Distributed Mixed Voltage Angle and Frequency Droop Control of Microgrid Interconnections with Loss of Distribution-PMU Measurements

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Abstract—Recent advances in distribution-level phasor measurement unit (D-PMU) technology have enabled the use of voltage phase angle measurements for direct load sharing control in distribution-level microgrid interconnections with high penetration of renewable distributed energy resources (DERs). In particular, D-PMU enabled voltage angle droop control has the potential to enhance stability and transient performance in such microgrid interconnections. However, these angle droop control designs are vulnerable to D-PMU angle measurement losses that frequently occur due to the unavailability of a GPS signal for synchronization. In the event of such measurement losses, angle droop controlled microgrid interconnections may suffer from poor performance and potentially lose stability. In this paper, we propose a novel distributed mixed voltage angle and frequency droop control (D-MAFD) framework to improve the reliability of angle droop controlled microgrid interconnections, thereby promoting the application of distribution-level PMUs in real-time control. In this framework, when the D-PMU phase angle measurement is lost at a microgrid, conventional frequency droop control is temporarily used for primary control in place of angle droop control to guarantee stability. We model the microgrid interconnection with this primary control architecture as a nonlinear switched system and design distributed secondary controllers to guarantee transient stability of the network. Furthermore, we incorporate provable performance guarantees such as robustness to generation-load mismatch and network topology changes in the distributed control design. We demonstrate the performance of this control framework by simulation on a test 123-feeder distribution network.

Index Terms—Phasor measurement units, microgrids, interconnected system stability, distribution systems, droop control.

I. INTRODUCTION

PHASOR measurement units (PMUs) have been extensively used in real-time wide-area monitoring, protection and control (WAMPAC) applications at the transmission level. However, in traditional distribution networks with one-way power flows and no active loads, real-time monitoring and control using phasor measurements has not been necessary for reliable operation [1]. Further, small angle deviations and consequently, poor signal-to-noise ratios, make real-time estimation of voltage phase angles at the distribution level an extremely challenging problem. Therefore, applications of distribution-level PMUs (D-PMUs) have so far been confined to the substation-level, with angle references being synchronized with the transmission network [2]. However, in future distribution systems, new architectures like microgrids with large-scale integration of renewable distributed energy resources (DERs) and flexible loads, as well as new economic paradigms like demand response, will require extensive real-time monitoring and control. In this context, D-PMUs (such as the µPMU) that can provide accurate measurements of small angle deviations (≈0.01°) and voltage magnitude deviations (≈0.0001 p.u.) with high sampling rates (≈120s⁻¹) have recently been developed [3], and are expected to be critical components of future power distribution infrastructure [1][4].

In particular, consider the problem of ensuring stability and reliability in distribution networks comprised of interconnected microgrids. Typically, such microgrid interconnections are controlled in a hierarchical manner with three layers of control [5][6] - (i) a primary control layer to ensure proper load sharing between microgrids, (ii) a secondary control layer to ensure system stability by eliminating frequency and voltage deviations, and (iii) a tertiary control layer to provide power reference set points for individual microgrids. The primary control layer commonly comprises of frequency droop and voltage droop controllers to regulate the real and reactive power outputs respectively of each microgrid at the point of common coupling (PCC). These droop control characteristics are implemented artificially using voltage-source inverter (VSI) interfaces that are designed such that individual microgrids emulate the dynamics of synchronous generators.

However, the use of frequency droop controllers in networks with a large penetration of low inertia VSI-interfaced microgrids has been demonstrated to result in numerous issues including chattering [7], loss of synchronization, and undesirable frequency deviations resulting from the trade-off between active power sharing and frequency accuracy [8]. With the availability of highly accurate D-PMUs, angle droop controllers that directly use the voltage phase angle deviation measurements at the PCC for active power sharing have been proposed to replace frequency droop controllers in the primary layer. These angle droop designs have been demonstrated to result in increased stability, smaller frequency deviations and faster dynamic response [9]–[13].

A key bottleneck in the widespread adoption of D-PMU based control designs for microgrids is the reliability of D-PMU voltage phase angle measurements. Phase angle measurements from D-PMUs require a global positioning system...
In this paper, we consider a network of interconnected microgrids operating under the MAFD framework. However, in contrast to [20], we propose a novel distributed secondary control design, where the secondary controller at every microgrid uses only local measurements from that microgrid and its immediate neighbors to guarantee network-wide stability of the microgrid interconnection under D-PMU measurement losses. We refer to this control architecture with MAFD primary control and the proposed distributed secondary control as the distributed MAFD (D-MAFD) framework.

The D-MAFD framework exploits the inherent dynamic coupling in the microgrid interconnection (represented by the power flow Jacobian) to allow for a distributed implementation of the secondary controller. The distributed design is advantageous as compared to the centralized secondary control design in [20], since it significantly reduces communication overhead costs in large-scale microgrid interconnections, and is more robust due to the lack of reliance on a single central controller. Additionally, in contrast to [20], the D-MAFD design incorporates provable performance guarantees including robustness to disturbances and network topology changes, with view of increasing the role of D-PMU measurements in islanding operations as well as plug-and-play architectures in future microgrid interconnections. Finally, we show through case studies that the proposed D-MAFD framework significantly enhances system stability in D-PMU measurement loss scenarios under conditions of generation-load mismatches and is robust to network topology changes induced by faults.

In summary, the D-MAFD framework will enhance the reliability, resilience and performance of D-PMU based control designs in microgrid interconnections, thereby promoting more widespread adoption of distribution-level PMUs for real-time control applications.

Notation: Let \( \mathbb{R} \) and \( \mathbb{R}^n \) denote the sets of real numbers and \( n \)-dimensional real vectors respectively. The \((i,j)\)-th element of a matrix \( A \in \mathbb{R}^{m \times n} \) is denoted by \( A_{ij} \) and the transpose is denoted by \( A' \in \mathbb{R}^{n \times m} \). The identity matrix is represented by \( I \), with dimensions clear from the context. A symmetric positive (negative) definite matrix \( P \in \mathbb{R}^{n \times n} \) is represented by \( P > 0 \) (\( P < 0 \)).
II. MIXED VOLTAGE ANGLE AND FREQUENCY DROOP
CONTROL (MAFD) MODEL

Consider a network of $N$ microgrids where each microgrid is connected to the network at the PCC. Define $N_i$ to be the neighbor set of the $i$-th microgrid, that is, the set of all microgrids to which the $i$-th microgrid is directly connected in the network. For convenience of notation, we also include $i$ in this set. Considering the coupled AC power flow model, the real and reactive power injections $P_{i}^{\text{inj}}(t)$ and $Q_{i}^{\text{inj}}(t)$, at the $i$-th microgrid at time $t$ are given by

$$
P_{i}^{\text{inj}}(t) = \sum_{k \in N_i} V_{j}^{t} V_{k}^{t} |Y_{jk}| \sin(\delta_{jk}(t) + \pi/2 - \angle Y_{jk})$$

$$Q_{i}^{\text{inj}}(t) = \sum_{k \in N_i} V_{j}^{t} V_{k}^{t} |Y_{jk}| \sin(\delta_{jk}(t) - \angle Y_{jk}),$$

where $V_{j}(t)$ and $\delta_{j}(t)$ are the voltage magnitude and phase angle at PCC respectively, $\delta_{jk}(t) = \delta_{j}(t) - \delta_{k}(t)$, and $Y_{jk}$ is the complex admittance of the line between the PCC buses of microgrids $j$ and $k$.

The primary control layer for every microgrid comprises of an angle droop and a voltage droop control loop, which regulate the real and reactive power injections of the microgrid respectively to track desired reference values $V_{i}^{\text{ref}}$ and $\delta_{i}^{\text{ref}}$ of the voltage magnitude and phase angle at the PCC respectively.

As mentioned in Section I, the use of angle droop primary control to directly regulate real power has several advantages such as increased frequency stability and power sharing accuracy [9]. The implementation of primary angle droop control schemes requires real-time angle measurements from D-PMUs placed at the microgrid PCCs, which in turn require a GPS signal for synchronization. However, since GPS signals are frequently lost, microgrid interconnections that primarily use angle droop control will suffer from poor performance and potential instability [20].

To address this issue, when D-PMU angle measurements are lost at certain microgrids, we employ a mixed angle and frequency droop control (MAFD) framework [20] for primary control, where classical frequency droop control is used in place of the angle droop control to temporarily regulate real power at those microgrids until D-PMU measurements are restored. Therefore, at any given time, some microgrids may operate with angle droop control while others operate with frequency droop control depending on the availability of D-PMU measurements. At every time $t$, each microgrid in the MAFD framework operates in one of two modes, denoted by $\sigma_{i}(t) \in \{1, 2\}$ - (i) angle droop control mode ($\sigma_{i}(t) = 1$), when real-time angle measurements are available from the D-PMU at that microgrid, and (ii) frequency droop control mode ($\sigma_{i}(t) = 2$), when D-PMU voltage angle measurements are lost or corrupted at that microgrid due to GPS signal loss or sensor malfunction (Fig. 1). The dynamics of the $i$-th microgrid in each of these modes is described as follows.

**Angle Droop Control Mode, $\sigma_{i}(t) = 1$:** In this mode, D-PMU angle measurements are available, and the microgrid operates with angle and voltage droop control laws given by

$$J_{\delta_{i}} \Delta \delta_{i}(t) = -D_{\delta_{i}} \Delta \delta_{i}(t) + \Delta P_{\text{ext}}^{i}(t) - \Delta P_{i}^{\text{inj}}(t)$$

$$J_{\delta_{i}} \Delta \dot{V}_{i}(t) = -D_{\delta_{i}} \dot{V}_{i}(t) + \Delta Q_{\text{ext}}^{i}(t) - \Delta Q_{i}^{\text{inj}}(t),$$

where $\Delta V_{i}(t) = V_{i}(t) - V_{i}^{\text{ref}}$ and $\Delta \delta_{i}(t) = \delta_{i}(t) - \delta_{i}^{\text{ref}}$, are the deviations of the PCC voltage magnitude from their reference values, $\Delta P_{i}^{\text{inj}}(t) = P_{i}^{\text{inj}}(t) - P_{i}^{\text{inj}}^{\text{ref}}$ and $\Delta Q_{i}^{\text{inj}}(t) = Q_{i}^{\text{inj}}(t) - Q_{i}^{\text{inj}}^{\text{ref}}$ are the deviations of the real and reactive power injections $P_{i}^{\text{inj}}(t)$ and $Q_{i}^{\text{inj}}(t)$ from their nominal values $P_{i}^{\text{inj}}^{\text{ref}}$ and $Q_{i}^{\text{inj}}^{\text{ref}}$ respectively, and $\Delta P_{\text{ext}}^{i}(t)$ and $\Delta Q_{\text{ext}}^{i}(t)$ are the generation-load mismatches at the $i$-th microgrid. The droop coefficients $J_{\delta_{i}}$, $D_{\delta_{i}}$, $J_{\delta_{i}}$, and $D_{\delta_{i}}$ can be implemented by programming the VSI interface at the PCC [11]. Additionally, the dynamics of the frequency error in the angle droop control mode is propagated as

$$\Delta \omega_{i}(t) = -\frac{D_{\delta_{i}}}{J_{\delta_{i}}} \dot{\delta}_{i}(t),$$

$$-\frac{1}{J_{\delta_{i}}} \dot{\Delta \delta_{i}}(t),$$

where $\Delta \omega_{i}(t) = \omega_{i}(t) - \omega_{i}^{\text{ref}}$ is the deviation of the frequency of the $i$-th microgrid from its reference value $\omega_{i}^{\text{ref}}$. The derivative $\Delta P_{i}^{\text{inj}}(t)$ is computed from (1).

**Frequency Droop Control Mode, $\sigma_{i}(t) = 2$:** When D-PMU angle measurements are not available, a frequency droop control law is used to regulate the real power, and the dynamics of the microgrid in this mode are given by

$$\Delta \delta_{i}(t) = \Delta \omega_{i}(t) \Delta P_{\text{ext}}(t)$$

$$J_{\delta_{i}} \Delta \omega_{i}(t) = -D_{\omega_{i}} \Delta \omega_{i}(t) + \Delta P_{\text{ext}}^{i}(t) - \Delta P_{i}^{\text{inj}}(t)$$

$$J_{\delta_{i}} \Delta \dot{V}_{i}(t) = -D_{\delta_{i}} \dot{V}_{i}(t) + \Delta Q_{\text{ext}}^{i}(t) - \Delta Q_{i}^{\text{inj}}(t),$$

where $J_{\delta_{i}}$ and $D_{\omega_{i}}$ are the frequency droop coefficients.

We now define the state, input and disturbance vectors of the $i$-th microgrid to be $x_{i}(t) = [\Delta \delta_{i}(t), \Delta \omega_{i}(t), \Delta V_{i}(t), \Delta P_{i}^{\text{inj}}(t), \Delta Q_{i}^{\text{inj}}(t), \Delta P_{\text{ext}}^{i}(t), \Delta Q_{\text{ext}}^{i}(t)]$, $u_{i}(t) = [\Delta P_{i}^{\text{inj}}(t), \Delta P_{\text{ext}}^{i}(t), \Delta Q_{\text{ext}}^{i}(t)]$, and $w_{i}(t) = [\Delta \omega_{i}(t), \Delta V_{i}(t), \Delta P_{\text{ext}}^{i}(t), \Delta Q_{\text{ext}}^{i}(t)]$ respectively. The output vector of the $i$-th microgrid is

$$y_{i}(t) = g_{\sigma_{i}(t)}^{i}(x_{i}(t), u_{i}(t)),$$

where $g_{\sigma_{i}(t)}^{i}(x_{i}(t), u_{i}(t))$ is the state function for $\sigma_{i}(t) = 1$ and $g_{\sigma_{i}(t)}^{i}(x_{i}(t), u_{i}(t))$ is the state function for $\sigma_{i}(t) = 2$. The dynamics of the $i$-th microgrid in the MAFD primary control framework can then be written as a nonlinear switched system

$$\dot{x}_{i}(t) = f_{\sigma_{i}(t)}^{i}(x_{i}(t), u_{i}(t), w_{i}(t))$$

$$u_{i}(t) = h^{i}(x_{i}(t)),$$

where the dynamics $f_{\sigma_{i}(t)}^{i}(x_{i}(t), u_{i}(t), w_{i}(t))$ in the angle droop control mode and $f_{\sigma_{i}(t)}^{i}(x_{i}(t), u_{i}(t), w_{i}(t))$ in the frequency droop control mode are defined by (2)-(4), the dynamics $f_{\sigma_{i}(t)}^{i}(x_{i}(t), u_{i}(t), w_{i}(t))$ in the frequency droop control mode are defined by (5)-(7), and $h^{i}(x_{i}(t))$ is given by the power flow model (1) independent of the switching mode $\sigma_{i}$. Define the augmented state vector for the microgrid interconnection as $x(t) = [x_{1}(t), x_{2}(t), \ldots, x_{N}(t)]$. Similarly, define the augmented input, disturbance and output vectors obtained by stacking the inputs, disturbances and outputs of all microgrids to be $u(t)$, $w(t)$ and $y(t)$ respectively. Finally, define the augmented switching vector $\sigma(t) = [\sigma_{1}(t), \ldots, \sigma_{N}(t)]$, where every element can take values of 1 or 2, indicating the availability or loss of D-PMU angle measurement at that microgrid. Let $\Sigma$ denote the set of all possible values of $\sigma(t)$.
switching vector. We now write the dynamics of the microgrid interconnection with MAFD as the nonlinear switched system

\[
\begin{align*}
\dot{x}(t) &= f_{\sigma(t)}(x(t), u(t), w(t)) \\
y(t) &= g_{\sigma(t)}(x(t), w(t)) \\
u(t) &= h(x(t)),
\end{align*}
\]

\(f_{\sigma(t)} = \begin{bmatrix} f_{\sigma_1(t)} \\ \vdots \\ f_{\sigma_N(t)} \end{bmatrix}, \\ g_{\sigma(t)} = \begin{bmatrix} g_{1\sigma(t)} \\ \vdots \\ g_{N\sigma(t)} \end{bmatrix}, \\ h = \begin{bmatrix} h^1 \\ \vdots \\ h^N \end{bmatrix}.
\]

In order to design a secondary controller that guarantees the stability of the microgrid interconnection with MAFD, we linearize the system model (10) around the origin to obtain a linearized switched system model

\[
\begin{align*}
\dot{x}(t) &= A_{\sigma(t)} x(t) + B_{\sigma(t)}^1 u(t) + B_{\sigma(t)}^2 w(t) \\
y(t) &= C_{\sigma(t)} x(t) + D_{\sigma(t)} w(t) \\
u(t) &= H x(t),
\end{align*}
\]

\[A_j = \frac{\partial f_j}{\partial x} \bigg|_{x=0, u=0}, \quad B_j^{(1)} = \frac{\partial f_j}{\partial u} \bigg|_{x=0, w=0}, \quad B_j^{(2)} = \frac{\partial f_j}{\partial w} \bigg|_{x=0, w=0},
\]

\[C_j = \frac{\partial g_j}{\partial x} \bigg|_{x=0, w=0}, \quad D_j = \frac{\partial g_j}{\partial w} \bigg|_{x=0, w=0}, \quad H = \begin{bmatrix} \frac{\partial h_1}{\partial x_1} & \cdots & \frac{\partial h_1}{\partial x_N} \\ \vdots & \ddots & \vdots \\ \frac{\partial h_N}{\partial x_1} & \cdots & \frac{\partial h_N}{\partial x_N} \end{bmatrix} x=0
\]

\[
\frac{\partial u_{ij}}{\partial x_k} = \begin{bmatrix} \frac{\partial P_{ij}}{\partial x_k} & \cdots & \frac{\partial P_{ij}}{\partial x_N} \\ \frac{\partial Q_{ij}}{\partial x_k} & \cdots & \frac{\partial Q_{ij}}{\partial x_N} \end{bmatrix}, \quad i, k \in \{1, \cdots, N\}.
\]

Note that \(H\) is the power flow Jacobian pertaining to the linearization of (1).

The MAFD primary control architecture allows for indirect control of the real power sharing in the microgrid interconnection in a decentralized manner even when D-PMU angle measurements are lost. With this architecture, a secondary controller must then be designed to eliminate the voltage and angle errors that arise due to open-loop droop control. In the following section, we propose a distributed secondary control design that uses only local information at each microgrid to regulate angle and voltage deviations.

### III. D-MAFD Secondary Control Design

Consider the microgrid interconnection with MAFD primary control as shown in Fig. 1. Given the dynamics in (10) and its linear approximation (11), we would like to design a secondary controller that eliminates voltage and angle deviations in the microgrid interconnection and guarantees stability in scenarios of D-PMU measurement losses. Typically, secondary control designs are centralized, that is, D-PMU measurements from the entire network are used by the microgrid central controller (MGCC) to eliminate voltage and angle deviations caused by the local primary controllers. However, MGCC based designs require more communication infrastructure and are less robust due to reliance on the single central controller [24]. Recently, distributed secondary control designs have also been proposed in the context of microgrid interconnections, wherein local control actions are taken at the individual microgrid level using local information communicated from other microgrids [25], [26].

In this section, we develop a distributed secondary control design for microgrid interconnections operating in the MAFD framework, where every microgrid locally determines its secondary control actions using information only from its immediate neighbors. The microgrid interconnection with MAFD primary control and distributed secondary control as shown in Fig. 1 is termed as the distributed MAFD (D-MAFD) framework. For this network, we would like to design a secondary output-feedback control law \(\tilde{u}(t) = K(\sigma(t))y(t)\), \(K_j \in \mathbb{R}^{2N \times 2N}, j \in \Sigma\), such that the microgrid interconnection (10) with \(u(t) \mapsto u(t) + \tilde{u}(t)\) is \(L_2\)-stable with respect to disturbances \(w(t)\) even when D-PMU angle measurements are unavailable in any number of microgrids in the network, that is, (10) switches arbitrarily between angle droop and frequency droop primary control modes of individual microgrids. Intuitively, the \(L_2\) stability property guarantees that the system outputs (angles, frequencies and voltages) are bounded for finite disturbances.

Using the concept of dissipativity, it can be shown [20] that a secondary controller that guarantees \(L_2\) stability of the microgrid interconnection with MAFD primary control subject to disturbances \(w(t)\) can be designed by solving linear matrix inequalities as follows.

**Proposition 1.** If there exists symmetric positive definite matrix \(Q_j > 0\), negative definite matrix \(R_j < 0\), and matrices \(U_j, V_j, S_j\) and \(R_j\) of appropriate dimensions such that (12) is satisfied for every switching vector \(j \in \Sigma\), then the control law \(u(t) \mapsto u(t) + \tilde{u}(t)\) where \(\tilde{u}(t) = K(\sigma(t))y(t)\) with

\[
K_j = V_j^{-1} U_j, \quad \forall j \in \Sigma,
\]

guarantees that the closed loop system with MAFD dynamics (10) is \(L_2\)-stable with respect to any disturbance \(w(t)\).

Using Proposition 1, a secondary controller can be designed for the microgrid interconnection with MAFD to guarantee stability even when D-PMU angle measurements are lost. However, the secondary controller thus obtained is centralized,

\[
M_j = \begin{bmatrix}
-P(A_j + B_j^{(1)} H) - (A_j + B_j^{(1)} H)P - B_j^{(1)} U_j C_j - C_j^t U_j^t B_j^{(1)'}, & -P B_j^{(2)} - B_j^{(1)} U_j D_j + C_j^t S_j - C_j^t Q_j^{1/2}
\end{bmatrix} > 0 \quad (12a)
\]

\[
P B_j^{(1)} = B_j^{(1)} V_j, \quad Q_j^{1/2} Q_j^{1/2} = -Q_j \quad (12b)
\]
that is, its implementation requires an MGCC that uses angle or frequency measurements from all microgrids and dispatches the control law to each microgrid. In the following subsection, we present an approach to synthesize a distributed secondary controller that can be locally implemented at each microgrid by including additional design constraints in Proposition 1.

A. Distributed Secondary Control Synthesis

In order to obtain a distributed secondary controller for the microgrid interconnection with MAFD, we impose a sparsity constraint on the structure of the controller matrices $K_j$, $j \in \Sigma$. We require that each microgrid $i$ only uses measurements from its immediate neighbors $N_i$. Therefore, the sparsity structure of the controller $K_j$ must be the same as that of the network Jacobian matrix $H$, that is,

$$K_j = V_j^{-1}U_j \in S_H,$$  

where $S_H$ is the set of all matrices with the same sparsity structure as the Jacobian matrix $H$. However, imposing (14) adds a nonlinear constraint that makes the design equations (12) difficult to solve using any standard matrix inequality solvers. It is therefore desirable to convert the structural constraint (14) to a form that maintains the linearity of the control design.

We address this problem by translating the structural constraint (14) into equivalent structural constraints on matrices $V_j$ and $U_j$, such that,

$$V_j \in S_v, \quad U_j \in S_H,$$  

where $S_v$ is the set of all diagonal matrices. The new structural constraints (15) ensure the linearity of the design by exploiting the fact that the inverse of a diagonal matrix is diagonal (that is, $V_j^{-1}$ is diagonal), and that the multiplication of any matrix with a diagonal matrix preserves its structure (that is, $K_j$ will have the same structure as $U_j$). These structural constraints ensure that the secondary control for each microgrid only uses information from its immediate neighbors (Fig. 1).

**Theorem 1.** The distributed secondary controller (13) obtained by solving design equations (12) along with (15) is sufficient to guarantee $L_2$ stability of the microgrid interconnection (10) in the D-MAFD framework with respect to disturbances $w(t)$ under arbitrary loss of D-PMU angle measurements.

This D-MAFD control framework is robust to disturbances by design, since the design conditions ensure that the microgrid interconnection (10) in this framework is $L_2$ stable with respect to the disturbance vector $w(t)$.

B. Robustness to network topology changes

In microgrid interconnections, topology changes may frequently occur due to islanding of some microgrids or line outages. In such scenarios, it is important to ensure that the stability of the microgrid interconnection with the new topology can be guaranteed without redesigning the existing controllers in the system, even if D-PMU angle measurement losses occur during islanding or reconnection of microgrids. We accomplish this objective by incorporating a robustness margin into the D-MAFD secondary control design as follows.

Let the perturbation in the system Jacobian due to the change in network topology be given by

$$\Delta H = H - H_{new},$$  

where $H_{new}$ is the Jacobian matrix after the network topology change. Then the stability of the microgrid interconnection with the new system topology with respect to disturbances $w(t)$ even under D-PMU measurement losses is guaranteed by the following robustness result.

**Theorem 2.** Given a network topology change with a new Jacobian matrix $H_{new}$, and $\Delta H$ as defined in (16), if there exists symmetric positive definite matrix $P > 0$, negative definite matrix $Q_j < 0$, and matrices $U_j, V_j, S_j$ and $R_j$ of appropriate dimensions such that (17) is satisfied with $\gamma = ||B_j^{(1)}\Delta H||_2 I$ for every $j \in \Sigma$, then the distributed control law $u(t) \rightarrow u(t) + \tilde{u}(t)$ where $\tilde{u}(t) = K_{\Sigma(t)} y(t)$ with

$$K_j = V_j^{-1}U_j, \quad \forall j \in \Sigma,$$

guarantees that the closed loop dynamics (10) is stable in the $L_2$ sense with respect to any disturbance $w(t)$ for the new network topology. Furthermore, the control law $u(t) \rightarrow u(t) + \tilde{u}(t)$ also guarantees stability of the closed loop system (10) for any new network topology with Jacobian matrix $H_{new}$ such that $||B_j^{(1)}\Delta H||_2 I < \gamma$, where $\Delta H = H - H_{new}$.

**Proof.** Consider the closed loop system (10) with a new Jacobian matrix $H_{new}$ and the control law $u(t) \rightarrow u(t) + \tilde{u}(t)$, where $\tilde{u}(t) = K_{\Sigma(t)} y(t), \forall j \in \Sigma$ is obtained from (17)-(18). Now consider the matrix $M_j$ in (12a) with its first term updated to $[M_j]_{11} = -P(A_j + B_j^{(1)} H) - (A_j + B_j^{(1)} H_{new}) P - B_j^{(1)} U_j C_j - C_j' U_j' B_j^{(1)} - 2\gamma P - PB_j^{(2)} B_j^{(1)} U_j D_j + C_j' S_j - C_j' Q_j^{1/2}$, and the control law $u(t) \mapsto u(t) + \tilde{u}(t)$, where $\tilde{u}(t) = K_{\Sigma(t)} y(t), \forall j \in \Sigma$ is obtained from (17)-(18).

$$M_j - \tilde{K}_j = \begin{bmatrix} -P(A_j + B_j^{(1)} H) - (A_j + B_j^{(1)} H_{new}) P - B_j^{(1)} U_j C_j - C_j' U_j' B_j^{(1)} - 2\gamma P - PB_j^{(2)} B_j^{(1)} U_j D_j + C_j' S_j - C_j' Q_j^{1/2} \\ -B_j^{(2)} P - D_j' U_j' B_j^{(1)} + S_j' C_j \\ -Q_j^{1/2} C_j \\ D_j' S + S_j' D_j + R_j - D_j' Q_j^{1/2} I \end{bmatrix} > 0$$

where $\gamma = ||B_j^{(1)}\Delta H||_2 I$. Clearly, since $\gamma \geq B_j^{(1)} \Delta H$, $M_j - \tilde{K}_j \geq 0$. If (17) holds, $M_j \geq \tilde{K}_j = 0 \Rightarrow M_j > 0$. Thus, using Theorem 1, if (17)-(18) holds, the closed loop system (10) is locally $L_2$ stable for the given network topology change with new Jacobian matrix $H_{new} = H + \Delta H$. It is then straightforward to show that this control law renders the closed loop system $L_2$-stable for any new network topology with Jacobian matrix $H_{new}$ such that $||B_j^{(1)}\Delta H||_2 I < \gamma$, where $\Delta H = H - H_{new}$.

$$\tilde{K}_j = \begin{bmatrix} -P(A_j + B_j^{(1)} H) - (A_j + B_j^{(1)} H_{new}) P - B_j^{(1)} U_j C_j - C_j' U_j' B_j^{(1)} - 2\gamma P - PB_j^{(2)} B_j^{(1)} U_j D_j + C_j' S_j - C_j' Q_j^{1/2} \\ -B_j^{(2)} P - D_j' U_j' B_j^{(1)} + S_j' C_j \\ -Q_j^{1/2} C_j \\ D_j' S + S_j' D_j + R_j - D_j' Q_j^{1/2} I \end{bmatrix} > 0$$

(17a)

$$PB_j^{(1)} = B_j^{(1)} V_j, \quad Q_j^{1/2} = Q_j^{1/2} Q_j^{1/2} = -Q_j, \quad V_j \in S_v, \quad U_j \in S_H$$

(17b)

(17c)
Selection of robustness margin: Theorem 2 presents a distributed secondary control design such that the microgrid interconnection in the D-MAFD framework is not only robust to a particular topology change, but also robust to any topology change that results in a smaller perturbation in the system Jacobian than the one used for the control design. Therefore, for maximal robustness, the D-MAFD secondary controller should be designed for the topology change that leads to the largest perturbation in the network Jacobian, that is, by selecting the robustness margin \( \gamma \) to be the maximum value of \( \|B_j^T \Delta \hat{H}\|_2 \) over all possible topology changes. However, in practice, such a choice of \( \gamma \) will require the computation of Jacobian matrices with respect to a very large number of network topologies, and may also be conservative. Therefore, the secondary control design using Theorem 2 can be carried out to guarantee \( (N-1) \)-robustness, that is, robustness in the scenario where a single microgrid is islanded or an outage takes place on a single line. In this case, an \( (N-1) \)-contingency analysis considering islanding or outage scenarios can be performed to select the worst-case robustness margin \( \gamma \) for the secondary control synthesis. With this design, robustness of the microgrid interconnection to topology changes can be guaranteed even if D-PMU measurement losses occur during islanding, outages or restoration operations.

IV. CASE STUDIES

We demonstrate the performance of the D-MAFD control framework by considering a test five-microgrid interconnection (Fig. 3) constructed from the 123-feeder test system shown in Fig. 2. For this network with MAFD primary control, we obtain a nonlinear switched system model of the form (10) with 32 switching modes. We then linearize the system around the power flow operating points in Table 1. We present two test scenarios to illustrate the performance of the D-MAFD framework - (i) under D-PMU measurement losses and disturbances, and (ii) during system topology changes due to line restoration after an outage.

A. Scenario 1: D-PMU measurement loss and load change

In this study, we assess the performance of the D-MAFD framework under D-PMU measurement losses. We use the linearized system model around the power flow solution for Condition (i) in Table 1 to design three controllers for comparison:

- **C1**: a distributed output-feedback secondary controller (D-MAFD controller) obtained by solving (12)-(15) for every \( j \in \Sigma \),
- **C2**: a centralized output-feedback secondary controller obtained by solving (12)-(13) for every \( j \in \Sigma \), and
- **C3**: a centralized secondary controller obtained by solving (12)-(13) for the microgrid interconnection where all microgrids continue to use angle droop control with the last available measurement even when D-PMU measurements are lost, that is, with the dynamics corresponding to the mode \( j = [1 \ldots 1] \) in (10).

The control design LMIs are solved using YALMIP [27]. We consider a test pattern of D-PMU measurement losses and a disturbance acting on all microgrids as shown in Fig. 4-A. For this test pattern, we simulate the nonlinear system dynamics with each of the designed controllers and make the following observations about their performance:

- We note that the sparsity structure of the D-MAFD secondary controller (Fig. 5) is the same as that of the network Jacobian, indicating that each microgrid only uses output measurements from its immediate neighbors.
- From the angle and voltage profiles in Fig. 6, we observe that the D-MAFD control design is successful in stabilizing the microgrid interconnection under the test D-PMU measurement loss scenario in the presence of disturbances. On the other hand, when all microgrids continue to use angle droop control with the last available measurement during D-PMU measurement loss, the

![Fig. 2. IEEE 123-feeder test network partitioned into five microgrids.](image)

![Fig. 3. Network parameters (p.u.) for 123-feeder five-microgrid test system.](image)

**TABLE I**

<table>
<thead>
<tr>
<th>Condition</th>
<th>( P_{inj}^{ref} ) (p.u.)</th>
<th>( Q_{inj}^{ref} ) (p.u.)</th>
<th>( P_{load}^{ref} ) (p.u.)</th>
<th>( Q_{load}^{ref} ) (p.u.)</th>
<th>( V_{inj}^{ref} ) (p.u.)</th>
<th>( \beta^{ref} ) (deg.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) SW1, SW2, SW3 closed</td>
<td>( \mu G_1 )</td>
<td>0.79</td>
<td>1.35</td>
<td>0.23</td>
<td>0.47</td>
<td>1.000</td>
</tr>
<tr>
<td>(ii) SW1 open</td>
<td>( \mu G_1 )</td>
<td>0.80</td>
<td>1.36</td>
<td>0.92</td>
<td>0.47</td>
<td>1.000</td>
</tr>
<tr>
<td>(iii) SW2 open</td>
<td>( \mu G_1 )</td>
<td>0.80</td>
<td>0.10</td>
<td>0.23</td>
<td>0.11</td>
<td>1.000</td>
</tr>
<tr>
<td>(iv) SW3 open</td>
<td>( \mu G_1 )</td>
<td>0.80</td>
<td>0.10</td>
<td>0.23</td>
<td>0.11</td>
<td>1.000</td>
</tr>
</tbody>
</table>
system suffers from poor transient performance, and the angle and voltage droop errors continue to increase from \( t = 12s \) until the disturbance is withdrawn at \( t = 14s \), indicating that angle droop control alone is unable to stabilize the system in this scenario.

- A comparison with the performance of the centralized secondary control design for this scenario indicates that the performance of the D-MAFD design is comparable (Fig. 7), despite the secondary controller in the D-MAFD framework using limited information from other microgrids in the network. This is an advantage, since distributed control designs are typically significantly outperformed by centralized designs.

**B. Scenario 2: Line reclosing after outage**

In order to demonstrate the robustness of the D-MAFD control design to changes in topology, we consider three fault scenarios that result in a line outage due to the opening of either \( SW_1 \), \( SW_2 \) or \( SW_3 \) in Fig. 3. The power flow solutions for each case are provided in Table 1. From a contingency analysis of the system, we determine that the opening of \( SW_2 \) (outage on the tie line between microgrids \( \mu G_1 \) and \( \mu G_5 \)) results in the worst case \( \gamma \) in Theorem 2. We design the D-MAFD controller corresponding to this outage by solving (17).

We study the performance of this D-MAFD control design when the faulted line is restored by reclosing \( SW_2 \) at \( t = 5s \), under the D-PMU measurement loss scenario shown in Fig. 4-B. Prior to the reclosing operation, the system initial condition corresponds to Condition (iii) in Table 1. After the reclosing operation, it is desired that the system returns to the original

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**Fig. 4.** A. D-PMU measurement loss pattern and disturbance pattern for Scenario 1. B. D-PMU measurement loss pattern for Scenario 2.

**Fig. 5.** Sparsity structure of the D-MAFD secondary controller for five-microgrid test system.

**Fig. 6.** A. Angle errors, and B. voltage errors of the D-MAFD design (C1) compared with a traditional angle droop controller (C3) for Scenario 1.

**Fig. 7.** A. Angle errors, and B. voltage errors of the D-MAFD design (C1) compared with a centralized secondary control design (C2) for Scenario 1.
operating point corresponding to Condition (i) in Table 1. From the angle and voltage error profiles for this scenario (Fig. 8-B), we observe that the D-MAFD control design maintains system stability even when a D-PMU measurement loss occurs during the reclosing operation.

We also evaluate the robustness of the designed controller by evaluating its performance for two other scenarios where $SW_1$ (Fig. 8-A) and $SW_3$ (Fig. 8-C) are reclosed after outages resulting from faults. We observe that the D-MAFD secondary controller designed for the worst-case line outage configuration ($SW_2$ open) is also successful in maintaining system stability in these two scenarios. This indicates that the D-MAFD framework is robust to network topology changes and does not require redesigning the secondary controller to maintain stability under different interconnection topologies.

V. CONCLUSION

We presented a mixed voltage angle and frequency droop control framework with a distributed secondary controller (D-MAFD framework) for distribution-level microgrid interconnections where loss of D-PMU angle measurements may result in degradation of stability and performance. The proposed D-MAFD framework provides a provably stable and robust solution to enhance the reliability of D-PMU based control designs for microgrid interconnections. Numerical case studies based on 123-feeder system confirm the efficacy of the proposed control scheme. Future work will involve investigating the control performance with respect to potential D-PMU data quality issues.

REFERENCES