

**Optimal Compensation with Earnings Manipulation:
Managerial Ownership and Retention**

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Abstract

The optimal managerial compensation contract is characterized in an environment in which the manager influences the distribution of earnings through an unobservable effort decision. Actual earnings, when realized, are private information observed only by the manager, who may engage in the costly manipulation of earnings reports. We derive the optimal contract that guarantees the manager non-negative profit for any earnings realization (interim individual rationality) to ensure manager retention. We find that the optimal contract induces under-reporting for low earnings and over-reporting for high earnings, and that the optimal contract may be implemented through a compensation package composed of a performance bonus based upon (manipulated) earnings and a stock option that is repriced to be strictly in the money for intermediate earnings realizations and at the money for low earnings realizations.

“Executives at dozens of public companies, including Starbucks, Google [and] Intel, are taking steps to lower the prices that their employees would have to pay to convert options into stock. The moves are usually described as important for retaining employees, especially as stock options that vest over several years look utterly worthless in the current market....But the moves leave shareholder advocates fuming....The process, in their view, is fundamentally unfair. Modifying the options means employees gain from stock price increases, while investors feel the brunt of stock price declines.” (The New York Times, March 27, 2009, p. B.1)

1. Introduction.

Management contracts commonly include some combination of performance bonuses, restricted stock grants, and stock options as mechanisms to align the interests of a manager with those of the shareholders.¹ In addition, some firms allow for the repricing of existing stock options as a tool to facilitate managerial retention.² The use of such practices raises concerns that the incentives managers face to manage earnings statements to improve their compensation comes at the expense of shareholders. Concern over the damaging effects of earnings management can be seen in the well-cited speech "The 'Numbers Game'" by Arthur Levitt during his term as chairman of the Securities and Exchange Commission (Levitt, 1998) as well as in the complaints of shareholders as evidenced by controversies involving the payment of bonuses to managers, such as the highly publicized cases in which managers have manipulated earnings

¹For a detailed description of the observed structure of managerial compensation arrangements, see Murphy (1999). The use of bonuses and options remains prevalent, as indicated by the 2008 *Wall Street Journal*/Hay Group CEO Compensation Study.

²Gilson and Vetsuypens (1993), Saly (1994), Brenner, Sundaram, and Yermack (2000), Chance, Kumar, and Todd (2000), Carter and Lynch (2001), and Chidambaran and Prabhala (2003) empirically study the types of firms that reprice manager stock options while Chen (2004) studies the factors that influence a firm's ex ante decision to allow for options repricing. He finds that firms that do not restrict the repricing of options experience less managerial turnover following a stock price decline than firms that do impose repricing restrictions.

statements in order to improve their compensation³ and more recently those cases involving Wall Street banks in the wake of the recent financial meltdown. Particular vitriol is reserved for the common practice of repricing a manager's under-water options to be in the money. Indeed, one representative of the investor community notes disapprovingly that “[o]ur members generally detest [repricing] and consider it antithetical to the whole concept of incentive compensation.”⁴

In this paper, we jointly study the role of bonuses, restricted stock grants, and options that can be repriced in manager compensation by deriving the optimal incentive contract in a model that formally incorporates the need to provide a manager with incentives to exert high effort, the ability of a manager to engage in costly earnings management, and the issue of manager retention. Contrary to the current view expressed above in the *Wall Street Journal* quote, we show that the optimal contract involves both upward and downward earnings management and can be implemented through a combination of performance bonuses and stock options that are repriced when realized earnings are low.

The problem we consider is that of a firm owner who wishes to hire a manager to run the business. The manager takes a private costly action that influences the probability distribution of actual earnings which, when realized, are observed only by the manager. The manager produces an earnings report, which may differ from the actual earnings if the manager is willing to incur falsification costs. Thus, we are examining a contracting environment with moral hazard followed by adverse selection, and the problem is to characterize the optimal incentive contract that balances the ex ante incentives of the manager to shirk with the ex post incentives to engage

³For example, see Young's 2004 story on the Qwest case.

⁴*The Wall Street Journal*, April 8, 1999, p. R.6.

in earnings manipulation.⁵ We also introduce the problem of managerial retention by requiring that the optimal contract be interim individually rational, so that the manager does not wish to leave the firm for any earnings realization. In so doing, the owner must also balance the incentives for the manager to shirk against the owner's incentive to limit the rents earned by the manager. We will show that it is precisely the concern for manager retention in an environment when the manager has private information about earnings, in addition to the standard moral hazard concern, that creates a role for options repricing in providing a manager with optimal effort and earnings management incentives.⁶

Our model is most closely related to Crocker and Slemrod (*RAND*, 2007), who derive the optimal ex ante individually rational contract under the costly state falsification of earnings and moral hazard. In that environment, an optimal contract induces only over-reporting of earnings

⁵While earnings manipulation can rise to the level of fraud (as in the Qwest case), it can also reflect the flexibility manager's have within GAAP guidelines. Badertscher et al (2009) find no evidence that firms who manage earnings down use conforming strategies any more than firms who manage earnings up (which is consistent with results of our analysis) and they also find evidence that firms who manage earnings down also employ non-conforming strategies. While nonconforming strategies, upon detection by auditors, would require the firm to restate its earnings, they do not generally trigger fraud statutes. This conclusion is consistent with Erickson et al (2006) who find no evidence that managerial incentive contracts affect the incidence of accounting fraud. Our analysis is also consistent with the recent work of Jayaraman and Milbourn (2011) who provide evidence that "firms consider the ex-post information manipulation effects of equity incentives and trade off these costs with the benefits of higher managerial effort when designing ex-ante compensation contracts."

⁶To the best of our knowledge, the mechanism design literature focusing on moral hazard followed by adverse selection with interim individual rationality constraints and commitment is limited. It appears that our analysis is the first to derive an optimal contract in this setting with a continuum of types and a continuum of effort levels. Laffont and Martimort (2002) analyze a two-type, two-effort-level model and provide no citations to related work. Fudenberg and Tirole (1990) and Netzer and Scheuer (2010) study renegotiation proof contracts in which a contract can be renegotiated (hence no commitment) after an agent has invested unobservable effort but before the outcome of the effort is realized. In these papers, the effort choice is private information at the renegotiation stage.

and entails bonuses paid to the manager which are increasing in the size of the earnings report, and the structure of the bonuses reflects an efficiency tradeoff between the effect such bonuses have on inducing higher levels of effort by the manager, on the one hand, and the incentives the bonuses generate for the falsification of earnings reports, on the other. While the ex ante individual rationality constraint permits full extraction of the manager surplus by the owner through the use of a lump sum transfer, one feature of their optimal contract is that, for some realized earnings, the manager may prefer to quit rather than continue with the firm. As in Crocker and Slemrod, we will consider the optimal contract under costly earnings falsification and moral hazard where the contract must be ex ante individually rational with respect to the manager's effort choice, but we will also address the retention issue directly by requiring that the contract also be interim individually rational with respect to the manager's earnings report. This latter requirement, which is necessary in order to guarantee that the manager not wish to leave the firm after observing the earnings outcome, introduces a surplus extraction role for the optimal bonus arrangement, which substantially changes the nature of the optimal contracting problem.

The efficient balancing of moral hazard and adverse selection in the presence of interim individual rationality creates countervailing incentives of the type examined by Lewis and Sappington (1989) and Maggi and Rodriguez-Clare (1995). Moderating the hidden action problem requires the owner to share firm profit with the manager, which gives the manager the incentive to over-report earnings, while the extraction of managerial surplus in the presence of hidden information and interim individual rationality requires the owner to engage in differential rent extraction, which gives the manager the incentive to under-report earnings. We show that the optimal contract exploits these competing effects.

In the case where the manager is given no ownership stake in the firm, we find that the optimal contract results in truthful earnings reports and zero gross manager rent (not accounting for the manager's effort cost) for actual earnings levels below a derived earnings threshold. Above this threshold, the manager over-reports earnings and earns positive gross rent. Alternatively, we find that endowing the manager with ownership shares in the firm by the granting of an option partially alleviates the moral hazard problem through the usual internalization channel, but it also increases the marginal rent the manager must earn to satisfy the incentive compatibility constraints and exacerbates the surplus extraction role of the bonuses in the optimal contract. With such partial ownership, the optimal contract exhibits the under-reporting of earnings and zero gross managerial rent below a certain earnings threshold. Above this threshold, the manager will continue to under-report earnings but will now earn a positive gross rent. Finally, there will be a second earnings threshold above which the manager over-reports earnings and earns gross rents that are increasing in the reported earnings. Thus, increasing the manager's ownership share reduces the extent of over-reporting induced by the optimal contract, and increases the extent of under-reporting of earnings by the manager.⁷

This structure of the optimal contract (conditional on the level of manager ownership) raises two key issues: The relationship between the manager's ownership share and owner expected profit, and the role of options and repricing. With regard to the former, we derive a simple test to determine the relationship between expected owner profit and the manager's

⁷While much of the earnings management literature focuses on over-reporting of earnings, Badertscher et al (2009) show that approximately 20% of earnings restatements are associated with *under-reported* earnings. They also find no evidence that firms are engaging in such under-reporting for tax reasons but rather for non-tax reasons. McAnally et al (2008) finds a lower incidence of earnings under-reporting but they exclude from their sample firms that granted options before an earnings announcement.

ownership stake. We show that if the expected earnings distortion, which is the expected difference between actual and reported earnings, is positive then the owner's expected profit is increasing in the manager's ownership share. Since, as noted above, with no manager ownership the optimal contract induces either correct reporting or over-reporting of earnings, our analysis implies that it is always optimal for the owner to endow a manager with some ownership.

With regard to the latter issue, we show that compensating the manager with options that are repriced for certain earnings realizations is preferable to restricted stock grants, an alternative approach that has been suggested by some observers.⁸ We find that the use of outright stock grants allows the manager to earn too much rent for some earnings realizations, so that to generate the optimal rent profile the manager must pay the owner (receive a negative bonus) at such earnings outcomes. In contrast, a contract that provides equity incentives via options that cannot be repriced pays the manager less than the optimal amount of rent for some earnings realizations and increases the cost of inducing any desired level of effort. However, providing equity incentives via options that are repriced at some lower earnings levels allows the owner to pay the manager her optimal rent without resorting to negative bonus payments. Together these two results indicate that, in an environment in which the optimal manager contract must create incentives that respond to *both* moral hazard and private information issues, options repricing can play an important role in ensuring manager retention. This conclusion is consistent with the empirical evidence in Chen (2004).⁹ Furthermore, when the optimal contract involves option

⁸"If Google is going to reprice when things go wrong, it should also limit the upside to employees. It would be easier simply to pay bonuses instead, tied to corporate performance, with a portion in stock that vests over time to aid retention." (*WSJ*, January 22, 2009)

⁹ In addition, we find that managerial stock ownership and the incidence of options repricing are co-determined in an optimal contract, in contrast to Chen (2004) in which manager

repricing, both repricing at the money and repricing strictly in the money can occur. This feature is consistent with the incidence of repricing found in Brenner et al (2000) of repricing at the money 77% of the time and repricing strictly in the money 19% of the time.

While there are extensive literatures on earnings management and on executive compensation, most focus almost exclusively on the effect of moral hazard. A smaller number of papers address the role of private manager information. The papers most closely related to our work are Dye (1988), and Peng and Röell (2009), both of which study incentive contracts under moral hazard when the manager possesses some private information.¹⁰ In Dye the manager has private information about actual earnings, while in Peng and Röell the private information involves the manager's costs of earnings manipulation. Despite the presence of private information in both of these cases, the contracts studied do not attempt to elicit this information from the manager and use it in the construction of the optimal contract.¹¹ The justification for not trying to directly incorporate the manager's private information is the claim (in footnote 2 of Dye's paper) that the Revelation Principle cannot be applied in optimal contracting papers in which the manager can manipulate earnings. This claim, which appears to

ownership determines the incidence of repricing.

¹⁰ Neither of these papers, however, address either the role of options (and, hence options repricing) or the issue of manager retention. One paper that does address the optimality of options is Acharya, John and Sundaram (2000) who consider a pure moral hazard environment in which the manager chooses effort at the beginning of each of two periods and the owner provides the manager with an option contract that can be exercised at the end of the second period. Repricing arises because of the ability of the manager to renegotiate the strike price prior to the second period after the value of first-period profit is publicly observed.

¹¹ In a similar vein Edmans et al (2009), while modeling earnings manipulation as a pure moral hazard problem, examine a multi-period environment in which the manager has private information in each period about actual earnings, yet the contracts examined do not attempt to elicit this private information.

have attained wide acceptance in the accounting and finance literatures, is simply not correct, a point that is made explicitly in Crocker and Slemrod (2007) and that we recount in note 22. Moreover, the analysis of Peng and Röell restricts attention to the subset of contracts that are linear in the firm's stock price.¹² In contrast, our approach characterizes the optimal contract without such a restriction on the class of admissible contracts.¹³

The paper proceeds as follows. In the next section we introduce the economic environment of the model and set up the optimal contracting problem. In Section 3 we provide an informal discussion of the structure of the optimal compensation contract, which is formally derived in Section 4. Section 5 contains our analysis of the effect of manager ownership on owner profit, and a final section contains concluding remarks.

2. The Model.

In this model there are two people who make decisions for a firm: an owner and a manager. The owner is responsible for setting managerial incentives and the manager is responsible for running the firm, which requires both an effort and the reporting of the firm's earnings to the owner. Effort is unobservable by the owner, which means $a \geq 0$ is a *hidden action*, and it is costly to the manager. Let $h(a)$ denote the manager's effort cost. We assume that

¹²A similar restriction characterizes the work of Goldman and Slezak (2006), who examine a model of information manipulation in an agency setting in which the manager takes a costly private action that inflates the profit signal seen by a third party. There is no private information in their setting, as the manager takes the action without any prior knowledge of the firm's profit.

¹³Garrett and Pavan (2009, 2010) develop a dynamic framework to derive optimal manager contracts with private manager information and moral hazard without restrictions on the set of feasible contracts. In Garrett and Pavan (2009), the manager accepts or rejects the contract in period 1 knowing only his first-period type and cannot quit thereafter and thus does not address the retention issue. In Garrett and Pavan (2010), the manager is allowed to quit and the owner is allowed to fire the manager in each period. Neither paper addresses the issue of earnings manipulation, nor does either paper address the issue of options repricing.

h is strictly increasing, strictly convex, and that $h(0) = h'(0) = 0$. The conditions on $h(0)$ and $h'(0)$ imply that the manager incurs no fixed costs of effort nor has a strictly positive initial marginal cost of effort that could result in the manager exerting zero effort in response to a range of positive incentive levels.

To induce the manager to choose a positive level of effort, the owner must offer a contract which ties the manager's compensation with the earnings of the firm. Following the literature, we consider two forms of compensation: performance-based compensation and endowing the manager with shares in the firm. To reflect the reality of most large corporations, we assume that only the manager observes the firm's true earnings, so that the value taken by x is *hidden information*. This means the owner cannot contract on the earnings, x , directly but only on the earnings reported by the manager, which we denote by R .

The time line of decisions and outcomes is illustrated in Figure 1. At time 0, the owner and the manager sign a contract of the form $(\alpha, B(R))$ where α denotes the manager's share of the firm's future profit, x , and $B(R)$ denotes the manager's performance-based compensation. At time 1, the manager is formally granted α .¹⁴ At time 2, the manager chooses an effort level a which then generates earnings x from the distribution $F(x|a)$ with strictly positive density $f(x|a)$ and support $[0,1]$. We assume that $F_a < 0$, so higher levels of manager effort shift the distribution of earnings to the right in the sense of first order stochastic dominance. The value of a is known only to the manager, and is therefore a hidden action. At time 3, the manager observes x , which is seen only by the manager and is therefore hidden information. At time 4, the manager decides either: (i) to quit, thereby surrendering any shares or options, and receive her outside option; or

¹⁴We will demonstrate below the difference between conferring this ownership through the granting of a stock option or through the granting of restricted stock.

(ii) to issue an earnings report, R . We assume that the cost to the owner of replacing the manager at this stage is sufficiently large that the owner prefers to always retain the manager. Based on this earnings report, the owner pays the manager $B(R)$ at time 5. At time 5, the owner may also reduce the strike price of the manager's options received at time 1. At time 6, the actual earnings of the firm are observed by the market but are not verifiable.¹⁵ This time period is needed to distinguish between the short-run compensation a manager receives, $B(R)$, and any long-run compensation that is usually tied to the long-run value of restricted stock or option grants. Any restricted shares or options the manager holds can be sold or exercised at this time. The non-verifiability of earnings assumption is an incomplete contracting assumption that precludes the owner from retroactively adjusting the manager's compensation or her number of shares based on actual earnings that are observed at some point in the possibly distant future. This simplifying assumption allows us to focus on the trade-off between effort incentives and rent extraction when the earnings report cannot influence the long-run market price, i.e., neither the market nor the

¹⁵ This is the same basic time line found in the finance literature papers referenced in the introduction with two exceptions: those papers treat times 0 and 1 as subdivisions of one time period and times 2 – 5 as subdivisions of a second period and we do not explicitly model any short-term relationship between R and the firm's stock price. Note that our formulation allows us to consider a larger class of contracts than those that are linear in the firm's stock price at $t=5$. In models in which a short-run price is calculated, the stock price is used to determine short-run (time 5) compensation; B in our model. Thus, calculating a short-run stock price is an intermediate step in determining how compensation B is affected by the earnings report, R . Since the manager is not allowed to sell any of her shares (or exercise her options) during time 5 and the owner does not have an incentive to manipulate the stock's short-term price as in Dye (1988)'s second or third models, a short-run stock price provides only one way to implement an optimal contract and is not essential for deriving the optimal contract. By allowing for any general relationship between the earnings report and short-run compensation, our model is consistent with any increasing monotonic relationship between R and the stock price. As a result, we abstract away from the issue of how one might write an optimal contract as a function of a short-term stock price (which in turn is determined by R) and focus instead on the direct relationship between compensation and R .

owner is misled about the value of the firm in time 6. We relax this assumption in section 6 to allow the earnings report to influence the long-run stock price and discuss how our results would be affected.¹⁶

Reporting earnings, R , that differ from actual earnings, x , imposes falsification costs $g(R-x)$ on the manager as it requires the manager to devote time and effort to managing the accounting to make such a report credible. In general, one would expect the falsification costs to be strictly convex in $R-x$, strictly increasing in $|R-x|$, and minimized at 0 such that $g(0)=0$. These properties imply the manager incurs no cost to issuing a truthful earnings report, and that under-reporting and over-reporting earnings are costly. To simplify the analysis and to allow us to focus more directly on the features of the optimal contract, we assume quadratic falsification costs, so that $g(R-x) = (R-x)^2 / 2$.¹⁷

The risk-neutral manager's utility given any value of α and any compensation contract $B(R)$ can be written as

$$\hat{V}(x, R, \alpha) = \alpha x + B(R) - g(R-x) - h(a) \quad (1)$$

and the risk-neutral owner's profit is

$$\Pi(x, R, \alpha) = (1-\alpha)x - B(R). \quad (2)$$

The owner's objective is to choose the indirect compensation $(\alpha, B(R))$ to maximize the expected value of $\Pi(x, R, \alpha)$ subject to several incentive constraints.¹⁸ Because the manager's ownership

¹⁶ We also could also have considered the possibility that the manager discounts any time-6 compensation. For ease of exposition we do not include a discount factor because doing so does not affect the qualitative features of the optimal contract.

¹⁷ We will use the general notation, $g(R-x)$, when it helps highlight the role of manipulation costs in the structure of the optimal contract.

¹⁸ The term "indirect" refers to the fact that the performance-based term $B(\cdot)$ depends

share is set before the manager chooses her effort and hence before earnings are realized, we can treat α as a parameter and derive the optimal compensation contract $B(R)$ for each value of α . We will determine the optimal value of α in a later section. We refer to the contract which solves the owner's problem for each value of α as the optimal *conditional contract*.¹⁹ For each value of α , a conditional contract induces an allocation that can be described by three components: an effort level, a , the level of manager utility, \hat{V} , and an earnings report, R , where the latter two depend on the firm's realized earnings, x .

Two comments are in order before proceeding. First, the risk neutrality of manager utility in the payments αx and B implies that the first-best solution for the owner is to sell the firm to the manager by setting $\alpha = 1$ in return for a lump sum payment, since doing so would internalize the effect of the manager's effort and earnings report choices on firm profits.²⁰ In this setting, however, we are concerned with managerial retention and so we proceed under the assumption that the manager has *no* resources with which to purchase the firm. This is a reasonable approximation for the case when manager wealth is very small relative to the firm's expected value, as it would be for large corporations. Second, the fact that the manager's shares are valued at αx is consistent with the idea that her shares do not vest (or the options cannot be exercised) until time t .²¹

indirectly on the firm's actual earnings through the manager's earnings report.

¹⁹ When maximizing a function $f(x,y)$, one can either jointly choose x and y or one can choose the optimal level of x for each value of y and then optimize over y . Both approaches are equivalent since it is the owner that chooses both variables. We use the conditional approach because it helps identify the way in which manager ownership influences the optimal bonus.

²⁰ Indeed, this is precisely the result in the analysis of Crocker and Slemrod (2007) who require that the contract be ex ante individual rational, which effectively permits a manager with sufficient resources to purchase the firm at time 0 through an up-front payment.

²¹ Similar assumptions are used in the first model of Dye (1998), and in the papers by Acharya, John and Sundaram (2000), Goldman and Slezak (2006), and Peng and Röell (2009).

The characterization of the optimal compensation contract must respect the informational structure of the model that is depicted in Figure 1. Since the manager selects an earnings report when she possesses private information about actual earnings, we will proceed by invoking the Revelation Principle which is a solution technique in which we recast the owner's problem as one in which the owner chooses a direct conditional contract instead of the indirect conditional contract, $B(R)$. Formally, for each value of α , a direct conditional contract consists of three components that mirror the allocation structure of this problem: a level of managerial effort the owner would like the manager to choose (prior to observing x), a ; an earnings report, $R(\theta)$, and a compensation schedule, $B(\theta)$, where θ is the manager's report of his type, x .²² Both R and B depend on θ since they are chosen after the manager has private information, but a does not depend on θ since it is chosen before the manager has private information about earnings. To the extent that the optimal indirect contract consists of a compensation schedule $B(R)$ that induces earnings manipulation, it will show up in the equivalent direct contract through the value of $R - x$.

²²Some readers may be familiar with an argument by Dye (1988) that zero earnings manipulation is not generally optimal in the presence of costly state falsification. In some literatures this result has been misinterpreted as invalidating the Revelation Principle in contracting models with costly state falsification. This interpretation is incorrect, as shown by Crocker and Slemrod (2007). Dye's argument demonstrated that one could not assume without loss of generality that the manager's earnings report is always equal to her private information (or true earnings). To apply the Revelation Principle correctly in this model involves using Myerson's (1982) generalized Revelation Principle by making the earnings report part of the contract. That is, the correct application of the Revelation Principle must distinguish between the manager's private information, x , and the earnings report made by the manager that is used to determine managerial compensation, $R(x)$. Whereas an indirect contract $B(R)$ would induce an optimal reporting strategy $R(x)$ and result in compensation $B(R(x))$, the equivalent direct revelation contract induces the manager to report his type truthfully to the contract ($\theta=x$) which in turn dictates that he issue the earnings report $R(x)$ and earn $B(R(x))$. This same misinterpretation led Lacker and Weinberg (1989) to assert that the Revelation Principle could not be applied in an insurance setting; an assertion that was corrected by Crocker and Morgan (1998). Gresik and Nelson (1994) correct a similar mistake in the analysis of multinational transfer price regulation.

By the Revelation Principle, we will restrict attention to direct contracts that induce the manager to choose the desired level of effort, a , and to issue a truthful type report (to the contract), so that $\theta = x$. Therefore, a direct conditional contract can be described by an effort level, $a(\alpha)$, a reporting function, $R(x; \alpha)$, and an indirect utility level for the manager, $V(x, a; \alpha)$. Given these values, (1) may be used to recover the optimal transfer function $B(x; \alpha)$ associated with the direct conditional contract. While here we have noted explicitly the reliance of the contract on α , we will for notational convenience drop explicit reference to α in these contract terms except where the clarification is helpful.

Because our model includes both moral hazard and adverse selection effects, any direct contract must satisfy several incentive compatibility and individual rationality constraints. Incentive compatibility generates four constraints, two of which apply to the manager's selection of θ , and two of which apply to the manager's choice of a . The choice of a type report is made after choosing a and after learning x , so that the manager chooses θ to maximize $\hat{V}(x, R(\theta), a; \alpha)$, whereas the ex ante choice of effort means the manager will choose a to maximize $E\hat{V}$.

Let $V(x, a; \alpha)$ denote the conditional indirect utility of the manager who optimally issues a truthful type report. That is,

$$V(x, a; \alpha) = \hat{V}(x, R(x; \alpha), a; \alpha). \quad (3)$$

Optimal truthful type reporting by the manager ($\theta = x$) requires that V satisfy two constraints:

$$V_x = \alpha + g'(R(x) - x) \quad (4)$$

and

$$R'(x) \geq 0. \quad (5)$$

Applying the Envelope Theorem to (3) and setting $\theta = x$ implies (4). Thus, the manager will earn a marginal rent that covers the change in the value of her shares plus the change in her manipulation costs.

Inequality (5) is the standard monotonicity condition in screening models with one-

dimensional decisions and supermodular payoffs that arises from the second order condition associated with the manager optimally choosing θ . Totally differentiating $\hat{V}_R(x, R(x), a; \alpha) = 0$ with respect to x implies $\hat{V}_{RR} \cdot R' + \hat{V}_{Rx} = 0$. Since $\hat{V}_{RR} \leq 0$ is the second-order condition for the optimal choice of θ and $\hat{V}_{Rx} = g''(R - x) > 0$, the earnings report function, R , must be non-decreasing. Thus, an incentive compatible contract will associate higher earnings, x , with higher earnings reports, R .

The last two incentive constraints deal with the manager's choice of productive effort, a . The manager will choose to invest a units of effort with $a > 0$ only if

$$\partial EV(x, a, \alpha) / \partial a = 0 \tag{6}$$

and

$$\partial^2 EV(x, a, \alpha) / \partial a^2 \leq 0. \tag{7}$$

Equation (6) is the first order condition for the manager's choice of a , and inequality (7) is the associated second order condition.

The manager's contract will also need to satisfy two individual rationality constraints. As in Crocker and Slemrod (2007), any contract must satisfy ex ante individual rationality, so that

$$EV(x, a; \alpha) \geq 0. \tag{8}$$

No manager would accept a contract that violates (8). In addition to (8), we explicitly model retention by adding the interim individual rationality constraint,

$$V(x, a; \alpha) + h(a) \geq 0. \tag{9}$$

This constraint captures the ability of the manager to quit after observing actual earnings, x , but before issuing an earnings report, R . Note that, since the manager has already chosen a prior to

²³ From the manager's perspective, the owner has already set the mechanism so choosing a different effort level does not change the reporting function, R , nor does it change his incentives to report truthfully.

observing x , the effort cost, $h(a)$, is a sunk cost.²⁴

To highlight the role of (9), define the manager's gross (of effort costs) indirect utility as

$$W(x; \alpha) \equiv \alpha x + B(R(x)) - g(R(x) - x) = V(x, \alpha; \alpha) + h(a). \quad (10)$$

Note that $W_x = V_x$ and that W does not depend directly on a since at the earnings report stage the effort choice is sunk. The effort choice, a , will however effect EW through the distribution F .

By using V to substitute B out of Π and W to substitute out V , the owner's problem can be written as choosing $(a, W(x), R(x))$ to

$$\begin{aligned} \max E(x - g - W) \text{ s.t. } & \text{a. } W_x(x) = \alpha + g'(R(x) - x) \\ & \text{b. } \partial EW / \partial a - h'(a) \leq 0 \\ & \text{c. } W(x) \geq 0 \\ & \text{d. } EW - h(a) \geq 0 \\ & \text{e. } R'(x) \geq 0 \\ & \text{f. } \partial^2 EW / \partial a^2 - h''(a) \leq 0. \end{aligned} \quad (11)$$

Constraints (11a) and (11e) are incentive compatibility constraints that ensure truth-telling ($\theta = x$) by the manager in the conditional direct revelation contract. Constraint (11b) is the same as (6), the manager's first-order condition for the choice of effort, a ; and constraint (11f) is the same as (7), the manager's second-order condition. Constraint (11c) is the interim individual rationality constraint, and (11d) is the ex ante individual rationality constraint.

Before proceeding, note that constraint (11a) implies

$$W(x; \alpha) = W(0) + \int_{t=0}^x [\alpha + g'(R(t) - t)] dt \quad (12)$$

which, after integrating by parts, yields

²⁴Career concerns might discourage the manager from quitting as long as $V+h$ is not too negative. While such concerns would strengthen the owner's rent-extraction ability, they would not change the qualitative features of the optimal contract we derive. We thank Tomasz Żylicz for bringing this issue to our attention.

$$EW(a; \alpha) = W(0) + \int_{t=0}^1 [\alpha + g'(R(t) - t)](1 - F(t | a)) dt, \quad (13)$$

so that

$$\partial EW(a; \alpha) / \partial a = - \int_{t=0}^1 [\alpha + g'(R(t) - t)] F_a(t | a) dt, \quad (14)$$

and

$$\partial^2 EW(a; \alpha) / \partial a^2 = - \int_{t=0}^1 [\alpha + g'(R(t) - t)] F_{aa}(t | a) dt. \quad (15)$$

We demonstrate in the Appendix that, given the Distribution Assumptions on F specified in section 4, an optimal contract satisfies $W_x = \alpha + g'(R(x) - x) \geq 0$ for all x . Thus, (15) is strictly negative since, by one of the Distribution Assumptions, F is convex in a . This means the second order condition (11f) will be non-binding. In section 6, we describe how the optimal contract changes when (15) binds.

As is commonly done in these settings, we will proceed by solving a modified version of (11) in which constraint (11e) is dropped (in addition to (11f)), and then check at the end to make sure that (11e) is satisfied. Call this modified problem (11'), which yields the Hamiltonian

$$\mathcal{H} = (x - g(R-x) - W)f + \varphi(\alpha + g'(R-x))$$

where φ is the co-state variable, W is the state variable, and R is the control. Using (14) yields the Lagrangian

$$\mathcal{L} = \mathcal{H} + \tau W - \mu[(\alpha + g'(R-x))F_a + h'(a)f] + \lambda f(W - h) \quad (16)$$

where $\tau(x)$ is the non-negative multiplier on the interim individual rationality constraint (11c), μ is the non-negative multiplier on the effort constraint (11b), and λ is the non-negative multiplier on the ex ante participation constraint (11d).²⁶

²⁵The notation $EW(a; \alpha)$ is used to emphasize that the manager's expected gross rent is a function of a and α as x is integrated out.

²⁶The reason $\mu \geq 0$ is as follows. Replace the right-hand side of (11b) with $\beta \geq 0$. An

3. Informal Discussion of Results

Before proceeding to characterize formally a solution to (16), we will describe the nature of our primary result, how this problem relates to others in the extant literature, and the role of countervailing incentives in the optimal contract. Under the Distribution Assumptions in section 4, we demonstrate that the optimal reporting function, R , satisfies

$$R - x = \frac{(1 - \lambda)(F - 1)}{f} - \frac{\mu F_a}{f} \quad (17)$$

when the interim individual rationality constraint (11c) does not bind and where λ is the multiplier associated with the ex ante individual rationality constraint (11d) and μ is the multiplier associated with the effort constraint (11b). The right-hand side of (17) is the difference of two non-positive terms, so that an optimal reporting function may entail either over- or under-reporting of earnings depending on which effect dominates.

In the special case where $F_a = 0$, manager effort has no effect on firm earnings and the contracting problem reduces to the costly state falsification environment examined by Crocker and Morgan (1998). In this setting, the parties face the standard tradeoff between efficiency and surplus extraction that is commonly observed when contracting in the presence of hidden information. When the parties face only an ex ante participation constraint, the solution to the contracting party is to sell the firm to the manager ($\alpha = 1$) for a lump sum payment equal to the firm's expected profit and then pay a bonus that is uniformly zero in reported earnings. Since the ex ante participation constraint permits full extraction of managerial surplus through lump sum transfers without efficiency cost, it is straightforward to show that $\lambda = 1$ and, from (17), $R = x$.

If instead the contracting parties face only an interim individual rationality constraint,

increase in β increases the marginal cost of inducing any given a and hence reduces owner expected profit. If Π denotes the owner's value function, then $\partial \Pi / \partial \beta = -\mu \leq 0$ (with strict inequality if $a > 0$). Thus, μ must be non-negative.

then $\lambda = 0$ and (17) reduces to $R - x = (F - 1)/f$. As long as F satisfies the monotone hazard rate property, so that $\partial((F - 1)/f) / \partial x \geq 0$ as depicted in Figure 2, then earnings are under-reported, the amount of under-reporting is monotonically decreasing in x , and $R'(x) \geq 0$. Moreover, as long as $W_x \geq 0$, interim individual rationality is satisfied by setting the bonus schedule so that $W(0) = 0$, which results in the manager earning information rents that are increasing in actual earnings, x .²⁷ Thus, in the presence of an interim individual rationality constraint, the optimal contract reflects the tradeoff between efficiency and surplus extraction that is commonly observed in settings with adverse selection, and because of surplus extraction the manager *under-reports* earnings.

In the case where $F_a \leq 0$, in which an increase in (unobservable) managerial effort shifts the distribution of earnings to the right, the optimal contract now has a moral hazard component. When the contracting parties face only the ex ante participation constraint, we are in the Crocker and Slemrod (2007) environment in which the use of lump sum transfers permits the frictionless extraction of managerial surplus, so that $\lambda = 1$. Then (17) reduces to $R - x = -\mu F_a / f$ and, as depicted in Figure 2, the optimal reporting function entails earnings *overstatement* by the manager. The optimal contract pays a bonus, B , to the manager which is increasing in the reported earnings, R . A bonus structure that is more sensitive to higher earnings reports gives the manager the incentive to take higher levels of the (private) costly action, a , but also increases the returns to the overstatement of earnings. Thus, the efficient contract reflects an efficiency tradeoff between the benefits of effort incentives and the costs of falsification.

The introduction of interim individual rationality adds a surplus extraction role to the optimal reporting contract and the associated bonus structure. Since the optimal contract in

²⁷Crocker and Slemrod ensure the monotonicity of W by assuming that $|g'| < 1$ and restricting their analysis to the case in which $\alpha = 1$. The problems encountered when W is nonmonotonic are discussed in section 6.

Crocker and Slemrod (2007) violates interim individual rationality, it follows that $\lambda < 1$ and the optimal reporting function satisfies (17), which is depicted in Figure 3. The optimal contract results in both under- and over-reporting, depending on the actual level of earnings, reflecting a tradeoff between surplus extraction and efficiency.²⁸ In addition, there is a problem in satisfying the interim individual rationality constraint since the values taken by V_x (and, hence W_x) implied by (17) are necessarily non-monotonic for sufficiently small values of α . For example, in the case of quadratic falsification costs, $W_x = 0$ implies that $R - x = -\alpha$, as depicted in Figure 3, while (17) implies that $R(0) = -(1-\lambda)/f(0)$ and $R(1)=1$. Thus, for α close to zero W_x will be strictly negative for x close to zero and strictly positive for some higher earnings. This non-monotonicity introduces countervailing incentives of the type first addressed by Lewis and Sappington (1989).

The countervailing incentives require an application of the approach developed by Maggi and Rodriguez-Clare (1995) to determine over what range of earnings levels the interim individual rationality constraint should bind in an optimal contract. The countervailing incentives imply an optimal contract with three features. First when earnings are below a threshold, \hat{x} , the manager earns zero gross profit ($W = 0$) and the contract under-reports earnings by the amount α . Second, when earnings are between \hat{x} and a second threshold, x^+ , manager profit is increasing in x , and the amount of under-reporting is decreasing. Third, for earnings levels above x^+ , manager profit is increasing in actual earnings and the contract over-reports earnings.

We now turn to a formal derivation of our results.²⁹

²⁸This motivation for under-reporting provides an alternative linkage between manager ownership and earnings management than suggested by McAnally et al (2008) who focus primarily on the impact of earnings management on future options grants.

²⁹ The formal analysis includes the possibility that the reporting function that solves (16) might not satisfy monotonicity condition (11e). Proposition 2, presented below, will show that

4. The Optimal Conditional Contract: A Formal Characterization

In order to characterize a solution to (16), we use several regularity assumptions regarding the behavior of the distribution function, F .

Distribution Assumptions:

- a. $F(x|a)$ is strictly decreasing, convex and continuously differentiable in a for all x and for all a .
- b. $f(x|a) > 0$ for all x and a and there exists $M > 0$ such that for all x and a , $f(x|a) < M$ and $f_x(x|a) < M$.
- c. $f_x(0|a) > 0$ for all $a \geq 0$.
- d. $(F(x|a) - 1) / f(x|a)$ is strictly increasing in x for all a .
- e. $(F(x|a) - 1) / f(x|a)$ is strictly concave in x for all a and $F_a(x|a) / f(x|a)$ is strictly convex in x for all a .
- f. $f_{ax}(x|a) \geq 0$ and $f_{xx}(x|a) \leq 0$, with at least one of the inequalities being strict, for all x and a .

Assumption (a) implies that higher manager effort induces a first-order stochastic improvement in the distribution of earnings (F decreasing in a) and results in diminishing marginal returns from effort (F convex in a). The convexity of F with respect to effort will ensure that the first-order approach is valid. Assumptions (b) and (c) are technical assumptions adopted to simplify several of the proofs. Assumption (b) restricts attention to densities that are bounded and have bounded first derivatives with respect to earnings. Assumption (c) requires that small but positive earnings are relatively more likely than zero earnings. Our proofs will indicate where these assumptions are used.

Assumption (d) is the standard monotone hazard rate assumption found in most adverse

the modifications to (16) needed to ensure that $R(x)$ is non-decreasing do not alter the qualitative properties of the optimal contract discussed in this section.

selection models, and is used to guarantee manager indifference curves that exhibit the single-crossing property. For a family of distributions indexed by a , it will be satisfied, for instance, as long as $\partial f/\partial x > 0$ for all a , and example 1 (below) provides an example of one such family. In this paper, assumption (d) is sufficient to support single-crossing only at sufficiently low effort levels. Because of the moral hazard component of this problem, the manager's indifference curves can fail to exhibit the single-crossing property at high enough levels of effort. Finally, assumptions (e) and (f) are a regularity conditions that imply the single-crossing property may only be violated for high earnings levels, (e), and that the constant rent curves are convex in x , (f).

Turning to the Lagrangean expression (16), the term τW is included because constraint (11a) reveals that W need not be strictly monotonic in x if the contract induces under-reported earnings (which implies $g' < 0$). In standard contract design problems when W is monotonic, one can replace the continuum of constraints represented by (11c) with a single constraint that sets either $W(0)$ or $W(1)$ equal to 0. Because the manager's indirect utility, W , may not be monotonic in x , the manager type that receives zero gross surplus ($W=0$) is endogenously determined. Introducing the term τW formally accounts for this endogeneity.

The potential non-monotonicity of W is due to two countervailing incentives created by the moral hazard and adverse selection effects in the presence of an interim individual rationality constraint. The first incentive (moral hazard) comes through the ownership term, α . Increasing α gives the manager a greater share of actual firm earnings and hence induces the manager to invest in higher effort. The second incentive (adverse selection) comes through the earnings report term, g' . When the direct contract reflects incentives to over-report earnings ($R(x) > x$), marginal manipulation costs will be increasing in x . This means that the owner can pay the manager a rent either by increasing the manager's ownership share *or* by inducing more over-reporting of earnings. When the direct contract reflects incentives to under-report ($R(x) < x$),

marginal manipulation costs will be decreasing in x . Now the ownership incentives and the under-reporting incentives work in opposite directions. These countervailing incentives give the owner the ability to combine increases in the manager's ownership share with incentives to under-report (via $R(\cdot)$) that result in zero marginal rent being paid to the manager. We will show that this type of countervailing incentive structure plays a key role in the optimal contract. Formally, the presence of the countervailing incentives means our analysis will employ the same techniques as found in Maggi and Rodriguez-Clare (1995).

Proposition 1. *Given Distribution Assumptions a-c, if there exists a piecewise continuous function, $\varphi(x)$, constraint multipliers, μ , λ , and $\tau(x)$, and a conditional contract (a, W, R) that satisfy*

$$R - x = (\varphi - \mu F_a) / f \text{ (almost everywhere),} \quad (18)$$

$$-(1-\lambda)f + \tau = -\varphi' \text{ (almost everywhere),} \quad (19)$$

$$a \left[\int_{t=0}^1 (\alpha + g'(R(t) - t)) F_a(t | a) dt + h'(a) \right] = 0, \quad (20)$$

$$\lambda(EW - h(a)) = 0 \text{ and } \lambda \geq 0, \quad (21)$$

$$\varphi(0) \leq 0, \varphi(1) \geq 0, \varphi(0)W(0) = \varphi(1)W(1) = 0, \tau \geq 0, \text{ and } \tau(x)W(x) = 0, \quad (22)$$

$$R'(x) \geq 0, \quad (23)$$

$$a \in \arg \max E[x - g(R - x) - W(x)] \quad (24)$$

and at points of discontinuity of $\varphi(x)$, which can occur only at a finite number of points, $\varphi(x)$ only jumps down, then (a, W, R) is an optimal conditional contract.³⁰

Proposition 1 is a translation of Theorems 1 and 2 (chapter 6) in Seierstad and Sydsæter

³⁰Proposition 1 provides sufficient conditions for an optimal conditional contract and thus does not rule out the possibility that all solutions to (18)-(22) may violate (23). We use this simpler formulation in the body of the paper to emphasize the key economic trade-offs in the optimal conditional contract. The more general formulation that explicitly incorporates monotonicity constraint (23) is developed in the appendix in the proof of Proposition 2 where the solution can involve standard ironing techniques.

(1987) to the specifics of (11'), with constraint (11e) added as (23) for completeness, that we can invoke because the Hamiltonian associated with (11) is concave in the control R and the state W and the constraint inequality is quasi-concave in W . Eq. (18) is the Euler equation and defines the optimal reporting function. The sign of the term $(\varphi - \mu F_a)/f$ determines for which earnings levels the contract induces over-reporting and for which earnings levels the contract induces under-reporting. The co-state variable, φ , will capture both the effort and manipulation distortions in the contract. To determine the manipulation incentive (captured by $R - x$), one must subtract out the effort effect, measured by the term $\mu F_a/f$. Thus, contracts that create strong incentives for the manager to invest in a larger amount of effort than she would otherwise choose correspond to a high value of μ and for a given value of φ , a large manipulation incentive. Eq. (20) is the manager's first-order condition with respect to effort. Condition (21) represents the complementary slackness conditions with regard to the ex ante individual rationality constraint. The conditions in (22) are the transversality conditions that will help determine which actual earnings level correspond to zero manager rents. The first four transversality conditions arise because interim individual rationality requires $W(0) \geq 0$ and $W(1) \geq 0$. The last two transversality conditions are standard complementary slackness conditions. Condition (24) ensures that the effort level induced by the contract is optimal from the owner's perspective. The derivation of this condition can be found in the Appendix (see eq. (A.1)).

It turns out that by studying conditions (18)-(22), one can learn quite a bit about the structure of optimal contracts conditional on any level of effort and any ownership share. Thus we follow the standard approach in the moral hazard literature by first solving for the optimal conditional contract for each level of effort and then optimizing over the level of effort.

The countervailing incentives allow for the possibility that the manager earns zero marginal rent over a range of earnings. Following the approach developed in Maggi and Rodriguez-Clare (1995), we begin by determining the contract properties that result in zero

marginal rent. To determine if such an outcome can be the result of an optimal contract, suppose the contract implies zero marginal rent for the manager on a non-degenerate interval of earnings, i.e., $W_x = 0$. Then (11a) implies $\alpha + g'(R-x) = 0$ or $R(x) - x = -\alpha \leq 0$ for all x in this interval. For an incentive compatible contract to result in zero marginal manager rents, it must induce the manager to under-report earnings so that the countervailing ownership and manipulation incentives exactly offset each other. Only in the case in which the manager owns no shares in the firm will incentive compatibility and zero marginal rents imply truthful reporting. Let $\hat{\varphi}(x)$ denote the value of the co-state variable associated with zero marginal rent. Thus, (18) implies

$$\hat{\varphi}(x) = \mu F_a(x|a) - \alpha f(x|a). \quad (25)$$

For all α , $\hat{\varphi}(x) < 0$ for all $x \in (0,1)$. The manager's rent is increasing when $\varphi(x) > \hat{\varphi}(x)$ and is decreasing when $\varphi(x) < \hat{\varphi}(x)$. Eq. (25) defines a feasible co-state variable as long as it also satisfies (19) and (22). With $\tau \geq 0$, (19) implies that $\varphi' \leq (1-\lambda)f$ which, in conjunction with (22) implies that³¹

$$(1-\lambda)(F-1) \leq \varphi(x) \leq (1-\lambda)F. \quad (26)$$

As long as $\hat{\varphi}$ falls within this range defined by (26), an optimal conditional contract can induce zero marginal rents for a range of earnings. If α and μ are sufficiently close to zero, $\hat{\varphi}$ will satisfy (26) for earnings below a level we denote by \hat{x} . For earnings above \hat{x} , $\hat{\varphi}$ will fall below $(1-\lambda)(F-1)$. Exploiting the countervailing incentives by setting $\varphi = \hat{\varphi}$ for $x \leq \hat{x}$ and setting $\varphi = (1-\lambda)(F-1)$ for $x > \hat{x}$ results in the reporting function from (18) of

$$R(x, a; \alpha) = \begin{cases} x - \alpha & \text{if } x \leq \hat{x} \\ x + \frac{(1-\lambda)(F(x|a)-1) - \mu F_a(x|a)}{f(x|a)} & \text{if } x > \hat{x} \end{cases} \quad (27)$$

³¹Integrating both sides from 0 to x , and noting that $(0) \leq 0$ from (22), yields the left inequality, while integration of both sides from x to 1 and noting that $\varphi(1) \geq 0$ from (22) yields the right hand inequality.

where the value of \hat{x} is endogenous and calculated as part of the optimal contract. Note that for any $\alpha > 0$, if μ is large enough, then $\hat{\phi}(x) < (1 - \lambda)(F - 1)$ for all $x < 1$. In this case, $\hat{x} = 0$.

For $\alpha = 0$, (27) implies the contract results in no distortion of low earnings and an upward distortion of high earnings. For $\alpha > 0$, (27) implies that the contract results in a downward distortion of low earnings and an upward distortion of high earnings. Truthful reporting when $\alpha > 0$ will only occur at $x = 1$ and at one other earnings level greater than \hat{x} . In addition for all α , the manager earns zero rent ($W = 0$) and not just zero marginal rent ($W_x = 0$) when $x < \hat{x}$ and positive rent when $x > \hat{x}$. Incentives that induce under-reporting can be attractive to the owner because they reduce the manager's information rent but they also reduce the manager's marginal effort incentive. By adjusting α , the owner can control the balance between these countervailing effects.

Example 1. Let $F(x|a) = (1 - a)x + ax^2$ for $0 \leq a \leq 1$, $h(a) = a^3 / 3$, $\alpha = 0$, and $\lambda = 0$.³² Figure 4 plots F , $F - 1$, and $\hat{\phi}$. F satisfies all of the Distribution Assumptions except (c) when $a = 0$. The technical issue described in Remark 1 of the Appendix, for which assumption (c) was introduced, does not arise in this example. For all a , F and $F - 1$ are increasing functions of x while $\hat{\phi}$ must be decreasing near $x = 0$ and increasing near $x = 1$. For this specific family of distributions, $\hat{\phi}$ and $F - 1$ intersect once on $(0, 1)$. The point of intersection of $\hat{\phi}$ and $F - 1$ is \hat{x} . Using (27) to define the reporting function, the optimal conditional contract for $\alpha = 0$ induces an effort level of .049 and results in the earnings manipulation shown in the top curve in Figure 5. Increasing α has the effect of shifting $\hat{\phi}$ down, thus reducing the range of earnings over which the manager earns zero rent, and changing \hat{x} slightly. Now zero manager rent is associated with under-reported earnings as illustrated by the lower curve in Figure 5 for $\alpha = .05$. In addition, under-reporting persists above \hat{x} even while the manager starts to earn positive rent. Over-reported

³²More precisely, we solved for an optimal contract in the example ignoring the ex ante participation constraint, and then checked to ensure that the solution satisfied (11d) so that $\lambda = 0$.

earnings arise only for the highest earnings levels but notice that the magnitude of the earnings manipulation is reduced. Effort rises to .055.

Example 1 highlights three interesting properties of an optimal contract: zero manager rent at low earnings levels induced by exploiting countervailing ownership and manipulation incentives, incentives for both under-reporting and over-reporting earnings, and a compensation schedule that incorporates both insurance and options features. We now prove that these are general properties of optimal conditional contracts by analyzing the conditions of Proposition 1.

Proposition 2. *Assume the Distribution Assumptions are satisfied. The optimal conditional contract induces a strictly positive level of effort and there exists earnings $x^+ \leq 1$ such that for all $x^+ \leq x \leq 1$ the contract induces weakly over-reported earnings ($R \geq x$) and the manager earns positive rent ($W(x) > 0$). If α is sufficiently close to zero, then there exists an earnings level, $\hat{x} > 0$, such that for all $x \leq \hat{x}$, the contract induces weakly under-reported earnings ($R \leq x$) and the manager earns zero gross rent ($W(x) = 0$) (with strict under-reporting for $\alpha > 0$) and for $\hat{x} < x < x^+$ the contract induces under-reported earnings and positive rent ($W(x) > 0$). For larger values of α , \hat{x} may equal 0.*

Proposition 2 establishes that under-reporting of low earnings is a robust feature of an optimal contract as illustrated in Figure 3. This robustness is important for the addressing the issue of repricing options as the repricing takes place precisely when the contract induces under-reporting. One can implement the pattern of over- and under-reporting described in Proposition 2 with the compensation schedule, $B(x)$, which given (1) and (10) is

$$B(x) = W(x) + g(R(x) - x) - \alpha x. \quad (28)$$

For $x \leq \hat{x}$, $W(x) = 0$ so $B(x) = g(-\alpha) - \alpha x$ and $B'(x) = -\alpha$. And for $x > \hat{x}$,

$$B(x) = g(R(x) - x) + \int_{\hat{x}}^x g'(R(t) - t) dt - \alpha \hat{x}$$

and $B'(x) = g'(R(x) - x)R'(x)$, which is negative when the contract induces under-reported

earnings and positive for over-reported earnings. Thus, $B(\cdot)$ is decreasing for earnings up to x^+ and increasing for earnings above x^+ and $B(x^+)$ is strictly negative for all $\alpha > 0$. This non-monotonic compensation function helps induce the desired level of effort as it effectively insures the manager against very low earnings levels ($x \leq \hat{x}$) and rewards the manager for high earnings ($x > x^+$). This discussion leads to the following proposition.

Proposition 3. *Because the optimal conditional contract induces (strictly) under-reported earnings below x^+ , the optimal compensation schedule will be (strictly) decreasing in earnings up to x^+ and strictly increasing in earnings above x^+ .*

Although $B(x) + \alpha x$ will be strictly positive for all x when $\alpha > 0$, negative values of $B(x)$ imply that the optimal contract requires the manager to pay the owner for intermediate values of x . This would be the case if α is conferred to the manager via a restricted stock grant.

Another way to interpret (28) to avoid explicit payments from the manager to the owner is to view it as a combination of an option that allows the manager to purchase α shares at a specified price after earnings are realized, a bonus schedule, and a repricing of the option's strike price for certain earnings realizations. The contract will not include a base wage since the fact that the manager is risk neutral and has an outside option equal to zero implies the base wage should equal zero. Viewed in this way, we will show that negative compensation shows up as a decrease in the value of the manager's shares.

The direct compensation and asset valuation of the manager's shares is described by $B(x) + \alpha x$. For $x \leq \hat{x}$,

$$B(x) + \alpha x = g(-\alpha) + \alpha(x - x) \quad (29)$$

which reduces to $g(-\alpha)$ and, for $x > \hat{x}$, (27) can be written as

$$B(x) + \alpha x = g(R(x) - x) + \alpha \left[x - (\hat{x} - (1/\alpha) \int_{t=\hat{x}}^x g'(R(t) - t) dt) \right]. \quad (30)$$

Since the manager's information rent for $x > \hat{x}$ equals $\int_{t=\hat{x}}^x (\alpha + g'(R(t) - t)) dt$ and is strictly

positive, the bracketed term in (30) is also strictly positive for all $x > \hat{x}$. Thus, changes in the value of the manager's shares can provide a channel through which manager rents are paid.

To observe the role of bonuses and options repricing in the optimal contract, consider a contract that includes giving the manager an option to purchase α shares of the firm at the strike price x^+ . For all $x \geq x^+$, (30) can be rewritten as

$$B(x) + \alpha x = g(R(x) - x) + \int_{t=x^+}^x g'(R(t) - t)dt + \int_{t=\hat{x}}^{x^+} (\alpha + g'(R(t) - t))dt + \alpha(x - x^+). \quad (31)$$

The first three terms on the right-hand side of (31) constitute a (positive) bonus payment that depends on the level of realized earnings. These terms serve to compensate the manager for the manipulation costs she incurs and to pay part of the manager's rent. The last term in (31) is the manager's gain from exercising the option which is in the money. This gain represents the remainder of the rent due the manager at earnings above x^+ . For earnings above x^+ , the use of a bonus payment and gains from an option in the money are complements as both increase with x .

For $\hat{x} < x < x^+$, the option is no longer in the money so a decomposition similar to (31) does not provide the manager with any rent via the option. For this range of earnings, the first term on the right-hand side of (30) represents a bonus to cover the manager's manipulation costs and the second term represents the gain to the manager from exercising an option with a revised strike price of $\hat{x} - \int_{\hat{x}}^x g' dt$ which is less than actual earnings, x . By repricing the option to a new lower price that is strictly in the money, the owner pays all of the manager's rent without resorting to a negative bonus. Note that the owner could avoid the need to reprice the option in this range of earnings by initially issuing an option with a strike price of \hat{x} . With a strike price of \hat{x} , (30) implies at x^+ that

$$B(x^+) + \alpha x^+ = \int_{\hat{x}}^{x^+} g'(R(t) - t)dt + \alpha(x^+ - \hat{x}). \quad (32)$$

Since the optimal contract induces under-reported earnings on (\hat{x}, x^+) , the first term on the right-hand side of (32) is strictly negative. This means an option with a strike price of \hat{x} allows the

manager to earn too much rent when earnings are just below x^+ and would still necessitate a *negative* bonus payment equal to $\int_{\hat{x}}^{x^+} g'(R(t) - t) dt$ to achieve the desired level of rent implied by the optimal contract. In fact, x^+ is the lowest strike price that does not require the use of negative bonus payments for some earnings levels.

Finally, for $x \leq \hat{x}$ manager rent is zero. As (29) indicates, this can be accomplished by paying the manager a bonus equal to $g(-\alpha)$ to cover her manipulation costs.³³ In addition, by repricing the option to be at the money, the net gain to the manager from exercising the repriced options is zero.³⁴

Returning to Example 1 and Figure 5, suppose $\alpha = .05$ and that one implements the optimal contract with options that can be repriced. Then conditional on earnings falling below x^+ (which would trigger a repricing), the probability of repricing the options to be exactly in the money is approximately .86 and the probability of repricing the options to be strictly in the money is approximately .14.

5. Optimal Firm Ownership by the Manager

The previous section demonstrated that the optimal contract conditional on the percent of the firm owned by the manager has certain features that are robust to the manager's stake in the firm. However, changes in α can be expected to affect not only the manager's behavior under the contract but also the owner's expected profit. In this section, we study the effect of changes in α and calculate the optimal level of manager ownership.

A change in α will affect expected owner profit via three channels: the direct change in the owner's share of the firm, the change in R and W associated with the optimal conditional

³³Although $g(-\alpha)$ is constant on $[0, \hat{x}]$, we refer to it as a constant bonus and not a fixed wage. If it was a fixed wage, it would have to be paid for all x which would make sense if $g(-\alpha)$ was the minimum bonus paid to the manager. This is not the case as $g(R(x^+) - x^+) = 0$.

³⁴ An equivalent outcome may be obtained by not repricing the options in which case they are not exercised.

contract to reflect a change in the manager's reporting incentives, and the change in the level of effort the owner wishes to induce. The effort channel includes the manager's response to stronger effort incentives created by increased ownership as well as the change in reporting incentives due to a shift in the earnings distribution. Since the owner chooses the level of effort to maximize her expected profit, the Envelope Theorem implies that the first-order effect of a change in α through the effort channel will be zero. Thus, expected owner profit from the optimal conditional contract for each α is

$$\hat{\Pi}(\alpha) = E(x - g(R(x; a^*(\alpha), \alpha) - x) - W(x; a^*(\alpha), \alpha)), \quad (33)$$

where $a^*(\alpha)$ denotes the level of effort induced by the optimal conditional contract for each α , and the Envelope Theorem implies

$$\hat{\Pi}'(\alpha) = E(-g'(R(x; a^*(\alpha), \alpha) - x)R_\alpha - W_\alpha(x; a^*(\alpha), \alpha)) \quad (34)$$

where R_α and W_α refer to the partial derivatives of R and W holding a fixed.

Given the Distribution Assumptions, Proposition 2 implies that $W = 0$ on $[0, \hat{x})$ and that the reporting function R is continuous with respect to x on $[0, 1]$. The fact that $W=0$ below \hat{x} means that (13) implies

$$EW_\alpha(a^*(\alpha), \alpha) = \int_{x=\hat{x}(\alpha)}^1 [1 + R_\alpha(x; a^*(\alpha), \alpha)](1 - F(x | a^*(\alpha)))dx \quad (35)$$

where the notation $\hat{x}(\alpha)$ reflects the effect of α on the earnings level at which the manager begins to earn positive rent. The continuity of R then allows us to write (34) as

$$\hat{\Pi}'(\alpha) = \int_0^{\hat{x}(\alpha)} (R - x)f(x | a^*(\alpha))dx - \int_{\hat{x}(\alpha)}^1 [(R - x)R_\alpha f(x | a^*(\alpha)) + (1 + R_\alpha)(1 - F(x | a^*(\alpha)))]dx. \quad (36)$$

Adding and subtracting $\int_{\hat{x}}^1 (R - x)fdx$ to the right-hand side of (36) then implies

$$\hat{\Pi}'(\alpha) = E(R - x) - \int_{\hat{x}}^1 (1 + R_\alpha)((R - x)f + 1 - F)dx, \quad (37)$$

where R_α is calculated from (27).³⁵

With effort fixed at $a^*(\alpha)$, the multipliers λ and μ must adjust to maintain equality of the manager's effort first-order condition, (20). Using (27), (20) implies for $a^*(\alpha) > 0$ that

$$- \int_{\hat{x}(a^*(\alpha), \alpha)}^1 (\alpha + (1 - \lambda(a^*(\alpha), \alpha))(F - 1) / f - \mu(a^*(\alpha), \alpha)F_a / f) F_a dx - h'(a^*(\alpha)) = 0 \quad (38)$$

where (38) makes explicit the dependence of λ and μ on a and α . Differentiating (38) with respect to α holding $a^*(\alpha)$ constant then implies

$$- \int_{\hat{x}}^1 (1 - \lambda_\alpha(F - 1) / f - \mu_\alpha F_a / f) F_a dx \equiv 0$$

or that

$$\int_{\hat{x}}^1 (1 + R_\alpha) F_a dx \equiv 0. \quad (39)$$

Finally, substituting the definition of R from (27) into the integrand in (37) yields

$$\hat{\Pi}'(\alpha) = E(R - x) + \int_{\hat{x}}^1 (1 + R_\alpha)(\lambda(F - 1) + \mu F_a) dx. \quad (40)$$

Eq. (39) implies that (40) reduces down to

$$\hat{\Pi}'(\alpha) = E(R - x) + \int_{\hat{x}}^1 \lambda(1 + R_\alpha)(F - 1) dx. \quad (41)$$

If $\lambda=0$, then (41) implies $\hat{\Pi}'(\alpha) = E(R - x)$. However, if $\lambda > 0$, then $EW(x; a^*(\alpha), \alpha) = h(a^*(\alpha))$

and (35) imply

$$EW_\alpha = \int_{\alpha}^1 (1 + R_\alpha)(1 - F) dx = 0, \quad (42)$$

so even if the ex ante individual rationality constraint binds, $\hat{\Pi}'(\alpha) = E(R - x)$. This means that

³⁵Eq. (27) is the correct formula for R if it yields a monotonic reporting function. The formula for R that accounts for the need to "iron" the solution to (27) is derived in the appendix and may involve a constant report \bar{R} at the highest earnings levels. For expositional purposes, we will assume in the text that the optimal reporting function is defined by (27). The more general derivation of R and R_α yields the same result but with more complicated expressions. A copy of this more general derivation is available from the authors on request.

the change in expected owner profit from increasing the manager's shares is solely a function of the average earnings distortion induced by the optimal conditional contract. From Proposition 2, this distortion is strictly positive when $\alpha = 0$ which leads to Proposition 4.

Proposition 4. *Given the Distribution Assumptions, the owner will want to endow the manager with a strictly positive share of the firm.*

The expression $\hat{\Pi}'(\alpha) = E(R - x)$ can be seen to reflect the trade-offs associated with making the manager a shareholder by rewriting it as

$$\hat{\Pi}'(\alpha) = -\alpha F(\hat{x} | a^*) + \int_{\hat{x}}^1 (R - x) dx. \quad (43)$$

The term, $-\alpha F$, reflects the misreporting costs the manager would incur from under-reporting earnings. The benefit to the owner of increasing α is the savings from paying lower marginal rents through less-overstated earnings. Thus, the optimal percentage of shares for the manager trades off the cost of earnings management at low earnings versus lower marginal rents paid to the manager at high earnings. It is achieved when on average the contract induces no earnings distortions.

Returning to Example 1, it is straightforward to demonstrate that the expected manipulation, $E(R-x)$, when $\alpha=0$ is strictly positive and when $\alpha=.05$ equals $-.027$. Thus, in the case of the example, the optimal ownership share of the manager is less than 1.

6. Concluding remarks.

We have characterized in this paper an optimal compensation contract for a manager who must take a hidden action which affects the probability distribution of firm profits. These profits, when realized, are themselves hidden information observable only to the manager, who may engage in earnings manipulation by making earnings reports which differ from the actual level of profits. In contrast with previous work, we model explicitly the managerial retention problem by allowing the manager to leave the firm whenever it is in her best interest to do so.

In this setting, the optimal compensation arrangement may be implemented using two tools: (i) an option to purchase shares of the firm that will be repriced to be in or at the money for lower earnings realizations, as well as (ii) performance bonuses based on (manipulated) earnings reports that are increasing in reported earnings once those reports exceed a well-defined earnings threshold. We also find that the optimal compensation arrangement results in managers under-reporting earnings for small levels of profit, and over-reporting earnings when actual profits are higher. Interestingly, and in contrast to conventional wisdom, giving managers a stake in the firm does not create the incentive to over-report earnings. Indeed, we find that an ownership share reduces over-reporting for high earnings and induces under-reporting for lower earnings realizations. With the optimal ownership share, the average earnings misstatement is zero.

While this pattern of earnings management appears to be at odds with the empirical accounting literature that finds evidence of earnings smoothing (over-reporting low earnings and under-reporting high earnings), we note that such studies (e.g. Badertscher (2009)) do not consider the rent extraction story that drives our results nor the potential role of repriced options while the limited dynamic nature of our model does not permit an earnings smoothing motive to arise. On the other hand, the earnings management pattern we derive does appear to be consistent with empirical evidence that suggests managers reduce their earnings reports in advance of option grants (see for example McAnally et al (2008), although for a very different reason. Carrying out a similar optimal contracting exercise in a more dynamic model that can include such motives is an important project that we will consider in future work.

Moreover, by abstracting away from these dynamic issues, we have been able to identify a new role for options repricing. To the best of our knowledge, our paper is the first to demonstrate that repricing of options, despite numerous claims to the contrary, can be part of an optimal contract when manager retention is an important concern for firms. Our analysis shows

that the ability to reprice options at certain earnings levels allows the owner to avoid paying a manager excessive rents as can be the case with restricted stock grants. For low earnings levels that are associated with zero manager rent, the optimal contract implies that the underwater options remain worthless or alternatively are repriced to be exactly at the money. For intermediate earnings levels that are associated with under-reporting and positive rent, the optimal contract can be implemented by paying all of the manager's rent by repricing the options to be strictly in the money. For high earnings associated with over-reporting, the options will be in the money. The manager will earn her rent through a combination of the profit from exercising the options and a bonus payment.

Our analysis also permits one to study the relationship between earnings management under an optimal contract and the manager's cost of earnings manipulation. For instance, suppose the manager's cost function is $g(R, x) = \beta(R - x)^2 / 2$. Eqs. (18) and (19) remain unchanged so the general structure of the optimal contract will not change. Zero marginal rent will now be associated with the co-state variable, $\hat{\varphi} = \mu F_a - \alpha f / \beta$. Thus, as β increases as a result of higher penalties for earnings management or enhanced oversight by the owner, the magnitude of under-reporting decreases and the range of earnings over which options are repriced to be strictly in the money, (\hat{x}, x^+) , gets smaller but will still exist as long as $\alpha > 0$. Thus, the general incentive properties that yield both under- and over-reporting and that support the use of options repricing would still persist.³⁶

Finally, we can consider the possibility that the manager's earnings report influences the long-run (time 6) stock price. Denote this price by $p(x, R)$ such that $0 < p_x < 1$, $p_R > 0$, and $p_{xR} > 0$. These assumptions are consistent with standard finance models of stock prices that view the stock price as an average of the firm's underlying true value, x , and the earnings

³⁶ We thank a reviewer of an earlier version of this paper for suggesting this comparative statics exercise.

announcement, R . Allowing R to influence the long-run stock price generates three effects. The first effect is that, with $p_x < 1$, the effect of stock ownership on the manager's marginal rents is muted. This reduces the amount of under-reporting associated with zero manager rent. The other two effects are related to the sign of $R - x$ which now depends on a term similar to that in (18) reflecting the rent extraction vs. effort incentive trade-off plus a term that depend on the direct effect of R on p . The second effect is that, with $p_{xR} > 0$, the way the trade-off between rent extraction and effort incentives in (18) affects the sign of $R - x$ is unchanged. The magnitude of this effect will increase for $\alpha > 0$. The third effect arises from $p_R > 0$. This term increases the magnitude of $R - x$ and can imply that the optimal contract induces only over-reporting if the owner also benefits from long-term distortions arising from R . However, as long as the long-term effect of R on p is small, the qualitative features of the optimal contract we study in this model remain unchanged.

Appendix

General information for the proofs.

Problem (11') is solved in two steps. In step 1, a is fixed and we use (16) to solve for the optimal earnings report function, R , and the optimal indirect manager utility function, W , as a function of a . In step 2, the optimal value of a is derived. We may write the owner's expected profit solely as a function of the conditionally optimal earnings report function, R . Noting that $\Pi = x - g - W$ and substituting out W using (13), in step 2 we need only choose a to maximize

$$E[x - g(R(x; a) - x) - W] = E[x - g(R(x; a) - x) + \frac{F(x|a) - 1}{f(x|a)}(\alpha + g'(R(x; a) - x))] - W(0). \quad (\text{A.1})$$

Step 1 is completed by using Proposition 1 which is a translation of Theorems 1 and 2 (chapter 6) in Seierstad and Sydsæter (1987). One issue in verifying that Proposition 1 is in fact an accurate translation involves the integral constraints, (20) and (21). These are constraints of the form $\int_{x=0}^1 m(R(x), x) dx \geq (=) 0$. One way to handle such constraints is to define a new state variable, $y(x)$, such that $y'(x) = m(R(x), x)$, $y(0) = 0$, and $y(1) \geq (=) 0$. If the co-state variable associated with y is $\varphi_y(x)$, the Maximum Principle conditions imply that $\varphi_y(x)$ must be a constant whose value has the same interpretation as a Lagrange multiplier with the integral constraint. Thus, the constant multiplier approach we use in relation to (20) and (21) is completely consistent with Theorems 1 and 2 of Seierstad and Sydsæter (1987).

Because of the pure state constraint, $W \geq 0$, it is possible for $\varphi(x)$ to be discontinuous. Seierstad and Sydsæter (p. 319) allow for this possibility but indicate that with the optimal contract $\varphi(x)$ can only jump down at a finite set of points. At all other points, φ must be continuous. Given α , sufficient conditions for an optimal conditional contract are (18)-(23). Even though we are choosing a arbitrarily in this step, (20) must still be satisfied so that this level of effort is optimal for the manager.

From the discussion in the text, (19) and (22) imply (26), hence $\lambda \leq 1$ (if $\lambda > 1$ the

inequalities in (26) are not consistent). Moreover, if $W(x) = 0$ on a non-degenerate interval, then $W_x = 0$ on such an interval and

$$\varphi(x) = \hat{\varphi}(x) \equiv \mu F_a(x|a) - \alpha f(x|a). \quad (\text{A.2})$$

By (26), $\varphi(x)$ can equal $\hat{\varphi}(x)$ only if $\hat{\varphi}(x) \geq (1-\lambda)(F(x|a)-1)$. Since $f(x|a) > 0$ for all x and a , this inequality holds if, and only if,

$$\Delta(x) \equiv (1-\lambda)(F(x|a)-1) / f(x|a) - \mu F_a(x|a) / f(x|a) + \alpha \leq 0.$$

Distribution Assumption (e) implies that $\Delta(x)$ is strictly concave. Note also that if $R(x)$ is defined by (27), then $R(x) - x = \Delta(x) - \alpha$.

If $\alpha = 0$, then $\hat{\varphi}(0) = \hat{\varphi}(1) = 0$ and for $x \in (0,1)$, $\hat{\varphi}(x) < 0$. If $\alpha > 0$, $\hat{\varphi}(x) < 0$ for all x . Thus, $\Delta(1) > \Delta(0)$, $\Delta(1) \geq 0$ and since f is bounded by Assumption (b), for α sufficiently small and for all $\lambda < 1$, $\Delta(0) < 0$. Then by the continuity and strict concavity of Δ there must exist a unique earnings value $\hat{x} > 0$ such that for all $x < \hat{x}$, $\Delta(x) < 0$, i.e., $\hat{\varphi}(x) > (1-\lambda)(F(x|a)-1)$. However, for α and λ sufficiently large, it is possible that $\Delta(0) \geq 0$. Holding λ , μ , and a fixed, this possibility suggests two cases, both of which are analyzed using arguments similar to those used by Maggi and Rodriguez-Clare (1995) for their "weakly convex case".

Case 1. $\Delta(0) \geq 0$. This case can only arise if $\alpha \geq (1-\lambda) / f(0|a) > 0$.

$\Delta(1) > \Delta(0)$ and the strict concavity of $\Delta(x)$ imply that $\Delta'(0) > 0$ and $\Delta(x) > 0$ for all $x > 0$. Given (26), the feasible co-state function associated with the lowest manager rent is $\varphi(x) = (1-\lambda)(F(x|a)-1)$. Since $\varphi(x) > \hat{\varphi}(x)$ for all $x > 0$, $W_x > 0$ which implies $W(0) = 0$ and $W(x) > 0$ for all $x > 0$. Note that $\Delta(0) < \alpha$. Thus, for x close to 0, $R(x)$ will be strictly less than x .

Case 2. $\Delta(0) < 0$. This case can only arise if $\alpha < (1-\lambda) / f(0|a)$.

As in Case 1, $\Delta(1) > \Delta(0)$ and the strict concavity of $\Delta(x)$ imply that $\Delta'(0) > 0$. Define \tilde{x} such that $\Delta(\tilde{x}) = 0$. Thus, $\Delta'(x) > 0$ on $(0, \tilde{x})$. Since both $\hat{\varphi}$ and F are continuous, Δ is strictly concave, and $\Delta(1) \geq 0$, \tilde{x} is well-defined, unique, and strictly positive. Also, define $\bar{x} = \inf\{x | \hat{\varphi}'(x) > (1-\lambda)f(x|a)\}$. If $\hat{\varphi}'(x) \leq (1-\lambda)f(x|a)$ for all x , define $\bar{x} = 1$. Finally,

define $\hat{x} = \min\{\tilde{x}, \bar{x}\}$.

From Distribution Assumption (c), $\hat{\varphi}'(0) < 0$ so \bar{x} must be strictly greater than zero. Distribution Assumption (f) implies that $\hat{x} = \tilde{x}$. Thus, we can define the co-state variable as $\varphi(x) = \hat{\varphi}(x)$ for $x \leq \hat{x}$, and define $\varphi(x) = (1-\lambda)(F(x|a)-1)$ for $x > \hat{x}$. This case is depicted in Figure 4. Thus, $W(x) = 0$ for all $x \leq \hat{x}$ and for all $x > \hat{x}$, $W(x) > 0$ and $W_x(x) > 0$. For this case, $R(x) < x$ on $(0, x^+)$ and $R(x) > x$ on $(x^+, 1)$.

Remark 1. Without Assumption (c), it is possible for $\hat{x} = 0$. In this situation, φ can neither equal $\hat{\varphi}$ nor $(1-\lambda)(F-1)$ for x close to zero. Then there will exist earnings $0 < x_0 < x_1$ such that for $x < x_0$, $\varphi(x) = (1-\lambda)(F(x|a)-1) + k$ where k is a positive constant and $W(x)$ will be positive. For $x > x_1$, $\varphi(x) = (1-\lambda)(F-1)$. For some $x > x_1$, $W(x) = 0$.

Remark 2. For completeness, we can describe the optimal conditional contract when $\hat{x} = \bar{x}$ so that one can see the role Distribution Assumption (f) is playing. In this case, define $\varphi(x) = \hat{\varphi}(x)$ and $W(x) = 0$ for $x \leq \hat{x}$. Because $\Delta(\hat{x}) < 0$, the co-state variable cannot jump down to $(1-\lambda)(F-1)$ at \hat{x} since for $x \in (\hat{x}, \tilde{x})$, $W_x(x) < 0$, which would imply $W < 0$ in this range. This situation is analogous to the situation studied by Maggi and Rodriguez-Clare (1995) in their "concave case".

Thus, define $\varphi(x) = (1-\lambda)(F(x|a) + k)$ for $x \in (\hat{x}, \tilde{x})$ where $k \in (-1, 0)$ solves

$(1-\lambda)(F(\hat{x}|a) + k) = \hat{\varphi}(\hat{x})$. This new earnings level, \tilde{x} , cannot equal 1 since this would mean $W(1) > 0$ and $\varphi(1) > 0$, which violates one of the transversality conditions in (22). Thus, at \tilde{x} , $\varphi(x)$ must jump down to $(1-\lambda)(F(x|a)-1)$. For all $x \in [\tilde{x}, 1]$, define $\varphi(x) = (1-\lambda)(F(x|a)-1)$.

If \tilde{x} is set too close to \hat{x} , this proposed definition of φ will imply that for some x on (\tilde{x}, \tilde{x}) , $W(x) < 0$. With this φ , $W(x)$ attains a local minimum at \tilde{x} . Thus, we require $W(\tilde{x}) \geq 0$. Between \tilde{x} and \tilde{x} , $W(x)$ is first increasing then decreasing. For $x > \tilde{x}$, $W(x)$ will be increasing. The decreasing segment may cause (11f) to be violated. If this occurs, then (18) becomes

$R - x = (\varphi - \mu F_a - p F_{aa}) / f$ where the multiplier $p \leq 0$ and $\hat{\varphi} = \mu F_a - \alpha f + p F_{aa}$. Notice that the modification to (18) implies that earnings reports will generally be higher so that underreporting

is attenuated and over-reporting is increased while the modification to $\hat{\phi}$ implies that range of earnings associated with underreporting is smaller but still exists as long as $F_{aa}(0|a)$ is sufficiently close to zero.

Proof of Proposition 2.

Based on the above analysis, for any $a > 0$ and for any $\lambda \leq 1$, (12) and (27) define a conditional contract that satisfies (18), (19), and (22) and hence also satisfies the properties of optimal conditional contracts described in Proposition 2 for Cases 1 and 2 as defined above. (For Case 1, $\hat{x} = 0$.) It remains for us to verify that (20), (21), and (23) are also satisfied and that an optimal conditional contract exists.

To check (21), note that for fixed α and a the maximum rents earned by the manager occur when $\lambda = 1$. Given that $h(a)$ is strictly increasing and strictly convex in a , Distribution Assumption (b) implies that EW is bounded while $h(\infty) = \infty$. Thus there exists $\hat{a} > 0$ such that (21) can never be satisfied even when $\lambda = 1$. For all $a < \hat{a}$, there exists a $\lambda < 1$ for which (21) will be satisfied given (27).

To check (20), the manager's first-order condition for effort, note that (11b) is strictly negative for $\mu = 0$ and strictly positive for $\mu = \infty$. Thus, for any $a > 0$, there exists a $\mu > 0$ that satisfies (20).

With regard to (23), for $x \leq \hat{x}$, $R'(x) = 1$ and for $x > \hat{x}$,

$$R'(x) = 1 + \partial((F-1)/f) / \partial x - \mu \cdot \partial(F_a/f) / \partial x. \quad (\text{A.3})$$

By Distribution Assumption (d), the second term in (A.3) is strictly positive. The third term can be either positive or negative. For a sufficiently close to zero, μ will be small enough to ensure that $R'(x) > 0$. The distribution in example 1 can be used to show that for a sufficiently large, (27) can imply $R'(x) < 0$ for some (high) x , which would violate the second order condition (23). Thus, this proof must also show that the solution to (11) when (23) binds for some x has the same features as contracts based on (27). A solution in this case will require the use of “ironing”

techniques (Fudenberg and Tirole, 1991) to characterize an optimal contract.

The more general formulation of the optimal control problem associated with (11) has the Hamiltonian

$$\mathcal{H} = (x - g - W)f + \varphi_W(\alpha + g') + \varphi_R R' \quad (\text{A.4})$$

and the Lagrangean

$$\mathcal{L} = \mathcal{H} + \tau W - \mu[(\alpha + g')F_a + h'f] + \lambda_1 f(W - h) + \lambda_2 R'$$

where W and R are now treated as state variables and R' is the control. φ_W and φ_R are the new co-state variables and $\lambda_2 \geq 0$ is the multiplier for constraint (23). Sufficient conditions analogous to those in Proposition 1 (again following Theorems 1 and 2 from chapter 6 in Seierstad and Sydsæter) are (20), (23),

$$R - x = \frac{\varphi_W - \mu F_a + \varphi_R'}{f}, \quad (\text{A.5})$$

$$-(1 - \lambda_1)f + \tau = -\varphi_W', \quad (\text{A.6})$$

$$\varphi_R + \lambda_2 = 0, \quad (\text{A.7})$$

$$\lambda_1(EW - h(a)) = 0 \text{ and } \lambda_1 \geq 0, \quad (\text{A.8})$$

$$\varphi_W(0) \leq 0, \varphi_W(1) \geq 0, \varphi_W(0)W(0) = \varphi_W(1)W(1) = 0, \tau(x) \geq 0, \tau(x)W(x) = 0, \quad (\text{A.9})$$

$$\varphi_R(0) = \varphi_R(1) = 0, \quad (\text{A.10})$$

$$\lambda_2(x) \geq 0 \text{ and } \lambda_2(x)R'(x) = 0. \quad (\text{A.11})$$

As with Proposition 1, φ_W and φ_R can jump down but not up at a countable number of values of x . Note that if the solution to (11') implies (23), then $\varphi_R(x) \equiv 0$ and (A.5)-(A.11) plus (20) and (23) collapses down to (18)-(23).

Note also that by (A.7), $\varphi_R \neq 0$ only if $R' = 0$ as then $\lambda_2 > 0$. By Distribution Assumption (e), solutions to (11') that violate (23) must do so on an interval from some x' to 1. And, since $R(1) = 1$ as defined by (27), it follows that, on $(x', 1)$, we must have $R(x) > 1$. That is, the "ironed" solution to (11) will induce $R = \bar{R} > 1$ on some interval \underline{x} to 1 where $\underline{x} < x'$.

The optimal conditional contract is constructed as follows. First, define \hat{x} and $\hat{\varphi}$ as above. $\hat{\varphi}$ now represents the value of the co-state variable, φ_W , for which $W_x = 0$ when $\varphi_R' = 0$. Then define $\varphi_W = \hat{\varphi}$ on $[0, \hat{x}]$ and define $\varphi_W = (1 - \lambda_1)(F - 1)$ on $(\hat{x}, 1]$. This is equivalent to the definition of φ above.

Second, define $x_1 = \min\{x \mid x + ((1 - \lambda_1)(F - 1) - \mu F_a) / f = 1\}$. x_1 represents the first earnings level at which the reporting function which solves (11') first equals 1. Since (27) implies $R(1) = 1$, x_1 always exists. If $x_1 = 1$, then (27) will define an increasing reporting function on $[0, 1)$ and $\varphi_R \equiv 0$. If $x_1 < 1$, then (27) must define a reporting function that is decreasing for x close to 1.

Third, choose \bar{R} from the interval $(1, \max_x x + ((1 - \lambda_1)(F - 1) - \mu F_a) / f)$. For each \bar{R} , there will be two solutions to the equation: $x + ((1 - \lambda_1)(F - 1) - \mu F_a) / f = \bar{R}$ on $(x_1, 1)$. Denote these two solutions by \bar{x}_1 and \bar{x}_2 such that $\bar{x}_1 < \bar{x}_2$. The various values of x that have been identified are noted in Figure A1.

Define $\varphi_R = 0$ for $x \leq \bar{x}_1$ and for $x > \bar{x}_1$ solve (A.5) for φ_R' when $R = \bar{R}$. This implies $\varphi_R'(x) = (\bar{R} - x)f - (1 - \lambda_1)(F - 1) + \mu F_a$. $\varphi_R'(x)$ will be negative on (\bar{x}_1, \bar{x}_2) and it will be positive on $(\bar{x}_1, 1]$. As a result,

$$\varphi_R(x) = -\lambda_2(x) = \int_{t=\bar{x}_1}^x [\bar{R} - t - ((1 - \lambda_1)(F - 1) - \mu F_a) / f] f dt \quad (\text{A.12})$$

on $(\bar{x}_1, 1]$. \bar{R} must be chosen so that $\varphi_R(\bar{x}_1) = \varphi_R(1) = 0$. Using (A.12), $\bar{R} = 1$ implies $\varphi_R(1) < 0$ and $\bar{R} = \max_x x + ((1 - \lambda_1)(F - 1) - \mu F_a) / f$ implies $\varphi_R(1) > 0$. Thus, by continuity there must exist a value of \bar{R} for which $\varphi_R(1) = 0$. Thus, ensuring that (23) is satisfied does not alter the quantitative properties of optimal conditional contracts given a and μ .

To summarize, for any $a > 0$, there exists $\mu > 0$ such that the optimal conditional contract must fall into one of four categories: Case 1 without ironing at the top, Case 1 with ironing at the top, Case 2 without ironing at the top, and Case 2 with ironing at the top. The optimal conditional contract in all 4 cases exhibits the pattern of under-reporting for low x described in

the statement of the proposition.

Turning to the issue of existence, the transition between these four cases is continuous and hence expected owner profit is a continuous function of a . If \bar{a} denotes the first-best level of effort, then the optimal level of effort will be less than or equal to \bar{a} . This means the owner's optimal choice of effort to induce exists since it maximizes a continuous function on $[0, \bar{a}]$. Since $h'(0) = 0$, the optimal value of a must be strictly positive. Since we have constructed an optimal conditional contract for each a , the existence of an optimal a for each α implies that an optimal conditional contract exists. ***Q.E.D.***

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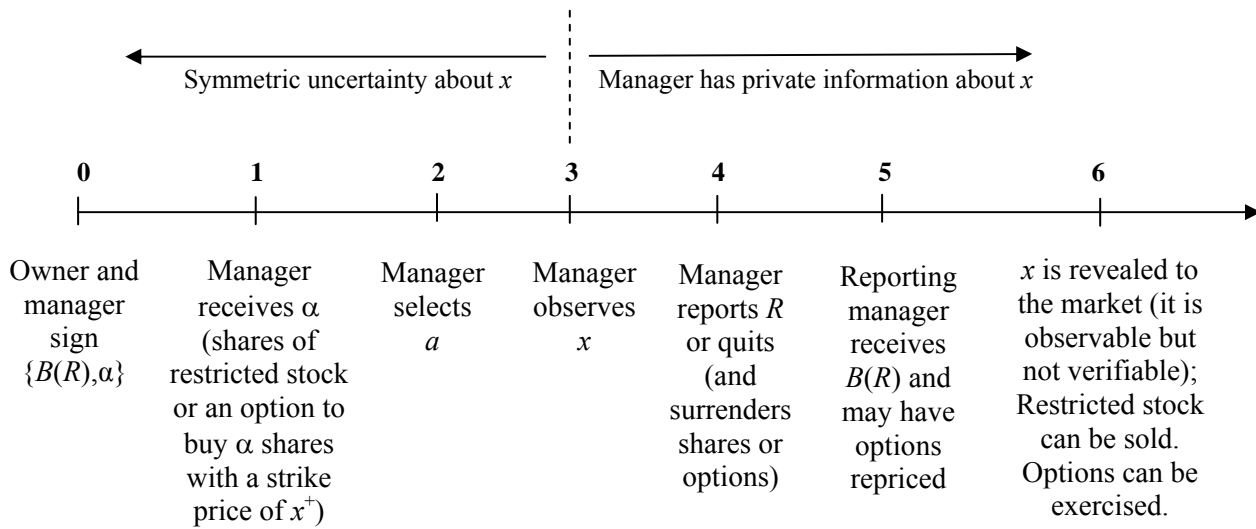


Figure 1: Time Line of Decisions and Observations

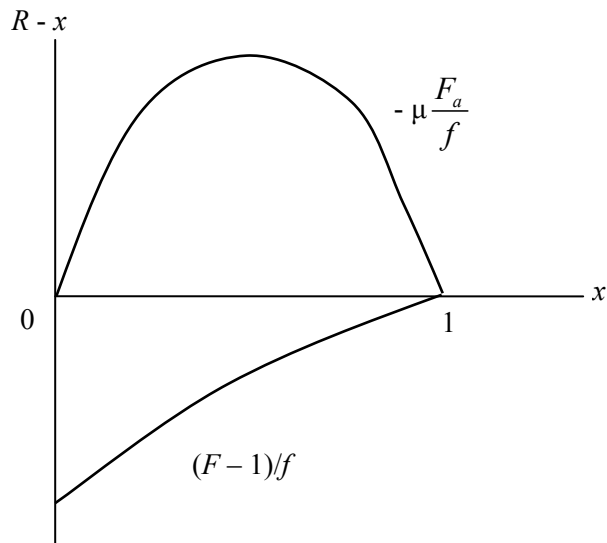


Figure 2: Rent extraction vs. effort incentive effects in the absence of an interim individual rationality constraint

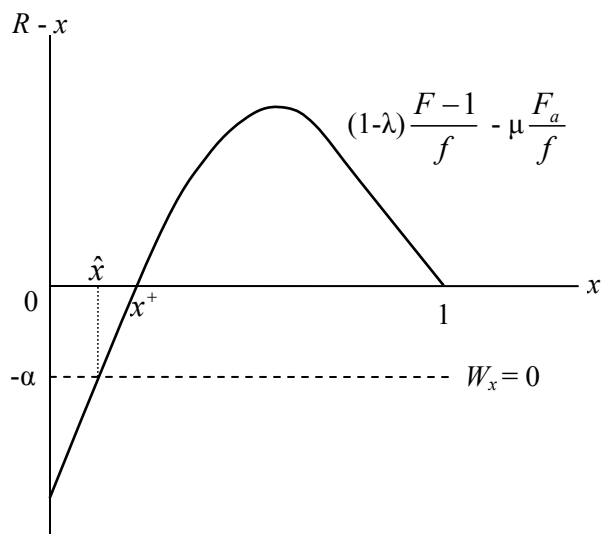


Figure 3: Rent extraction vs. effort incentive effects with an interim individual rationality constraint

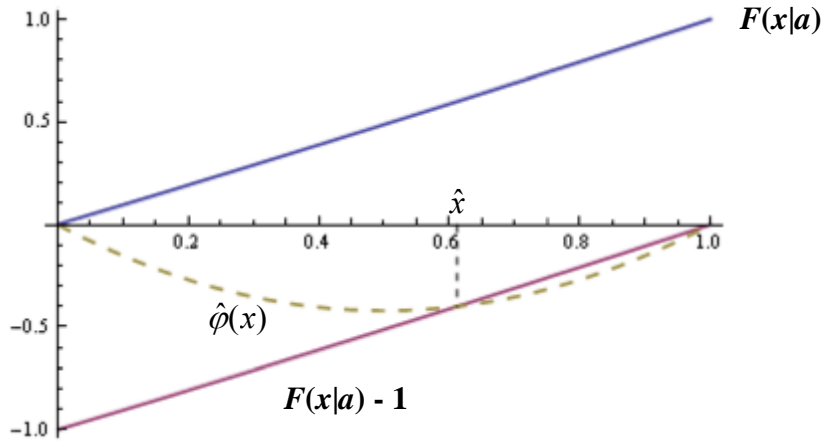


Figure 4: The trade-off between zero manager rent and earnings manipulation in Example 1.

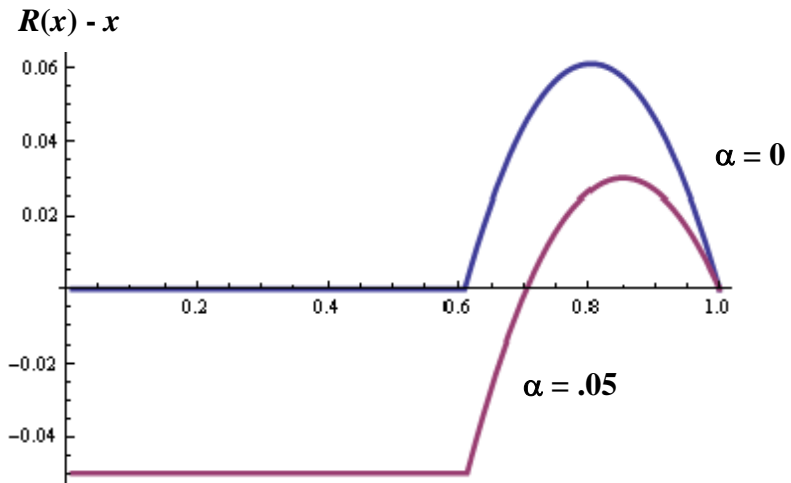


Figure 5: Optimal earnings manipulation when $\alpha = 0$ and when $\alpha = .05$.

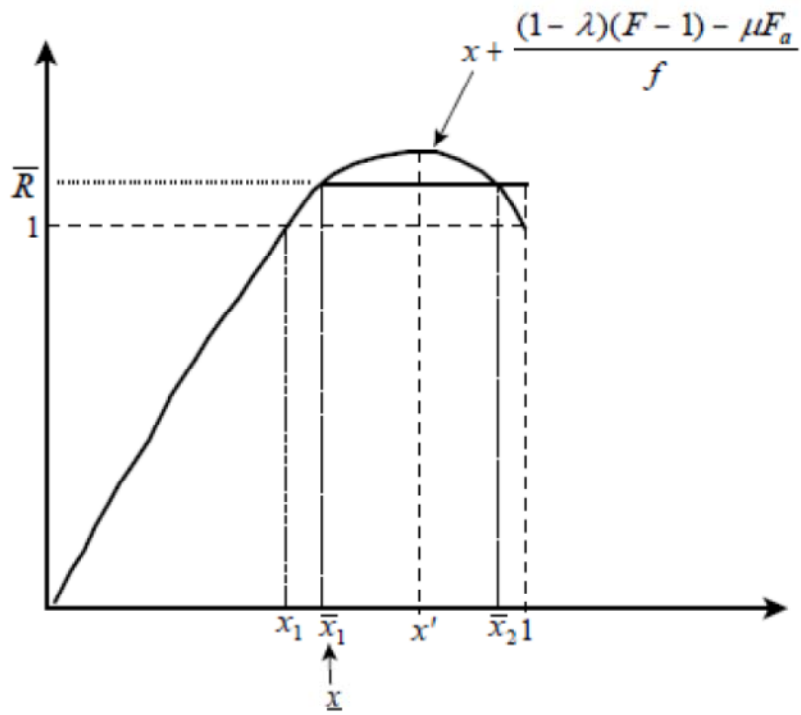


Figure A1. Optimal earnings manipulation when ironing at the top is required.