu}{\sigma}\right)\mathfrak{g}(s)\,ds = 0$$

Define the vector $\hat{s}$ of "standardized scenarios" that solves this system of equations for $\mu = 0$ and $\sigma = 1$. The first order conditions for given μ and σ imply that $\hat{g}_i = \mu + \sigma\hat{s}_i$. It follows in particular that span is $\hat{g}_n - \hat{g}_1 = \sigma(\hat{s}_n - \hat{s}_1)$. ■

Appendix F Detailed summary statistics

In this appendix, we report summary statistics for the answers to the questions in our survey module. Table 13 pools all firm-quarter observations and reports mean, standard deviation, and key quantiles for this pooled sample. The numbers here reflect variation both in the time series and in the cross section of firms. For Table 14, we compute, for each individual firm, the time-series mean and standard deviations. The panel reports mean, standard deviation and quantiles of the cross-sectional distributions of firm-level statistics. The number of observations for (functions of) forecast errors naturally drops because, in order to compute firm-level forecast errors, we need to observe the expected sales growth rate and the realized sales growth rate of a firm in two consecutive quarters; for details see Appendix C. In addition, we present in this appendix the same two summary statistics tables split by our three firm characteristics: firm size (Tables 15 to 22), trend (Tables 23 to 28), and turbulence (Tables 29 to 36).

Table 13: Summary statistics of survey answers and derived variables, pooled

Variable	N	Mean	Std. Dev.	P10	P25	P50	P75	P90
Sales growth rate in the previous quarter	2,762	1.71	14.69	-15	-5	2	10	15
Forecast sales growth rate for the current quarter	2,710	2.22	10.63	-10	0	2	5	10
Worst case sales growth rate for the current quarter	2,762	-4.75	11.82	-20	-10	-2	0	5
Best case sales growth rate for the current quarter	2,762	7.36	12	0	2	5	10	20
Span between worst and best case forecast	2,762	12.11	9.89	3	5	10	15	25
Forecast error	1,664	-.22	13.98	-15	-5	0	5	13
Forecast error from random walk model	1,664	.43	17.17	-14	-5	0	5	15
Forecast error from iid model	1,664	.51	10.82	-9.11	-3.21	.22	4.05	10.67
Absolute forecast error	1,664	8.69	10.95	0	2	5	10	20

Notes: P10 to P90 denote the corresponding percentiles of the distribution. The summary statistics for all variables listed in the table above are based on the *baseline sample span* with 2,762 firm-quarter observations, with two exceptions: First, we do not have the answer to question 2.b for all of these firm-quarter observations, which is why we are left with 2,710 firm-quarter observations with a forecast sales growth rate for the current quarter. Second, the summary statistics for forecast error variables are based on the *baseline sample forecast* with a forecast error, leading to 1,664 firm-quarter observations (see Table 11 in Appendix C). For comparison, we also compute the forecast error from a random walk model and from an *iid* process, respectively, based on the same sample as for the firms' forecast errors.

Table 14: Summary statistics of survey answers and derived variables, by firm

Variable	N	Mean	Std. Dev.	P10	P25	P50	P75	P90
Time-series mean by firm								
Mean by Firm: Sales growth rate in the previous quarter	400	1.74	7.87	-6.67	-1.92	1.69	5.3	10.29
Mean by Firm: Forecast sales growth rate for the current quarter	399	2.61	6.57	-3.75	-.6	2.25	5	9
Mean by Firm: Worst case sales growth rate for the current quarter	400	-4.48	7.4	-13	-8.31	-3.35	-.15	2.72
Mean by Firm: Best case sales growth rate for the current quarter	400	7.87	8.02	.81	3.41	6.67	10.61	16.67
Mean by Firm: Span between worst and best case forecast	400	12.34	7.35	5.09	7.15	10.57	15.55	22.33
Mean by Firm: Forecast error	389	-.23	10.48	-10	-3.57	0	3.5	8
Mean by Firm: Forecast error from random walk model	389	.6	9.74	-8.33	-3.22	0	3.8	9.67
Mean by Firm: Forecast error from iid model	389	.55	7.67	-6.8	-2.35	.2	2.63	8.67
Mean by Firm: Absolute forecast error	389	9.44	9.55	2.25	4	7	11.67	17.57
Time-series standard deviation by firm								
Std. Dev. by Firm: Sales growth rate in the previous quarter	397	11.42	9.22	3.44	5.48	8.59	13.7	23.49
Std. Dev. by Firm: Forecast sales growth rate for the current quarter	396	7.36	7.1	1.6	2.89	5.24	9.72	14.92
Std. Dev. by Firm: Worst case sales growth rate for the current quarter	397	8.09	7.13	2.12	3.18	6.28	10.8	16.33
Std. Dev. by Firm: Best case sales growth rate for the current quarter	397	8.13	7.78	2.16	3.43	5.87	10.31	15.52
Std. Dev. by Firm: Span between worst and best case forecast	397	5.85	5.05	2.04	2.87	4.67	7.5	10.61
Std. Dev. by Firm: Forecast error	338	10.17	9.79	2.29	4.04	7.46	12.73	20.21
Std. Dev. by Firm: Forecast error from random walk model	338	12.37	13.38	2.07	4.04	8.09	14.57	29.69
Std. Dev. by Firm: Forecast error from iid model	338	7	7.19	.71	2.3	4.51	9.9	16.42
Std. Dev. by Firm: Absolute forecast error	338	6.44	6.28	1.41	2.51	4.33	8.08	13.63

Notes: P10 to P90 denote the corresponding percentiles of the distribution. The summary statistics for all variables in the upper panel are based on the *baseline sample span* with 400 firms, with two exceptions: First, one firm did not answer question 2.b, which is why we are left with 399 firm-quarter observations with a forecast sales growth rate for the current quarter. Second, the summary statistics for forecast error variables are based on the *baseline sample forecast* with a forecast error, leading to 389 firms (see Table 11 in Appendix C). For comparison, we also compute the forecast error from a random walk model and from an *iid* process, respectively, based on the same sample as for the firms' forecast errors. As for the summary statistics in the lower panel, not all firms have a sufficient number to compute a standard deviation. This explains the difference in number of firms between the upper and lower panel.

Table 15: Summary statistics of survey answers and derived variables from tiny firms, pooled

Variable	Obs	Mean	Std. Dev.	P10	P25	P50	P75	P90
Sales growth rate in the previous quarter	131	-2.72	18.06	-30	-10	0	10	15
Absolute value of sales growth rate in the previous quarter	131	13.07	12.71	0	3	10	20	30
Forecast sales growth rate for the current quarter	130	-1.25	14.87	-20	-10	0	5	15
Worst case sales growth rate for the current quarter	131	-12.37	18.47	-50	-20	-10	0	5
Best case sales growth rate for the current quarter	131	7.48	17.27	-10	0	5	15	30
Span between worst and best case forecast	131	19.85	16.39	4	10	15	30	40
Forecast error	86	-2.7	18.5	-30	-10	0	5	15
Forecast error from random walk model	86	-.44	21.54	-20	-5	0	5	17
Forecast error from iid process	86	1	13.92	-16.67	-6.67	2.31	7.5	14.67
Absolute forecast error	86	12.65	13.7	0	3	10	15	35

Table 16: Summary statistics of survey answers and derived variables from tiny firms, by firm

Variable	Obs	Mean	Std. Dev.	P10	P25	P50	P75	P90
Time-series mean by firm								
Mean by Firm: Sales growth rate in the previous quarter	19	-2.6	10.73	-20	-10.5	-1.36	7.14	9.71
Mean by Firm: Forecast sales growth rate for the current quarter	19	1.86	7.1	-9	-3.57	4.43	6.92	10.63
Mean by Firm: Worst case sales growth rate for the current quarter	19	-8.22	8.61	-26	-13.83	-5.83	-2.29	2.5
Mean by Firm: Best case sales growth rate for the current quarter	19	9.94	8.1	.58	6.67	10	15.38	18.75
Mean by Firm: Span between worst and best case forecast	19	18.16	7.71	9	13.5	17.86	21.67	32.5
Mean by Firm: Forecast error	19	-3.53	11.8	-15	-12	-1.77	3.5	8
Mean by Firm: Forecast error from random walk model	19	-.78	8.19	-15	-5.5	-1	2.6	12.73
Mean by Firm: Forecast error from iid process	19	1.34	9.22	-11.67	-4.5	1.5	7.27	12.5
Mean by Firm: Absolute forecast error	19	12.5	9.95	2.5	6.67	8.67	17.5	33.6
Time-series standard deviation by firm								
Std. Dev. by Firm: Sales growth rate in the previous quarter	19	16.74	10.54	4.54	8.76	13.59	22.36	36.06
Std. Dev. by Firm: Forecast sales growth rate for the current quarter	19	12.02	10.14	3.31	4.21	10.49	15.3	37.47
Std. Dev. by Firm: Worst case sales growth rate for the current quarter	19	12.86	9.02	3.36	5	12.22	17.96	25.03
Std. Dev. by Firm: Best case sales growth rate for the current quarter	19	13.53	12.43	2.48	5.73	10.31	14.56	42.43
Std. Dev. by Firm: Span between worst and best case forecast	19	9.77	6.31	4.28	5.48	8.23	12.2	16.26
Std. Dev. by Firm: Forecast error	18	12.89	10.13	3.21	3.56	10.37	18.85	28.28
Std. Dev. by Firm: Forecast error from random walk model	18	16.42	21.97	2.71	4.73	10.78	15.28	35.23
Std. Dev. by Firm: Forecast error from iid process	18	11.42	11.03	2.89	4.51	7.07	13.66	24.75
Std. Dev. by Firm: Absolute forecast error	18	8.67	7.22	2.08	3.21	4.97	13.78	19.68

Table 17: Summary statistics of survey answers and derived variables from small firms, pooled

Variable	Obs	Mean	Std. Dev.	P10	P25	P50	P75	P90
Sales growth rate in the previous quarter	646	2.4	16.55	-15	-7	3	10	20
Absolute value of sales growth rate in the previous quarter	646	11.44	12.18	1	5	10	15	22
Forecast sales growth rate for the current quarter	619	3.7	10.9	-5	0	3	8	15
Worst case sales growth rate for the current quarter	646	-4.53	11.53	-20	-10	-2	0	5
Best case sales growth rate for the current quarter	646	9.72	12.44	0	3	10	15	20
Span between worst and best case forecast	646	14.25	10.27	4	7	10	20	30
Forecast error	801	-.15	12.97	-12	-5	0	5	11
Forecast error from random walk model	801	.56	17.22	-13	-5	0	5	15
Forecast error from iid process	801	.43	10.29	-8.44	-3.2	.33	3.73	9.78
Absolute forecast error	801	8.19	10.05	0	2	5	10	20

Table 18: Summary statistics of survey answers and derived variables from small firms, by firm

Variable	Obs	Mean	Std. Dev.	P10	P25	P50	P75	P90
Time-series mean by firm								
Mean by Firm: Sales growth rate in the previous quarter	108	2.54	9	-7	-3.06	1.35	6.53	12.5
Mean by Firm: Forecast sales growth rate for the current quarter	108	3.53	7.64	-4.87	-.48	3	6.8	12.67
Mean by Firm: Worst case sales growth rate for the current quarter	108	-5.01	8.39	-15.83	-9.33	-3.02	-.35	4.2
Mean by Firm: Best case sales growth rate for the current quarter	108	9.88	9.5	1.2	4.08	7.57	13.31	20
Mean by Firm: Span between worst and best case forecast	108	14.89	8.71	6.67	8.09	12.5	19.45	27.5
Mean by Firm: Forecast error	104	1.33	14.44	-13.25	-5.28	.63	5.5	13
Mean by Firm: Forecast error from random walk model	104	.2	9.93	-10.18	-4.5	0	5	10
Mean by Firm: Forecast error from iid process	104	.02	8.38	-9.5	-2.78	.39	3	9.17
Mean by Firm: Absolute forecast error	104	11.66	12.45	2.75	5	8.75	15	20
Time-series standard deviation by firm								
Std. Dev. by Firm: Sales growth rate in the previous quarter	108	12.64	10	3.97	6.7	9.54	15.18	24.61
Std. Dev. by Firm: Forecast sales growth rate for the current quarter	107	7.19	6.45	1.15	2.74	5.79	9.69	14.48
Std. Dev. by Firm: Worst case sales growth rate for the current quarter	108	8.01	6.22	1.89	3.13	6.7	11.06	18.22
Std. Dev. by Firm: Best case sales growth rate for the current quarter	108	8.15	6.72	2.51	3.68	6.13	10.42	15.32
Std. Dev. by Firm: Span between worst and best case forecast	108	6.71	4.67	2.3	3.39	5.75	8.49	13.93
Std. Dev. by Firm: Forecast error	80	12.33	10.83	2.86	5.87	9.92	15.48	22.18
Std. Dev. by Firm: Forecast error from random walk model	80	12.78	11.57	2.1	4.8	9.03	16.76	26.56
Std. Dev. by Firm: Forecast error from iid process	80	6.59	6.33	0	2.09	4.94	8.87	16.54
Std. Dev. by Firm: Absolute forecast error	80	7.85	6.8	1.41	3.18	6.43	9.73	18.58

Table 19: Summary statistics of survey answers and derived variables from medium-sized firms, pooled

Variable	Obs	Mean	Std. Dev.	P10	P25	P50	P75	P90
Sales growth rate in the previous quarter	1,296	1.71	14.29	-14	-5	3	9	15
Absolute value of sales growth rate in the previous quarter	1,296	9.83	10.51	1	3	7	12	20
Forecast sales growth rate for the current quarter	1,280	2.13	10.44	-9	0	2	5	10
Worst case sales growth rate for the current quarter	1,296	-4.6	11.79	-20	-10	-2	0	5
Best case sales growth rate for the current quarter	1,296	6.92	11.43	0	2	5	10	20
Span between worst and best case forecast	1,296	11.53	8.74	4	5	10	15	20
Forecast error	801	-.15	12.97	-12	-5	0	5	11
Forecast error from random walk model	801	.56	17.22	-13	-5	0	5	15
Forecast error from iid process	801	.43	10.29	-8.44	-3.2	.33	3.73	9.78
Absolute forecast error	801	8.19	10.05	0	2	5	10	20

Table 20: Summary statistics of survey answers and derived variables from medium-sized firms, by firm

Variable	Obs	Mean	Std. Dev.	P10	P25	P50	P75	P90
Time-series mean by firm								
Mean by Firm: Sales growth rate in the previous quarter	180	1.85	7.73	-6.06	-1.18	2	5.53	10.92
Mean by Firm: Forecast sales growth rate for the current quarter	179	2.67	6.47	-3.07	-.57	2.27	5.4	8.25
Mean by Firm: Worst case sales growth rate for the current quarter	180	-4.13	7.33	-12.58	-8	-3.33	0	2.76
Mean by Firm: Best case sales growth rate for the current quarter	180	7.63	7.64	.94	3.41	7	10.48	16.67
Mean by Firm: Span between worst and best case forecast	180	11.76	6.33	5.33	7.15	10.67	14.59	20.21
Mean by Firm: Forecast error	176	-.92	9.47	-10	-3.45	-.37	2.5	7.5
Mean by Firm: Forecast error from random walk model	176	1.39	10.62	-7.29	-2.48	.17	3.62	9.56
Mean by Firm: Forecast error from iid process	176	1.11	7.84	-5.14	-1.94	.35	2.99	9.5
Mean by Firm: Absolute forecast error	176	9.07	8.84	2	4	6.27	11.29	18
Time-series standard deviation by firm								
Std. Dev. by Firm: Sales growth rate in the previous quarter	178	11.26	9.32	3.4	5.38	8.18	13.57	23.08
Std. Dev. by Firm: Forecast sales growth rate for the current quarter	178	7.38	7.68	1.83	2.89	5.13	9.02	15.14
Std. Dev. by Firm: Worst case sales growth rate for the current quarter	178	8.25	8.02	2.19	3	6.13	10.8	15.62
Std. Dev. by Firm: Best case sales growth rate for the current quarter	178	8.12	8.23	1.95	3.46	5.73	10.26	16.43
Std. Dev. by Firm: Span between worst and best case forecast	178	5.51	5.27	2	2.74	4.46	7.07	9.67
Std. Dev. by Firm: Forecast error	161	9.67	9.79	2.5	3.97	7.07	11.77	18.65
Std. Dev. by Firm: Forecast error from random walk model	161	12.32	13.9	2.12	3.54	7.53	13.76	31.12
Std. Dev. by Firm: Forecast error from iid process	161	6.82	7.17	1	2.23	4.04	8.76	15.5
Std. Dev. by Firm: Absolute forecast error	161	6.13	6.1	1.26	2.35	4.24	7.78	12.02

Table 21: Summary statistics of survey answers and derived variables from large firms, pooled

Variable	Obs	Mean	Std. Dev.	P10	P25	P50	P75	P90
Sales growth rate in the previous quarter	689	1.9	12.6	-10	-4	3	7	12
Absolute value of sales growth rate in the previous quarter	689	8.24	9.72	1	3	5	10	15
Forecast sales growth rate for the current quarter	681	1.69	9.52	-8	-1	2	5	10
Worst case sales growth rate for the current quarter	689	-3.79	9.91	-15	-10	-2	0	5
Best case sales growth rate for the current quarter	689	5.95	11.07	-2	1	5	10	15
Span between worst and best case forecast	689	9.74	8.77	3	5	8	10	20
Forecast error	427	.02	12.33	-10	-4	0	5	10
Forecast error from random walk model	427	.07	16.42	-11	-5	0	5	12
Forecast error from iid process	427	.14	10.22	-9.31	-3.21	-.21	3	7.13
Absolute forecast error	427	6.97	10.16	0	2	5	9	15

Table 22: Summary statistics of survey answers and derived variables from large firms, by firm

Variable	Obs	Mean	Std. Dev.	P10	P25	P50	P75	P90
Time-series mean by firm								
Mean by Firm: Sales growth rate in the previous quarter	93	1.47	5.57	-5.25	-1	1.5	4.75	8
Mean by Firm: Forecast sales growth rate for the current quarter	93	1.55	5.09	-4	-.5	1.43	3.6	6.67
Mean by Firm: Worst case sales growth rate for the current quarter	93	-3.77	5.72	-10.71	-7.22	-3.43	-.37	1.67
Mean by Firm: Best case sales growth rate for the current quarter	93	5.56	6.05	0	3.09	4.8	7.75	11.29
Mean by Firm: Span between worst and best case forecast	93	9.33	5.64	3.67	5.38	8	11.5	16
Mean by Firm: Forecast error	90	.01	5.2	-5	-2.6	.33	3.14	5.33
Mean by Firm: Forecast error from random walk model	90	-.17	7.85	-9	-3.08	-.5	2.88	7.31
Mean by Firm: Forecast error from iid process	90	-.09	5.99	-6.05	-2.83	-.32	1.63	3.62
Mean by Firm: Absolute forecast error	90	6.96	5.46	2.13	3.4	5.5	9.5	13.38
Time-series standard deviation by firm								
Std. Dev. by Firm: Sales growth rate in the previous quarter	92	9.21	6.99	3.22	4.91	7.42	10.83	16.5
Std. Dev. by Firm: Forecast sales growth rate for the current quarter	92	6.56	5.47	1.69	2.88	4.42	9.19	13.58
Std. Dev. by Firm: Worst case sales growth rate for the current quarter	92	6.9	5.3	1.81	3.14	5.66	9.04	12.06
Std. Dev. by Firm: Best case sales growth rate for the current quarter	92	7.01	6.37	2.04	2.92	4.75	9.32	13.61
Std. Dev. by Firm: Span between worst and best case forecast	92	4.67	4.19	1.69	2.37	3.66	5.58	7.81
Std. Dev. by Firm: Forecast error	79	8.37	8.13	2.02	3.31	5.55	11.6	17.68
Std. Dev. by Firm: Forecast error from random walk model	79	11.11	11.43	1.41	3.54	7.78	13.44	27.37
Std. Dev. by Firm: Forecast error from iid process	79	6.79	6.78	.71	2.3	4.04	10.05	16.44
Std. Dev. by Firm: Absolute forecast error	79	5.14	5.57	1.39	2.14	3.54	5.8	11.55

Table 23: Summary statistics of survey answers and derived variables from 'bad' trend firms, pooled

Variable	Obs	Mean	Std. Dev.	P10	P25	P50	P75	P90
Sales growth rate in the previous quarter	664	-5.92	16.97	-21	-15	-5	2	10
Absolute value of sales growth rate in the previous quarter	664	12.72	12.7	1	5	10	15	25
Forecast sales growth rate for the current quarter	648	-.36	13.75	-15	-5	0	5	10
Worst case sales growth rate for the current quarter	664	-9.89	15.79	-25	-20	-10	0	5
Best case sales growth rate for the current quarter	664	6.53	15.61	-10	0	5	10	20
Span between worst and best case forecast	664	16.42	12.92	5	10	15	20	30
Forecast error	386	-4.28	17.58	-22	-10	-3	5	14
Forecast error from random walk model	386	4.28	22.01	-15	-5	2	10	29
Forecast error from iid process	386	4.95	13.19	-8.15	-.9	3.64	10.92	19.33
Absolute forecast error	386	12.11	13.43	0	4	8	15	25

Table 24: Summary statistics of survey answers and derived variables from 'bad' trend firms, by firm

Variable	Obs	Mean	Std. Dev.	P10	P25	P50	P75	P90
Time-series mean by firm								
Mean by Firm: Sales growth rate in the previous quarter	100	-6.11	7.99	-13.56	-9.39	-5.29	-2.55	-1.52
Mean by Firm: Forecast sales growth rate for the current quarter	99	.29	9.03	-8.6	-3.57	-.83	3.33	10.67
Mean by Firm: Worst case sales growth rate for the current quarter	100	-9.1	10.08	-19.36	-14.02	-9.09	-3.58	.71
Mean by Firm: Best case sales growth rate for the current quarter	100	7.16	10.04	-1.18	1	6.14	11.15	18.75
Mean by Firm: Span between worst and best case forecast	100	16.26	8.18	7.67	10.48	14.51	20	28.64
Mean by Firm: Forecast error	97	-5.49	11.2	-17.5	-11.75	-3.5	.33	5.5
Mean by Firm: Forecast error from random walk model	97	5.27	12.92	-7.5	0	3	9.56	20
Mean by Firm: Forecast error from iid process	97	5.83	8.95	-3.5	.71	2.63	11	17.4
Mean by Firm: Absolute forecast error	97	12.6	9.5	4	6	10	15.71	25
Time-series standard deviation by firm								
Std. Dev. by Firm: Sales growth rate in the previous quarter	98	14.62	10.49	5.42	8.01	10.81	18.78	29.81
Std. Dev. by Firm: Forecast sales growth rate for the current quarter	97	10.19	9.53	2.52	4.58	7.45	12.32	19.94
Std. Dev. by Firm: Worst case sales growth rate for the current quarter	98	11.57	9.56	3.56	5.77	9.57	14.05	22.45
Std. Dev. by Firm: Best case sales growth rate for the current quarter	98	11.18	10.86	2.88	4.8	7.75	13.23	22.45
Std. Dev. by Firm: Span between worst and best case forecast	98	8.15	7.68	2.89	3.93	6.41	9.12	13.93
Std. Dev. by Firm: Forecast error	81	13.02	11.15	4.04	6.08	10.16	16.89	22.23
Std. Dev. by Firm: Forecast error from random walk model	81	16.25	17.25	3.54	5	9.57	18.19	41.12
Std. Dev. by Firm: Forecast error from iid process	81	9.76	8.69	2.12	3.54	7.07	13.14	23.76
Std. Dev. by Firm: Absolute forecast error	81	8.61	7.55	2.52	3.54	6.41	11.31	17.21

Table 25: Summary statistics of survey answers and derived variables from 'normal' trend firms, pooled

Variable	Obs	Mean	Std. Dev.	P10	P25	P50	P75	P90
Sales growth rate in the previous quarter	1,472	1.61	10.11	-10	-4	2	6	11
Absolute value of sales growth rate in the previous quarter	1,472	7.28	7.19	1	3	5	10	15
Forecast sales growth rate for the current quarter	1,447	1.81	7.93	-5	0	2	5	10
Worst case sales growth rate for the current quarter	1,472	-4.05	8.8	-15	-7.5	-2	0	3
Best case sales growth rate for the current quarter	1,472	6.13	8.94	0	2	5	10	15
Span between worst and best case forecast	1,472	10.18	7.83	3	5	9	15	20
Forecast error	940	-.03	10.43	-10	-5	0	5	10
Forecast error from random walk model	940	0	12.55	-10	-5	0	5	12
Forecast error from iid process	940	.2	8.2	-6.71	-2.06	.29	3.06	7.28
Absolute forecast error	940	6.73	7.96	0	2	5	9	15

Table 26: Summary statistics of survey answers and derived variables from 'normal' trend firms, by firm

Variable	Obs	Mean	Std. Dev.	P10	P25	P50	P75	P90
Time-series mean by firm								
Mean by Firm: Sales growth rate in the previous quarter	201	1.53	3.45	-1.25	0	1.6	3.14	4.88
Mean by Firm: Forecast sales growth rate for the current quarter	201	2	4.51	-2.5	0	1.7	3.8	5.63
Mean by Firm: Worst case sales growth rate for the current quarter	201	-3.93	4.98	-10	-7	-3.3	-.75	1
Mean by Firm: Best case sales growth rate for the current quarter	201	6.39	6.53	1.67	3.18	5.1	8.13	11.5
Mean by Firm: Span between worst and best case forecast	201	10.32	6.46	4.27	5.8	8.71	12.67	18.64
Mean by Firm: Forecast error	197	-.22	5.42	-5	-2.75	-.4	1.67	6
Mean by Firm: Forecast error from random walk model	197	.12	5.99	-5.83	-2	0	2.5	7.5
Mean by Firm: Forecast error from iid process	197	.19	4.65	-4.11	-1.39	.23	2.2	4.7
Mean by Firm: Absolute forecast error	197	6.85	5.64	1.71	3.2	5.43	8.71	13.75
Time-series standard deviation by firm								
Std. Dev. by Firm: Sales growth rate in the previous quarter	200	8.86	6.32	2.9	4.89	7.54	10.92	16.15
Std. Dev. by Firm: Forecast sales growth rate for the current quarter	200	5.8	4.78	1.31	2.49	4.39	7.35	12.36
Std. Dev. by Firm: Worst case sales growth rate for the current quarter	200	6.48	4.96	1.59	2.58	5.23	8.76	12.4
Std. Dev. by Firm: Best case sales growth rate for the current quarter	200	6.19	4.87	1.92	2.67	5.06	7.8	11.65
Std. Dev. by Firm: Span between worst and best case forecast	200	4.54	2.94	1.64	2.33	3.93	5.76	8.73
Std. Dev. by Firm: Forecast error	178	8.15	6.94	2.08	3.54	6.5	10.61	15.67
Std. Dev. by Firm: Forecast error from random walk model	178	9.35	8.63	1.41	3.46	7.07	12.5	18.87
Std. Dev. by Firm: Forecast error from iid process	178	5.6	5.67	.71	2.06	3.82	6.45	13.67
Std. Dev. by Firm: Absolute forecast error	178	4.95	4.23	1.15	2.12	3.6	6.99	10.54

Table 27: Summary statistics of survey answers and derived variables from 'good' trend firms, pooled

Variable	Obs	Mean	Std. Dev.	P10	P25	P50	P75	P90
Sales growth rate in the previous quarter	626	10.03	16.59	-5	3	10	15	25
Absolute value of sales growth rate in the previous quarter	626	13.34	14.06	2	5	10	15	29
Forecast sales growth rate for the current quarter	615	5.89	11.41	-5	2	5	10	15
Worst case sales growth rate for the current quarter	626	-.95	11.28	-11	-5	0	5	10
Best case sales growth rate for the current quarter	626	11.15	13.06	0	5	10	15	25
Span between worst and best case forecast	626	12.1	9.05	4	6	10	15	20
Forecast error	338	3.91	16.49	-10	-2	2	9	19
Forecast error from random walk model	338	-2.79	20.84	-17	-7	-2	2	13
Forecast error from iid process	338	-3.7	12.3	-14.57	-7	-3.18	-.17	6.33
Absolute forecast error	338	10.21	13.51	0	2	5	12	24

Table 28: Summary statistics of survey answers and derived variables from 'good' trend firms, by firm

Variable	Obs	Mean	Std. Dev.	P10	P25	P50	P75	P90
Time-series mean by firm								
Mean by Firm: Sales growth rate in the previous quarter	99	10.1	5.46	5.2	6.57	8.83	12.33	17.86
Mean by Firm: Forecast sales growth rate for the current quarter	99	6.15	5.78	0	3.14	5.4	8.25	13.57
Mean by Firm: Worst case sales growth rate for the current quarter	99	-.92	5.9	-8	-3.57	-.6	2.1	6.17
Mean by Firm: Best case sales growth rate for the current quarter	99	11.58	7.37	4.5	7.4	10	14.36	23.57
Mean by Firm: Span between worst and best case forecast	99	12.5	6.55	6.2	7.67	10.7	16.25	22.5
Mean by Firm: Forecast error	95	5.12	14.37	-5	1	3.57	8	14.5
Mean by Firm: Forecast error from random walk model	95	-3.16	10.42	-15.5	-6.5	-3	1	6.67
Mean by Firm: Forecast error from iid process	95	-4.09	8.1	-13.83	-6.83	-2.7	-.79	3.33
Mean by Firm: Absolute forecast error	95	11.58	13.74	3	4.33	7.75	14	25
Time-series standard deviation by firm								
Std. Dev. by Firm: Sales growth rate in the previous quarter	99	13.44	11.21	3.43	5.59	9.62	18.23	31.13
Std. Dev. by Firm: Forecast sales growth rate for the current quarter	99	7.76	7.39	1.6	3	5.32	9.79	16.26
Std. Dev. by Firm: Worst case sales growth rate for the current quarter	99	7.9	6.89	2.24	2.99	5.49	10.8	19.12
Std. Dev. by Firm: Best case sales growth rate for the current quarter	99	9.02	7.87	2.51	3.61	5.92	11.51	20
Std. Dev. by Firm: Span between worst and best case forecast	99	6.21	4.25	2.38	2.95	5	8.21	12.04
Std. Dev. by Firm: Forecast error	79	11.8	12.53	1.41	3.78	7.59	17.97	25
Std. Dev. by Firm: Forecast error from random walk model	79	15.19	15.95	2.12	4.24	9.19	17.68	44.55
Std. Dev. by Firm: Forecast error from iid process	79	7.34	7.77	.71	2.52	3.69	10.6	18.48
Std. Dev. by Firm: Absolute forecast error	79	7.57	7.76	1.15	2.28	4.68	10.49	18.63

Table 29: Summary statistics of survey answers and derived variables from low turbulence firms, pooled

Variable	Obs	Mean	Std. Dev.	P10	P25	P50	P75	P90
Sales growth rate in the previous quarter	748	2.83	5.73	-5	0	3	5	10
Absolute value of sales growth rate in the previous quarter	748	4.99	3.99	0	2	5	7	10
Forecast sales growth rate for the current quarter	739	2.32	4.36	-2	0	2	5	7
Worst case sales growth rate for the current quarter	748	-1.87	5.3	-10	-5	0	1	4
Best case sales growth rate for the current quarter	748	5.36	5.09	0	2	5	8	10
Span between worst and best case forecast	748	7.23	4.97	3	4	6	10	14
Forecast error	453	.49	4.64	-5	-2	0	3	6
Forecast error from random walk model	453	-.49	4.7	-5	-3	0	2	5
Forecast error from iid process	453	-.47	3.68	-4.92	-2.18	-.29	1.4	3.78
Absolute forecast error	453	3.51	3.07	0	1	3	5	8

Table 30: Summary statistics of survey answers and derived variables from low turbulence firms, by firm

Variable	Obs	Mean	Std. Dev.	P10	P25	P50	P75	P90
Time-series mean by firm								
Mean by Firm: Sales growth rate in the previous quarter	102	3.02	4.81	-2	.38	2.79	5.11	9.71
Mean by Firm: Forecast sales growth rate for the current quarter	101	2.57	3.27	-1.1	.83	2.27	4	6.5
Mean by Firm: Worst case sales growth rate for the current quarter	102	-1.84	4.16	-6.89	-3.87	-1.27	.63	2.83
Mean by Firm: Best case sales growth rate for the current quarter	102	5.61	3.64	1.88	3.15	4.95	7.57	10.83
Mean by Firm: Span between worst and best case forecast	102	7.46	3.7	3.67	4.8	6.76	9	12
Mean by Firm: Forecast error	100	.53	3.32	-3.27	-1.08	.35	2.13	3.8
Mean by Firm: Forecast error from random walk model	100	-.36	3.23	-3.95	-2.12	-.63	1.86	3.42
Mean by Firm: Forecast error from iid process	100	-.58	2.86	-3.86	-2.05	-.38	1.19	2.32
Mean by Firm: Absolute forecast error	100	3.6	2.27	1	2	3.5	4.66	5.93
Time-series standard deviation by firm								
Std. Dev. by Firm: Sales growth rate in the previous quarter	101	3.93	1.31	2.27	2.88	4.02	4.97	5.42
Std. Dev. by Firm: Forecast sales growth rate for the current quarter	101	2.78	1.79	1.1	1.6	2.54	3.45	4.57
Std. Dev. by Firm: Worst case sales growth rate for the current quarter	101	3.41	2.08	1.25	2.1	2.87	4.49	5.96
Std. Dev. by Firm: Best case sales growth rate for the current quarter	101	3.48	2.1	1.64	2.17	2.97	4.21	5.97
Std. Dev. by Firm: Span between worst and best case forecast	101	3.24	1.85	1.46	2.07	2.58	4	5.48
Std. Dev. by Firm: Forecast error	90	3.54	2.12	.71	2.02	3.42	5.13	6.46
Std. Dev. by Firm: Forecast error from random walk model	90	3.66	2.41	.71	2	3.18	4.95	6.74
Std. Dev. by Firm: Forecast error from iid process	90	2.5	1.78	0	1.1	2.31	3.54	5.06
Std. Dev. by Firm: Absolute forecast error	90	2.21	1.44	.55	1.14	2.13	3.08	4.03

Table 31: Summary statistics of survey answers and derived variables from medium low turbulence firms, pooled

Variable	Obs	Mean	Std. Dev.	P10	P25	P50	P75	P90
Sales growth rate in the previous quarter	689	.46	8.72	-10	-5	1	5	10
Absolute value of sales growth rate in the previous quarter	689	7.02	5.18	1	3	5	10	14
Forecast sales growth rate for the current quarter	672	1.36	7.19	-6	-1	2	5	10
Worst case sales growth rate for the current quarter	689	-4.29	8.36	-15	-10	-3	0	5
Best case sales growth rate for the current quarter	689	5.39	7.81	-2	1	5	10	15
Span between worst and best case forecast	689	9.68	6.66	4	5	10	10	19
Forecast error	403	-.53	8.49	-10	-5	0	5	10
Forecast error from random walk model	403	1.16	10.28	-9	-4	0	6	13
Forecast error from iid process	403	.66	7.11	-5.3	-2.2	.33	3.5	8.36
Absolute forecast error	403	6.23	5.79	0	2	5	9	13

Table 32: Summary statistics of survey answers and derived variables from medium low turbulence firms, by firm

Variable	Obs	Mean	Std. Dev.	P10	P25	P50	P75	P90
Time-series mean by firm								
Mean by Firm: Sales growth rate in the previous quarter	98	.72	5.92	-5.86	-1.43	.29	3	7.08
Mean by Firm: Forecast sales growth rate for the current quarter	98	1.72	4.87	-4.14	-.73	1.77	4.4	6.25
Mean by Firm: Worst case sales growth rate for the current quarter	98	-3.98	5.76	-11.33	-6.67	-3.43	-.17	1.6
Mean by Firm: Best case sales growth rate for the current quarter	98	5.76	5.65	0	2.57	5.67	8.13	11.2
Mean by Firm: Span between worst and best case forecast	98	9.74	4.99	5.4	6.86	8.71	11.5	15.5
Mean by Firm: Forecast error	96	-.43	6.61	-5.67	-3.05	0	3.17	6
Mean by Firm: Forecast error from random walk model	96	1.45	8.29	-5	-1.64	.39	4	8
Mean by Firm: Forecast error from iid process	96	.81	6.28	-4.63	-1.54	.57	2.37	5.62
Mean by Firm: Absolute forecast error	96	6.72	4.64	2.83	4.37	5.81	8.05	11.33
Time-series standard deviation by firm								
Std. Dev. by Firm: Sales growth rate in the previous quarter	98	7.31	1.25	6.03	6.47	7.51	8.11	8.62
Std. Dev. by Firm: Forecast sales growth rate for the current quarter	98	5.36	3.44	1.47	3.01	4.85	7.09	9.01
Std. Dev. by Firm: Worst case sales growth rate for the current quarter	98	5.91	3.51	2.26	3.41	5.43	7.5	11.34
Std. Dev. by Firm: Best case sales growth rate for the current quarter	98	5.92	3.6	2.5	3.24	5.22	7.5	11.06
Std. Dev. by Firm: Span between worst and best case forecast	98	4.49	3.33	2.08	2.69	3.51	5.38	7.58
Std. Dev. by Firm: Forecast error	82	7.25	3.79	3.54	4.36	7.02	8.66	11.15
Std. Dev. by Firm: Forecast error from random walk model	82	8.02	5.21	2.5	4.43	7.44	10.41	13.65
Std. Dev. by Firm: Forecast error from iid process	82	4.8	4	1.41	2.12	3.71	6.45	8.76
Std. Dev. by Firm: Absolute forecast error	82	4.67	2.66	1.41	2.83	4.29	6.21	7.35

Table 33: Summary statistics of survey answers and derived variables from medium high turbulence firms, pooled

Variable	Obs	Mean	Std. Dev.	P10	P25	P50	P75	P90
Sales growth rate in the previous quarter	681	1.16	11.66	-15	-10	2	10	15
Absolute value of sales growth rate in the previous quarter	681	9.51	6.83	2	5	10	15	20
Forecast sales growth rate for the current quarter	661	1.34	9.52	-10	-2	2	5	10
Worst case sales growth rate for the current quarter	681	-6.59	10.98	-20	-10	-5	0	5
Best case sales growth rate for the current quarter	681	7.12	9.85	0	2	5	10	20
Span between worst and best case forecast	681	13.71	8.75	5	8	10	20	25
Forecast error	409	-.51	12.42	-15	-9	0	5	15
Forecast error from random walk model	409	.41	14.05	-15	-5	0	8	17
Forecast error from iid process	409	.77	9.99	-8.82	-3.18	1	5.2	10.92
Absolute forecast error	409	9.27	8.27	0	3	7	14	20

Table 34: Summary statistics of survey answers and derived variables from medium high turbulence firms, by firm

Variable	Obs	Mean	Std. Dev.	P10	P25	P50	P75	P90
Time-series mean by firm								
Mean by Firm: Sales growth rate in the previous quarter	100	1.15	5.84	-6.07	-2.55	1.28	4.75	7.38
Mean by Firm: Forecast sales growth rate for the current quarter	100	1.57	5.46	-5.04	-1.31	1.44	4.67	7.81
Mean by Firm: Worst case sales growth rate for the current quarter	100	-6.14	6.5	-13.79	-9.77	-5.69	-1.44	.7
Mean by Firm: Best case sales growth rate for the current quarter	100	7.34	6.09	1.13	3.27	7.11	10.48	13.79
Mean by Firm: Span between worst and best case forecast	100	13.48	6.25	7	8.94	12.56	17	21.75
Mean by Firm: Forecast error	96	-.75	7.8	-10	-5	-1	4.07	8
Mean by Firm: Forecast error from random walk model	96	1	9.05	-8	-4.57	0	5.75	10
Mean by Firm: Forecast error from iid process	96	.86	6.57	-4.94	-2.35	.26	3.33	7.27
Mean by Firm: Absolute forecast error	96	9.51	4.49	4.33	6.56	8.78	12.45	16
Time-series standard deviation by firm								
Std. Dev. by Firm: Sales growth rate in the previous quarter	100	10.88	2.27	8.81	9.71	10.82	12.25	13.26
Std. Dev. by Firm: Forecast sales growth rate for the current quarter	99	7.45	4.44	2.65	4.43	6.66	9.98	12.76
Std. Dev. by Firm: Worst case sales growth rate for the current quarter	100	8.5	4.92	2.5	5.14	8.06	11.37	13.51
Std. Dev. by Firm: Best case sales growth rate for the current quarter	100	7.61	4.44	2.94	4.9	6.77	10.16	12.42
Std. Dev. by Firm: Span between worst and best case forecast	100	6.14	2.63	2.76	4.24	5.82	8.15	9.65
Std. Dev. by Firm: Forecast error	81	10.97	5.48	3.42	7.07	10.59	14.52	18.38
Std. Dev. by Firm: Forecast error from random walk model	81	11.7	6.98	4.63	7.07	10.15	15.36	18.12
Std. Dev. by Firm: Forecast error from iid process	81	7.13	5.47	.96	3.54	5.45	11.21	13.44
Std. Dev. by Firm: Absolute forecast error	81	7.05	4.31	2.36	3.5	6.43	8.92	12.73

Table 35: Summary statistics of survey answers and derived variables from high turbulence firms, pooled

Variable	Obs	Mean	Std. Dev.	P10	P25	P50	P75	P90
Sales growth rate in the previous quarter	644	2.3	25.67	-30	-15	3	15.5	30
Absolute value of sales growth rate in the previous quarter	644	19.37	17	3	9	15	25	40
Forecast sales growth rate for the current quarter	638	3.9	17.48	-15	-5	5	10	20
Worst case sales growth rate for the current quarter	644	-6.65	18.66	-30	-15	-5	3	10
Best case sales growth rate for the current quarter	644	12.05	19.73	-10	0	10	20	30
Span between worst and best case forecast	644	18.7	13.51	6	10	15	25	35
Forecast error	399	-.4	23.67	-27	-11	0	10	25
Forecast error from random walk model	399	.75	29.94	-31	-14	0	15	40
Forecast error from iid process	399	1.2	17.86	-20.31	-9	.83	11	21
Absolute forecast error	399	16.45	17	1	5	10	22	40

Table 36: Summary statistics of survey answers and derived variables from high turbulence firms, by firm

Variable	Obs	Mean	Std. Dev.	P10	P25	P50	P75	P90
Time-series mean by firm								
Mean by Firm: Sales growth rate in the previous quarter	100	2.02	12.43	-10.95	-5.62	2.61	8.14	16.85
Mean by Firm: Forecast sales growth rate for the current quarter	100	4.54	10.21	-6.04	-1.21	3.67	8.88	17.08
Mean by Firm: Worst case sales growth rate for the current quarter	100	-5.99	10.77	-16.02	-12.71	-7.46	0	6.45
Mean by Firm: Best case sales growth rate for the current quarter	100	12.75	11.97	.65	5.14	11.67	17.75	25.5
Mean by Firm: Span between worst and best case forecast	100	18.74	8.13	9.92	13	16.7	22.75	31.25
Mean by Firm: Forecast error	97	-.3	18.12	-20	-8.25	-.37	6.75	15
Mean by Firm: Forecast error from random walk model	97	.37	14.88	-15	-7.5	-.87	7.13	15.33
Mean by Firm: Forecast error from iid process	97	1.16	12.08	-13.83	-6.19	.5	8.67	17.33
Mean by Firm: Absolute forecast error	97	18.09	14.26	6.5	9.67	14.44	20.71	35
Time-series standard deviation by firm								
Std. Dev. by Firm: Sales growth rate in the previous quarter	98	23.82	10.36	14.25	16.44	20.85	27.87	40.41
Std. Dev. by Firm: Forecast sales growth rate for the current quarter	98	14	9.98	4.09	6.76	11.68	17.02	27.54
Std. Dev. by Firm: Worst case sales growth rate for the current quarter	98	14.68	9.71	4.35	8.16	12.24	19.12	26.08
Std. Dev. by Firm: Best case sales growth rate for the current quarter	98	15.66	11.15	5.01	8.73	12.58	20.41	29.05
Std. Dev. by Firm: Span between worst and best case forecast	98	9.6	7.68	3.76	4.92	7.98	11.73	16.26
Std. Dev. by Firm: Forecast error	85	19.24	14.07	6.08	9.71	15.42	22.82	39.59
Std. Dev. by Firm: Forecast error from random walk model	85	26.41	18.43	6.69	11.55	23.53	37.48	53.41
Std. Dev. by Firm: Forecast error from iid process	85	13.77	9.33	3.35	7.07	11.81	18.38	26.37
Std. Dev. by Firm: Absolute forecast error	85	12.04	8.8	3.46	6.06	10.36	17.02	23.02

Appendix G Additional regressions for the cross section

Table 37: Regressions of time-series averages of absolute forecast errors, forecast errors, and unconditional volatilities on firm characteristics, by firm

Dependent variable:	(1) avg. abs. FE	(2) avg. abs. FE	(3) avg. abs. FE	(4) avg. FE	(5) avg. FE	(6) avg. FE	(7) volatility	(8) volatility	(9) volatility	(10) volatility
Dummy small firms	-0.841			4.866			-2.552			0.232
	(2.546)			(3.003)			(2.336)			(1.504)
Dummy medium-sized firms	-3.437			2.617			-4.062*			0.484
	(2.331)			(2.743)			(2.218)			(1.427)
Dummy large firms	-5.543**			3.545			-6.313***			0.490
	(2.306)			(2.704)			(2.226)			(1.399)
Dummy 'bad' sales growth trend		5.746***			-5.271***			5.964***		1.727**
		(1.044)			(1.200)			(1.222)		(0.732)
Dummy 'good' sales growth trend		4.732***			5.343***			4.434***		1.593**
		(1.464)			(1.522)			(1.191)		(0.636)
Dummy medium low turbulence			3.123***			-0.956			3.347***	3.261***
			(0.525)			(0.752)			(0.151)	(0.205)
Dummy medium high turbulence			5.908***			-1.273			7.081***	6.819***
			(0.512)			(0.862)			(0.182)	(0.250)
Dummy high turbulence			14.49***			-0.827			20.25***	19.78***
			(1.466)			(1.869)			(1.071)	(1.005)
Constant	12.50***	6.852***	3.600***	-3.534	-0.219	0.527	15.55***	8.976***	3.912***	2.895**
	(2.233)	(0.403)	(0.227)	(2.648)	(0.387)	(0.332)	(2.102)	(0.433)	(0.124)	(1.393)
No. of observations	389	389	389	389	389	389	400	400	400	400
No. of firms	389	389	389	389	389	389	400	400	400	400
No. of parameters (excl. intercept)	3	2	3	3	2	3	3	2	3	8
R-squared	0.036	0.077	0.32	0.013	0.13	0.0020	0.029	0.080	0.67	0.68

Notes: avg. abs. FE denotes the time-series average of the firm-level absolute forecast error, avg. FE is the time-series average of the firm-level forecast error, and volatility denotes the unconditional volatility, that is, the time-series standard deviation of firm-level sales growth. Results from OLS regressions. Standard errors in parentheses, clustered by firm; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Appendix H Uncertainty and change by firm characteristics

In this appendix, we show that the V-shaped relationship between sales growth and subjective uncertainty, first shown in Figure 3 in Section 3.1, holds separately, and in a quantitatively similar manner, for all firm-level subgroups: the four firm size groups, the three growth trend groups, and the four turbulence groups. To be specific, the solid lines in the following figures represent nonparametric regression lines. They are the predictions from a kernel-weighted local polynomial regression of degree zero with an Epanechnikov kernel where the bandwidth was selected based on the rule of thumb suggested by Fan and Gijbels (1996). The dashed lines depict the predicted values from a piecewise linear regression of subjective uncertainty on past sales growth, with a break at zero.

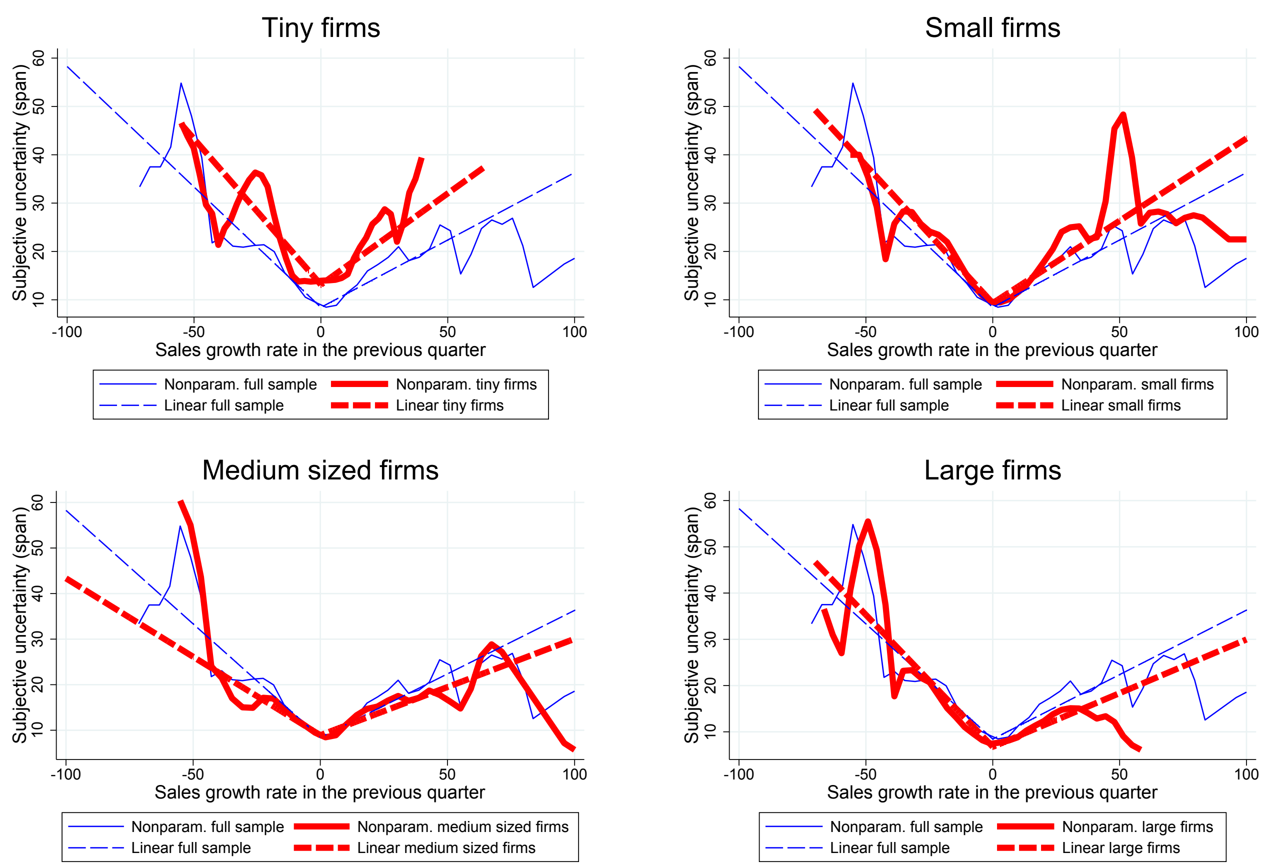


Figure 10: Relationship between subjective uncertainty (span) and sales growth in the previous quarter for four different firm size groups: tiny, small, medium, and large firms (full sample = blue and thin lines, subsample = red and bold lines).

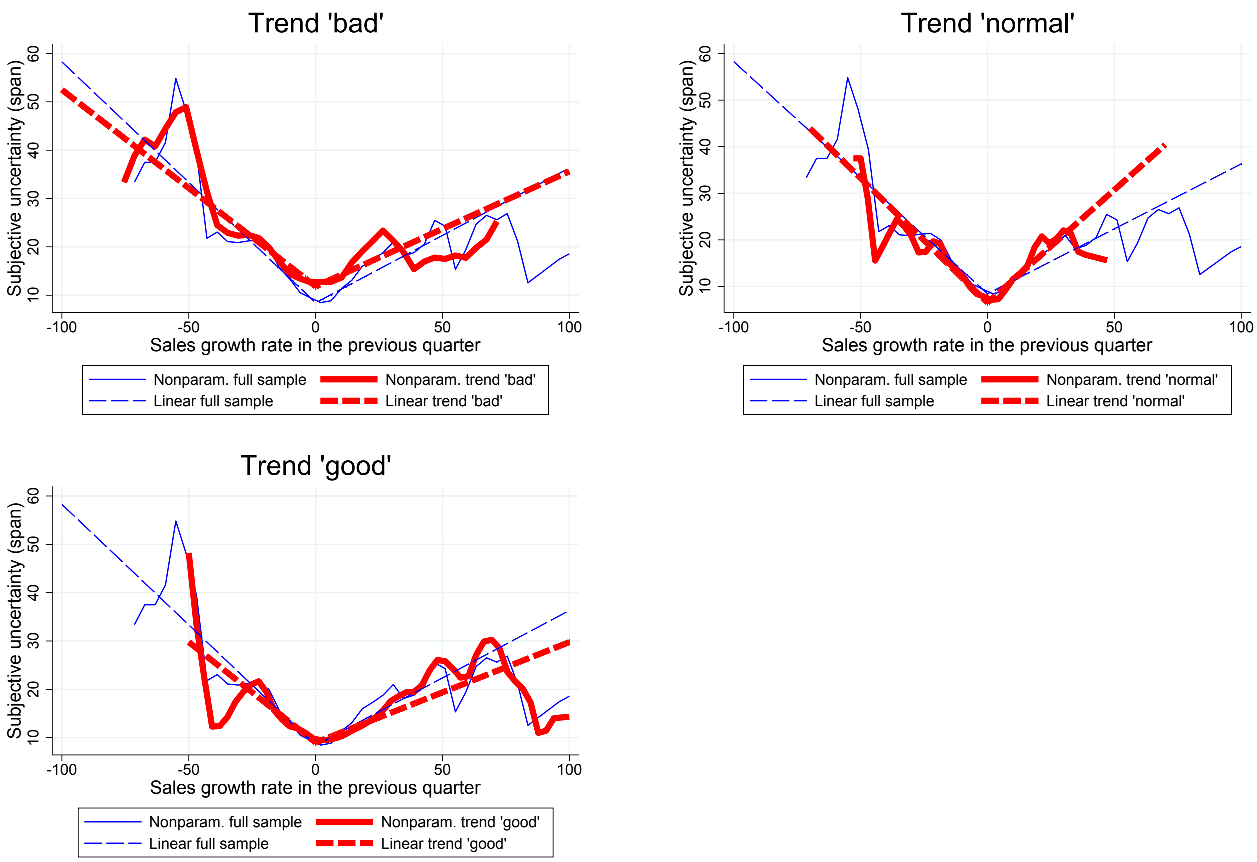


Figure 11: Relationship between subjective uncertainty (span) and sales growth in the previous quarter for three different firm trend growth groups: 'bad', 'normal', and 'good' trend growth (full sample = blue and thin lines, subsample = red and bold lines).

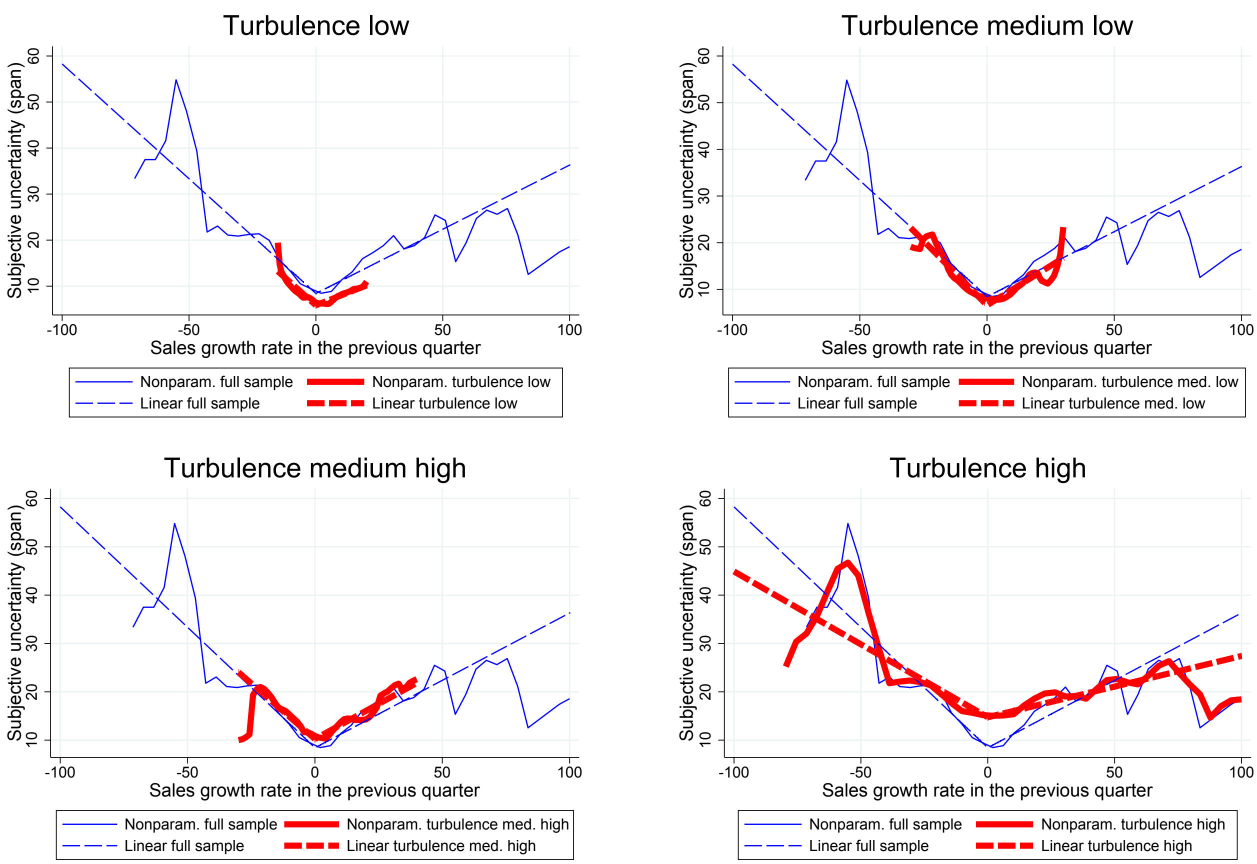


Figure 12: Relationship between subjective uncertainty (span) and sales growth in the previous quarter for four different firm volatility groups: low, medium low, medium high, and high volatility (full sample = blue and thin lines, subsample = red and bold lines).

Appendix I Modelling conditional volatility

In this appendix, we provide details on the two preliminary steps summarized in Section 7, that is, on how we estimate the dynamics of "statistical" uncertainty experienced by firms. The first step, documented in Appendix I.1, is to calculate two sets of (unbiased) forecast errors: one based on the subjective forecast made by firms, corrected for firm-specific bias, and one based on a statistical model forecast. The second step, documented in Appendix I.2, is to estimate dynamic models of conditional volatility for both the bias-adjusted subjective forecast errors and for the statistical forecast errors.

I.1 Bias-adjusted subjective and statistical forecast errors

In order to clean observed forecast errors of firm-specific bias, we estimate regressions of survey-provided forecast errors on fixed characteristics and use the resulting residuals as our cleaned errors. In terms of the representation (5) from Section 3.2, this removes the part of the bias $b(s_t^i, x^i)$ that depends on fixed characteristics x^i. We do this in order to focus on belief *dynamics*, that is, we are interested in the response of span to temporary surprises experienced by firms, not surprises firms routinely experience because they make biased forecasts. It turns out that the results are very similar, had we not removed this average bias.

Specifically, we regress the survey-provided forecast errors on our previously defined size, trend, and turbulence dummies as well as their pairwise interactions. To prevent overfitting, we apply the LASSO estimator to select a subset of relevant regressors.[26] We choose the LASSO tuning parameter τ by minimizing Mallows's C_p statistic as suggested by Efron, Hastie, Johnstone and Tibshiranie (2004). The LASSO then selects 11 predictors. In particular, denote the high and low growth dummies by gd_1 and gd_3, the medium low, medium high and high volatility dummies by vd_2, vd_3, and vd_4, and the small, medium and large size dummies by sd_2, sd_3, and sd_4, respectively. The selected predictors are then gd_1, gd_3, as well as the interactions $sd_2 \cdot gd_1$, $sd_3 \cdot gd_1$, $sd_3 \cdot gd_3$, $sd_2 \cdot vd_2$, $sd_2 \cdot vd_3$, $sd_3 \cdot vd_4$, $sd_4 \cdot vd_4$, $gd_1 \cdot vd_4$, $gd_3 \cdot vd_4$.

To obtain a bias-adjusted forecast error, we compute the OLS residuals of a regression of the forecast error on these predictors. In doing so, we follow the recommendations in Belloni and Chernozhukov (2013) who argue that the LASSO should select the relevant regressors and OLS should estimate the regression coefficients. We note that the distributional properties of these bias-adjusted forecast errors are very similar to those of the raw forecast error directly from the survey data. In addition, the result, documented in Section 6, that the V-shaped nexus between previous-quarter sales growth and subjective uncertainty is robust to including forecast errors, and also the result that previous-quarter sales growth drive out forecast errors, both hold when we use these bias-adjusted forecast errors.

To provide an alternative benchmark for firm-level forecast errors, we construct a set of statistical forecast errors by using statistical forecasting models an econometrician would pre-

[26]The LASSO is a standard shrinkage estimator popular in "big data" analysis as it recovers the correct (sparse) model with high probability. By requiring that the L_1 norm of the coefficient vector does not exceed a certain threshold, say, τ, the LASSO restricts many coefficients to zero and thus helps to balance the bias-variance tradeoff. This is why the LASSO and related estimators are widely applied in data-rich environments.

sumably consider. In particular, we regress sales growth on its own lag as well as size, growth, and turbulence dummies. We allow for an asymmetric response to past growth, as in our span regressions. We also note that, since trend and turbulence are defined using sample moments, they are, strictly speaking, not part of the information set of a firm. At the same time, firms have longer samples than ours that speak to their trend growth and volatility. Our assumption here is that trend and turbulence reflect medium-term prospects known to firms.

The regression coefficients of these various forecasting models are reported in Table 38. The specifications in columns (1)-(3) allow for an asymmetric effect of past sales growth, whereas the specifications in columns (4)-(6) restrict this effect to be symmetric. The forecast errors from all six specifications are highly correlated, with correlation coefficients at 0.93 or above. Since the model selection criteria AIC and BIC favor specification (1), we report, in what follows, the results based on that specification.

Table 38: Regressions of sales growth on past sales growth and firm characteristics

Dependent variable: sales growth in quarter t	(1)	(2)	(3)	(4)	(5)	(6)
Negative sales growth in quarter $t-1$	-0.269*	-0.141	-0.130			
	(0.141)	(0.145)	(0.128)			
Positive sales growth in quarter $t-1$	-0.0357	0.0392	0.0495			
	(0.0907)	(0.0978)	(0.0856)			
Sales growth in quarter $t-1$				-0.132***	-0.0365	-0.0247
				(0.0414)	(0.0455)	(0.0462)
Dummy small firms	5.314***	7.798***		5.298***	7.776***	
	(1.815)	(2.255)		(1.747)	(2.192)	
Dummy medium-sized firms	4.996***	7.408***		5.082***	7.432***	
	(1.669)	(2.136)		(1.607)	(2.073)	
Dummy large firms	5.000***	7.228***		5.013***	7.159***	
	(1.689)	(2.127)		(1.632)	(2.057)	
Dummy 'bad' sales growth trend	-7.143***			-6.726***		
	(0.967)			(0.936)		
Dummy 'good' sales growth trend	8.005***			8.253***		
	(1.095)			(1.112)		
Dummy medium low turbulence	-1.717***	-2.068**		-1.434***	-1.827*	
	(0.572)	(0.959)		(0.500)	(0.929)	
Dummy medium high turbulence	0.427	-0.882		0.973	-0.411	
	(0.833)	(1.117)		(0.642)	(0.968)	
Dummy high turbulence	-0.143	-0.274		1.343	0.954	
	(1.561)	(1.691)		(1.149)	(1.384)	
Intercept	-3.243**	-5.059**	1.164	-2.881*	-4.663**	2.015***
	(1.610)	(2.092)	(0.811)	(1.534)	(2.011)	(0.439)
No. of obs.	1,329	1,329	1,329	1,329	1,329	1,329
No. of firms	292	292	292	292	292	292
R-squared	0.12	0.021	0.0047	0.12	0.018	0.001
AIC	10696.7	10837.8	10848.1	10702.2	10839.9	10851.5
BIC	10753.8	10884.5	10863.7	10754.1	10881.5	10861.9

Notes: results from OLS regressions. They are based on the *baseline sample forecast* as defined in Table 11 in Appendix C which includes all quarter-firm observations for which a forecast is available but not necessarily a span, that is, 1,664 observations. In addition, because we want to compute forecast errors with these statistical forecasts and use them in dynamic models, we have to eliminate isolated forecast errors that have no forecast error surrounding them and thus end up with 1,329 observations. Standard errors in parentheses, clustered by firm. * p < 0.10, ** p < 0.05, *** p < 0.01.

I.2 Estimating conditional volatility

We now proceed to estimate conditional volatility models on both types of forecast errors: the biased-corrected subjective forecast errors and the statistical forecast errors. We have operationalized subjective uncertainty with span, the difference between best and worst case scenarios. A natural "objective" counterpart would be the length of a forecast interval constructed by the econometrician, for example, the difference between an upper and lower quantile of the conditional distribution of forecast errors. In the broad class of distributions which belong to the location-scale family that forecast interval length is simply a multiple of the distribution's standard deviation. We, therefore, choose the conditional standard deviation of forecast errors as our measure of "objective" uncertainty. We select and estimate a conditional volatility model that optimally describes the data as indicated by information criteria.

Let e^i_{t+1} be the (bias-adjusted) forecast error of firm i in the quarter beginning in t, and denote its conditional standard deviation by $\hat{\sigma}^i_t$, which is the econometrician's implementation of $\hat{\sigma}\left(s^i_t, x^i\right)$ from equation (5). Our choice of functional form mirrors our analysis of subjective uncertainty: We write $\hat{\sigma}^i_t$ as a function of past growth and fixed firm characteristics, summarized in a vector w^i_t, as well as a function of past forecast errors and $\hat{\sigma}^i_{t-1}$. We thus use a restricted version of the power GARCH model. Whereas the unrestricted power GARCH model conditions $(\hat{\sigma}^i_t)^p$ on past information, $\mathcal{I}^i_t$, where p is a power coefficient to be estimated, we impose the restriction $p = 1$ to model the conditional standard deviation.

Our conditional volatility model then has the general form:[27]

$$e^i_{t+1} = \hat{\sigma}^i_t \hat{\varepsilon}^i_{t+1}, \qquad \hat{\varepsilon}^i_{t+1} | \mathcal{I}^i_t \sim N(0,1) \tag{8}$$

with a conditional standard deviation equation:

$$\hat{\sigma}^i_t = \exp(\beta_0 + \beta_1' w^i_t) + \alpha_1(|e^i_t| + \gamma e^i_t) + \alpha_2 \hat{\sigma}^i_{t-1}. \tag{9}$$

The conditional volatility equation (9) allows, in some specifications, for an asymmetric effect of the past absolute forecast error measured by the coefficient γ, because asymmetry was found to be relevant in explaining subjective uncertainty.[28] The conditional volatility equation also contains two types of explanatory variables through an exponential link function which ensures that conditional volatilities are always positive. The first type consists of size, trend, and turbulence dummies which are essentially time-invariant and thus control for different levels of conditional volatility for subgroups of firms. Our analysis in the main text indicated that these dummies are sufficient to capture the bulk of time-invariant heterogeneity in subjective uncertainty. The second type includes positive and negative sales growth in the previous quarter which we found to be relevant to explain the dynamics of subjective uncertainty.

[27] In the estimation, the mean equation (8) includes an intercept, μ, to account for a nonzero sample mean that arises because we apply the bias adjustment of the forecast error to all 1,329 observations but have to estimate the volatility model on an effective sample of those 949 observations for which a lag is available.

[28] The empirical unconditional distribution of the bias-adjusted subjective forecast errors is essentially symmetric with a sample skewness of 0.1. A test of the null hypothesis that the population skewness is zero cannot be rejected (p-value of 0.2). Taking both the lack of skewness and the fact that asymmetry was relevant in explaining subjective uncertainty on balance, we, therefore, experiment with symmetric and asymmetric specifications.

To find a reliable parsimonious specification, we estimate several restricted versions of (8)-(9) by maximum likelihood. Specification (1) adds no additional control variables ($\beta_1 = 0$), specifications (2)-(4) allow, respectively, only for size, trend, and turbulence dummies, specification (5) allows for only positive and negative sales growth rate in the previous quarter, and (6) adds all variables together. All specifications are estimated either assuming symmetric effects of past forecast errors ($\gamma = 0$) or allowing for asymmetry (γ unrestricted).

To select among these specifications, we use two information criteria, AIC and BIC, which are commonly used in applied work with GARCH models. In finite samples, the BIC typically favors overly sparse models, while the AIC picks models with a more generous number of parameters. Hence, the models chosen by AIC and BIC may be thought of giving upper and lower bounds in terms of richness of parametrization.

Table 39: Model selection criteria for different specifications of the conditional volatility model

	Symmetric ($\gamma = 0$)			Asymmetric ($\gamma \neq 0$)		
Specification	k	AIC	BIC	k	AIC	BIC
(1) no controls	4	7,256.39	7,275.81	5	7,253.13	7,277.41
(2) only size dummies	7	7,249.95	7,283.94	8	7,248.10	7,286.94
(3) only growth trend dummies	6	7,173.30	7,202.43	7	7,174.46	7,208.45
(4) only turbulence dummies	7	6,901.40	6,935.38	8	6,903.38	6,942.22
(5) only sales growth rate	6	7,106.90	7,136.03	7	7,104.63	7,138.62
(6) all controls	14	6,871.59	6,939.57	15	6,870.96	6,943.79

Notes: k denotes the number of parameters. All specifications are estimated by maximum likelihood using 949 observations and 380 pre-sample observations on which we condition as explained in Appendix C. Model selection is for the bias-corrected subjective forecast errors.

The selection results for the bias-corrected subjective forecast errors reported in Table 39 suggest that the inclusion of turbulence dummies, whether by themselves in specification (4) or jointly with the other control variables in specification (6), is essential for model fit: All other specifications generate much larger information criteria. Deciding between specifications (4) and (6) is less obvious. In both the symmetric and the asymmetric case, the AIC favors the inclusion of all controls while the BIC picks the turbulence dummies alone. However, the differences in terms of AIC are large (29.81 and 32.42) while the differences in terms of BIC are small (4.19 and 1.57). Given that the BIC tends to select overly parsimonious models and based on the classification of Kass and Raftery (1995) that only BIC differences of more than six are "strong", we prefer, on balance, specification (6).

Since neither information criterion gives us clear guidance whether to prefer the symmetric or the asymmetric specification, we report, in Table 40, the coefficient estimates for both. It turns out that the asymmetry parameter γ is not statistically different from zero while the estimates of the other coefficients are largely unaffected by restricting it to zero. We thus conclude that the symmetric specification (6) is a sufficient description of the conditional volatility process that drives the data.

For the statistical forecast errors we fit the same symmetric and asymmetric volatility models as for the firms' subjective forecast errors, see columns (3) and (4) of Table 40. Again, the asymmetry parameter is not significantly different from zero, and restricting it to zero leaves the other parameter estimates essentially unchanged. Therefore, we take again the symmetric specification as a sufficient description of the conditional volatility process that characterizes the statistical forecast errors.

Table 40: Conditional volatility equation (9) estimated by maximum likelihood

Dependent variable:	Firms' forecast errors		Statistical forecast errors	
	(1)	(2)	(3)	(4)
Mean equation				
Intercept (μ)	0.298	0.291	0.171	0.122
	(0.252)	(0.251)	(0.270)	(0.281)
Volatility equation: baseline parameters				
Lagged absolute FE (α_1)	0.0852*	0.102*	0.00830	0.00918
	(0.0511)	(0.0538)	(0.0682)	(0.0579)
Lagged volatility (α_2)	0.235***	0.215***	0.236**	0.229***
	(0.0874)	(0.0799)	(0.0921)	(0.0824)
Asymmetry (γ)	0	0.478	0	3.359
	(.)	(0.317)	(.)	(21.85)
Volatility equation: parameters of predetermined regressors				
Dummy medium low volatility	0.504***	0.506***	0.518***	0.517***
	(0.0867)	(0.0886)	(0.0725)	(0.0708)
Dummy medium high volatility	0.794***	0.797***	0.873***	0.879***
	(0.0873)	(0.0874)	(0.0684)	(0.0674)
Dummy high volatility	1.336***	1.322***	1.519***	1.520***
	(0.110)	(0.111)	(0.101)	(0.0964)
Negative sales growth in $t-1$	-0.0286***	-0.0309***	-0.0259***	-0.0278***
	(0.00910)	(0.00879)	(0.00792)	(0.00801)
Positive sales growth in $t-1$	0.0131**	0.0107*	0.00710	0.00468
	(0.00515)	(0.00576)	(0.00522)	(0.00605)
Dummy small firms	-0.139	-0.138	-0.142	-0.127
	(0.118)	(0.119)	(0.0867)	(0.0884)
Dummy medium-sized firms	-0.223**	-0.230**	-0.182**	-0.170**
	(0.105)	(0.105)	(0.0744)	(0.0766)
Dummy large firms	-0.248**	-0.269**	-0.170**	-0.156*
	(0.111)	(0.111)	(0.0822)	(0.0871)
Dummy 'bad' sales growth trend	0.194**	0.167*	-0.0281	-0.0456
	(0.0882)	(0.0858)	(0.0652)	(0.0673)
Dummy 'good' sales growth trend	0.199**	0.209**	0.0648	0.0818
	(0.0948)	(0.0958)	(0.0815)	(0.0857)
Intercept (β_0)	1.101***	1.125***	1.204***	1.199***
	(0.180)	(0.167)	(0.151)	(0.146)
Number of observations	949	949	949	949
Number of firms	292	292	292	292

Notes: All specifications are estimated by maximum likelihood using 949 observations and 380 pre-sample observations on which we condition as explained in Appendix C. Standard errors in parentheses, clustered by firm. * p < 0.10, ** p < 0.05, *** p < 0.01.