R&D investment, asymmetric costs, and research joint ventures

Alejandro Montecinos-Pearce* Thomas Gresik†‡§

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Abstract

This paper investigates how an initial asymmetry in production costs affects the welfare differences between non-cooperative R&D investment and a research joint venture, through cost-reducing R&D investment decisions. Using a duopoly model in which R&D investment generates spillovers and builds absorptive capacity, we find that if there exists a critical spillover value for which welfare under the two regimes is the same, then increasing the initial cost asymmetry has an ambiguous effect on this critical spillover value. This ambiguity arises because the relative size of the welfare reduction due to an increase in the initial cost asymmetry is determined by the rate at which absorptive capacity is built. Increasing this rate amplifies the welfare losses from an increase in the cost asymmetry under non-cooperation and mitigates the welfare losses under a RJV.

JEL Classification Codes: O31, L13, C72

Keywords: R&D investment, absorptive capacity, cost asymmetry, research joint venture.

*Escuela de Negocios, Universidad Adolfo Ibáñez

†Department of Economics, University of Notre Dame

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§Contact: amontecinos@uai.cl(Montecinos-Pearce); tgresik@nd.edu(Gresik)

1 Introduction

When a firm invests in R&D, the investment may cause a positive spillover that benefits its competitors, and the size of the spillover can affect whether a research joint venture (RJV) or non-cooperative R&D investment (NC) yields higher welfare. The early literature on RJVs shows that increasing the spillover increases welfare under an RJV.[1] Starting with d'Aspremont and Jacquemin (1988), the theoretical literature has compared the benefits of RJVs and NC for identical firms in a duopoly that choose cost reducing R&D investment in the presence of a spillover.[2] Moreover, in comparing the benefits of RJVs versus NC, this literature has also tried to identify the circumstances under which RJVs generate more aggregate R&D investment than under NC. The main goal of this paper is to determine how an initial cost asymmetry affects the welfare comparisons of RJVs versus NC in the presence of a spillover. We extend the literature initiated by d'Aspremont and Jacquemin (1988) by introducing an initial cost asymmetry and by adopting the Leahy and Neary (2007) focus on welfare differences to compare RJVs to non-cooperative R&D investment.

The introduction of a cost asymmetry is motivated by the empirical literature that documents the presence of cost differences among firms within an industry.[3] Yet the impact of such intra-industry cost heterogeneity on R&D investments and its welfare implications cannot be analyzed with the current theoretical models, as they assume firms always have identical costs. In addition, with asymmetric initial production costs, the traditional rankings of RJVs and NC based on aggregate R&D investment will no longer coincide with broader welfare rankings.

[1]See De Bondt (1996) for a survey of the early literature on spillovers in innovative activities.

[2]López-Pueyo et al. (2008) finds evidence that cost reducing R&D investment of one firm causes spillover benefits to other firms in the same industry. Examples of such spillovers occur in the food, textiles, chemical products, plastic products, and machinery industries.

[3]Olley and Pakes (1996) find evidence of heterogeneity in productivity in the telecommunication industry. Roberts and Tybout (1996) and Pavcnik (2002) also find evidence of this same heterogeneity in a sample of production plants in Colombia and Chile, respectively. There is empirical evidence of an association between firm heterogeneity and RJV formation. Majewski and Williamson (2002) document that the presence of asymmetries between firms causes complementarities, which motivate heterogenous firms to form a RJV. Moreover, Röller, Siebert, and Tombak (2007) provide evidence that links firms' asymmetries and RJV formation. Namely, RJVs are usually formed between firms with similar initial marginal costs.

In our model, we endow one firm with a larger marginal cost of production and analyze the equilibrium effects when the magnitude of this initial asymmetry is close to zero. Following Kamien and Zang (2000), we assume that the exogenous spillover parameter considered by d'Aspremont and Jacquemin (1988) is the maximum possible fraction of R&D investment that spills over from one firm to the other. The actual spillover is endogenously determined by each firm's investment in absorptive capacity, where absorptive capacity captures the idea that a firm learns from a competitor's R&D only if the firm also conducts R&D.[4]

Our analysis focuses primarily on small initial cost asymmetries. We then discuss how several extensions to our model affect our results.[5] We find that when the rate at which absorptive capacity is built is high, an increase of the initial cost asymmetry reduces the critical spillover value and encourages RJV formation.[6] This occurs for two reasons. First under NC, when the competitor builds absorptive capacity at a high rate, the incentive for a firm to limit the spillover benefits of its own R&D investment increases, and aggregate equilibrium R&D investment is reduced. Second, the benefits that the competitor derives from the positive externality under a RJV are internalized by the firms. Thus, when the rate of absorptive capacity acquisition is high, an increase in the initial cost asymmetry will favor RJVs and lead to a reduction in the welfare-based critical spillover value. The effect

[4]RJV and NC regimes under which firms conduct cost reducing R&D investment with spillovers have been studied since d'Apremont and Jacquemin (1988). In their model, d'Apremont and Jacquemin assume that identical firms choose their R&D investment and then they compete in a final good market. In addition to analyzing the effects of R&D spillovers, Kamien, Müller, and Zang (1992) and Amir, Evstigneev, and Wooders (2003) also consider endogenous information sharing. A key extension to this set up introduces endogenous spillovers (Kamien, Müller, and Zang (1992) and Kamien and Zang (2000)). More recently, the focused changed to demand conditions under which endogenizing the spillover effect drives up the incentives to invest in R&D (Grünfeld (2003)), the determination of R&D appropriability through the firm's choice of R&D approaches (Wiethaus (2005)), and distinguishing R&D investment oriented towards inventive activities and R&D investment aimed to learn from others (Hammerschmidt (2009)). Until Leahy and Neary (2007) this early literature studied the normative implications of identical firms R&D investment by focusing on aggregate R&D investment. However, Leahy and Neary use a model with symmetric costs to compare RJVs with NC in terms of welfare differences.

[5]Salant and Shaffer (1999) show that ex ante identical firms may choose to invest different amounts in a RJV. We note that their application to RJVs does not include absorptive capacity. In our model, asymmetric R&D investments do not arise.

[6]This positive link between firms' initial cost asymmetries and RJV formation is in contrast to the finding in Röller, Siebert, and Tombak (2007) who show that initial firm asymmetries create a disincentive for firms to form a RJV. This difference arises because their model includes no R&D spillover effects.

is reversed when the rate at which absorptive capacity is built is low.

Also, our analysis identifies the key channels through which cost asymmetries affect welfare comparisons between a RJV and NC, thereby providing a useful framework for empirical work. We show that in the presence of a small cost asymmetry and sufficiently large demand, equilibrium welfare under a RJV is increasing in the exogenous spillover parameter while equilibrium welfare under NC can be non-monotonic.[7] Under a RJV, as the exogenous spillover parameter increases, R&D investment increases, implying that equilibrium welfare under a RJV necessarily increases because the direct marginal cost reduction effect of the spillover is reinforced by more R&D investment thereby increasing aggregate production. Under NC, as the exogenous spillover parameter increases, R&D investment decreases and results in less equilibrium marginal cost reduction which reduces aggregate production. However, an increase of the exogenous spillover parameter also directly reduces the marginal cost of production and hence increases aggregate production, because holding R&D investment fixed, larger spillovers allow each firm to take more advantage of the competitor's R&D investment. Thus, the direct marginal cost reduction effect increases welfare while the decrease in R&D investment increases the marginal cost of production thereby decreasing welfare. When the exogenous spillover parameter is small, the direct marginal cost reduction effect dominates and NC welfare is increasing in the spillover parameter. However, when the exogenous spillover parameter is large, the R&D investment effect dominates and NC welfare is decreasing in the the spillover parameter.

The rest of the paper is organized as follows. In section 2, the model is introduced. Then in section 3, the main effects of a cost asymmetry on welfare are analyzed. Finally in section 4, we discuss the results and conclude.

[7]Our analytic results formalize Leahy and Neary's (2007) numerical examples, in which non-monotonic equilibrium welfare under NC can arise even with symmetric production costs.

2 The Model

We build upon the model pioneered by d'Aspremont and Jacquemin (1988), and used more recently by Leahy and Neary (2007) and Hammerschmidt (2009), in which two firms compete in a two-stage game. In the first stage the two firms ($i, j \in \{1, 2\}$ and $i \neq j$) simultaneously choose R&D investment. We will consider two scenarios: NC in which the firms choose their R&D investments non-cooperatively and RJV in which the firms choose their R&D investments to maximize industry profits. Then, in the second stage, the firms engage in Cournot competition in the market for an homogeneous product. The solution concept we use in both scenarios is subgame perfection.

The inverse demand function is $p = a - q_1 - q_2$, where $a > 0$ is a demand parameter that determines its size, and q_i is the quantity produced by firm $i \in \{1, 2\}$. Each firm has a constant return to scale production function, which renders a total cost of production $C_i = c_i q_i$, where c_i is firm i's unit cost of production. We introduce an initial cost asymmetry by assuming in the absence of any R&D investment that $c_2 - c_1 = \varepsilon$, where $\varepsilon \geq 0$. Thus ε represents the magnitude of the initial cost advantage that firm 1 has over firm 2.[8] Each firm i is able to reduce its initial unit cost of production through the cost reduction function $f(z_i)$. Including the cost-reducing effects of each firm's R&D investment we can write $c_1 = c - f(z_1)$ and $c_2 = c + \varepsilon - f(z_2)$, where $z_i = x_i + \gamma(x_i)\theta x_j$ is firm i 's effective investment in R&D. Note that larger values of ε imply higher average production costs in the market. Firm i can reduce its marginal production cost directly by investing $x_i \geq 0$ in R&D, and through an endogenously determined spillover effect $\gamma(x_i)\theta x_j$. This term has three components. The first component is firm j 's R&D investment, x_j. The second component is the exogenous spillover parameter, $\theta \in [0, 1]$. It represents the maximum fraction of firm j 's R&D investment, x_j, that can spill over to firm i. The third component is firm i 's absorptive capacity $\gamma(x_i) : R_+ \to [0, 1]$. It determines firm i's ability to take advantage of the maximum spillover, θx_j, from firm j 's R&D investment by incorporating it into its effective

[8] We consider alternative ways to formulate an initial cost asymmetry in section 3.5.

R&D investment, z_i. The absorptive capacity function $\gamma(x_i)$ is strictly increasing, strictly concave, and continuously differentiable. It is also assumed that the cost reduction function $f(z_i) : R_+ \to [0, s]$ is strictly increasing, strictly concave, and continuously differentiable. The number s is the maximum possible level of cost reduction and $0 < s < c$.

Leahy and Neary (2007) consider a specification of the endogenous spillover term that encompasses some special cases considered in the literature, such as Kamien and Zang (2000). Our formulation of the endogenous spillover differs from that found in Leahy and Neary (2007) in two ways. First the Leahy and Neary formulation uses a parameter that represents the difficulty of absorbing the rival's R&D investment. This absorbing difficulty parameter in Leahy and Neary (2007) is equivalent to $1 - \theta$. Second, the model presented here allows for an additional dimension of absorptive capacity acquisition related to the rate at which absorptive capacity is built. This new dimension is introduced in section 3.3 through a parameter λ by adopting Hammerschmidt's (2009) absorptive capacity function $\gamma(x_i) = (x_i/1 + x_i)^\lambda$ for $\lambda \in [0, 1]$. With Hammerschimdt's function lower values of λ correspond to a more concave function, which means that initial R&D investments will allow a firm to acquire absorptive capacity more quickly.

Together these model components imply that firm i earns a profit of $\pi_i(q_i, q_j, x_i, x_j, \theta, \varepsilon) = p(q_i + q_j)q_i - c_i(x_i, x_j, \theta, \varepsilon)q_i - x_i$. The solution of the second stage problem corresponds to a standard Cournot game. We denote firm i 's subgame equilibrium production quantity by $q_i(x_i, x_j, \theta, \varepsilon)$ where $q_1(x_i, x_j, \theta, \varepsilon) = 1/3\,(a - c + \varepsilon + 2f(z_1) - f(z_2))$ and $q_2(x_i, x_j, \theta, \varepsilon) = 1/3\,(a - c - 2\varepsilon + 2f(z_2) - f(z_1))$, and firm i 's second-stage indirect profit function by

$$\begin{aligned}\Pi_i(x_i, x_j, \theta, \varepsilon) = \quad & p\left(q_i(x_i, x_j, \theta, \varepsilon) + q_j(x_j, x_i, \theta, \varepsilon)\right) q_i(x_i, x_j, \theta, \varepsilon) \\ & - c_i(x_i, x_j, \theta, \varepsilon) q_i(x_i, x_j, \theta, \varepsilon) - x_i.\end{aligned}$$

Notice that a larger initial cost asymmetry increases firm 1's output and decreases firm 2's output in every subgame defined by x_1 and x_2. The subgame equilibrium quantities also reveal that an increase in one firm's effective R&D investment increases its subgame equilibrium quantity more than it decreases its competitor's quantity. With the linear demand specification $\Pi_i(x_i, x_j, \theta, \varepsilon) = q_i^2(x_i, x_j, \theta, \varepsilon) - x_i$ so that changes in θ affect Π_i only through changes in firm i's output.

The NC case is defined by the reduced form non-cooperative game $G^{RNC} \equiv \{X_1, X_2; \Pi_1, \Pi_2\}$, where for all i firm i 's strategy space is $X_i = \{x_i : x_i \epsilon R_+\}$. We assume that a unique, stable, pure strategy equilibrium to G^{RNC} for each (θ, ε) exists, and we refer to it as $(x_1^*(\theta, \varepsilon), x_2^*(\theta, \varepsilon))$, where for all $i \neq j, x_i^{BR}(x_j, \theta, \varepsilon) = \arg\max_{x_i \geq 0} \Pi_i$ and $x_i^*(\theta, \varepsilon) = x_i^{BR}\left(x_j^*(\theta, \varepsilon), \theta, \varepsilon\right)$. Because $\Pi_i = q_i^2 - x_i$ in every subgame defined by (x_1, x_2) the subgame perfect equilibrium R&D investment levels are defined by $2q_i \partial q_i / \partial x_i = 1$ for $i = 1, 2$.

The RJV case is defined by the reduced form cooperative problem

$$\begin{aligned} \max_{x_1, x_2 \geq 0} \Pi_1 + \Pi_2 &= p\left(q_1\left(x_1, x_2, \theta, \varepsilon\right) + q_2\left(x_2, x_1, \theta, \varepsilon\right)\right)\left(q_1\left(x_1, x_2, \theta, \varepsilon\right) + q_2\left(x_1, x_2, \theta, \varepsilon\right)\right) \\ &\quad - (C_1 + C_2) - (x_1 + x_2) \end{aligned} \tag{1}$$

A solution to (1) is (x_1^{*c}, x_2^{*c}). Since $q_i\left(x_i, x_j, \theta, \varepsilon\right)$ is bounded and continuous in $(x_i, x_j, \theta, \varepsilon)$, a solution to (1) for each (θ, ε) exists, and we assume it is unique.

The non-cooperative equilibrium production and the cooperative production decisions are $q_i^*(\theta, \varepsilon) = q_i\left(x_i^*(\theta, \varepsilon), x_j^*(\theta, \varepsilon), \theta, \varepsilon\right)$ and $q_i^{*c}(\theta, \varepsilon) = q_i\left(x_i^{*c}(\theta, \varepsilon), x_j^{*c}(\theta, \varepsilon), \theta, \varepsilon\right)$ respectively. To simplify notation we use $x_i^* = x_i^*(\theta, \varepsilon), x_i^{*c} = x_i^{*c}(\theta, \varepsilon)$, $q_i^* = q_i^*(\theta, \varepsilon)$, and $q_i^{*c} = q_i^{*c}(\theta, \varepsilon)$ when convenient. We denote equilibrium firm profits by $\Pi_i^*(\theta, \varepsilon) = \Pi_i(x_i^*, x_j^*, \theta, \varepsilon)$ and $\Pi_i^{*c}(\theta, \varepsilon) = \Pi_i(x_i^{*c}, x_j^{*c}, \theta, \varepsilon)$. Aggregate R&D investment is defined by $X = x_1 + x_2$. Welfare is defined by $W = CS(Q) + \pi_i(q_1, q_2, x_1, x_2, \theta, \varepsilon) + \pi_2(q_2, q_1, x_2, x_1, \theta, \varepsilon)$, where aggregate production Q is defined by $Q = q_1 + q_2$, and consumer surplus $CS(Q)$ is defined by $CS(Q) = \int_0^Q p(t)\, dt - p(Q)\, Q$. Evaluating the aggregate R&D investment and welfare functions in the non-cooperative and cooperative equilibria yields the respective equilibrium aggregate R&D investment and welfare levels: $X^* = x_1^* + x_2^*$, $X^{*c} = x_1^{*c} + x_2^{*c}$, $W^* = CS\left(Q^*\right) + \Pi_1^* + \Pi_2^*$ and $W^{*c} = CS\left(Q^{*c}\right) + \Pi_1^{*c} + \Pi_2^{*c}$, where $Q^* = q_1^* + q_2^*$ and $Q^{*c} = q_1^{*c} + q_2^{*c}$.[9]

Finally, we define the set of welfare-based critical spillover values as $\Theta^*\left(\varepsilon\right) = \left\{\theta \in [0,1] \middle| W^*\left(\theta, \varepsilon\right) = W^{*c}\left(\theta, \varepsilon\right)\right\}$. Given an initial cost asymmetry ε, this set of spillover values contains all the spillover values $\theta \in [0,1]$ for which equilibrium welfare under a RJV is equal to equilibrium welfare under NC. At this stage of the development of the model we define $\Theta^*\left(\varepsilon\right)$ as a set to allow for multiple welfare-based critical spillover values. Proposition 3 provides sufficient conditions under which there exists a unique critical spillover value. In order to compare our results to the previous literature we can also define the set of investment-based critical spillover values $\Theta'\left(\varepsilon\right) = \left\{\theta \in [0,1] \middle| X^*\left(\theta, \varepsilon\right) = X^{*c}\left(\theta, \varepsilon\right)\right\}$. Lemma 1 shows that in the absence of any cost asymmetry these two sets are identical when demand is sufficiently large.

Lemma 1 *There exists $\overline{a} > 0$ such that for all $a > \overline{a}$ and $\theta \in \Theta^*(0)$, $x^*\left(\theta, 0\right) = x^{*c}\left(\theta, 0\right)$ and $\Theta'(0) = \Theta^*(0)$.*

Our reading of the literature suggests that RJVs and NC are compared in terms of aggregate R&D investment because the first papers in the literature also had a specific focus on R&D investment. Lemma 1 shows that in absence of any cost asymmetry, adopting a welfare approach would imply the same critical spillover values as would an aggregate R&D investment approach. However, this is not generally true in the presence of an initial cost asymmetry, so in the rest of the paper we will consider welfare differences between RJVs and NC.

[9]The assumptions on inverse demand, production costs, the cost reduction function, and the absorptive capacity function, ensure that the equilibrium values of X, Π_i, CS, Q, and W are continuous and differentiable.

3 R&D Investment and Welfare

Three sets of results are presented in this section all focusing on initial cost asymmetries close to zero. First, we show how welfare changes when the spillover parameter θ changes in the presence of a small asymmetry. Second, we show how welfare changes when ε changes. Third, we show that if a critical spillover value exists, then it is unique and we show how it changes with the size of ε. All proofs are in the Appendix. We then discuss these effects for larger cost asymmetries.

3.1 Marginal welfare effect of the exogenous spillover θ

Since d'Aspremont and Jacquemin (1988), the literature has acknowledged that the welfare differences between a RJV and NC depends on the magnitude of the spillover parameter θ. Using the fact that $\Pi_i = q_i^2 - x_i$, direct calculation of $\partial W^*/\partial\theta$ and $\partial W^{*c}/\partial\theta$ yields[10]

$$\begin{aligned} \frac{\partial W^*}{\partial\theta} = &\left\{CS'\left(q_1^* + q_2^*\right)\cdot\frac{\partial(q_1+q_2)}{\partial x_1} + 2q_2^*\frac{\partial q_2}{\partial x_1}\right\}\frac{\partial x_1^*}{\partial\theta} \\ &+\left\{CS'\left(q_1^* + q_2^*\right)\cdot\frac{\partial(q_1+q_2)}{\partial x_2} + 2q_1^*\frac{\partial q_1}{\partial x_2}\right\}\frac{\partial x_2^*}{\partial\theta} \\ &+CS'\left(q_1^* + q_2^*\right)\cdot\frac{\partial(q_1+q_2)}{\partial\theta} + 2q_1^*\frac{\partial q_1}{\partial\theta} + 2q_2^*\frac{\partial q_2}{\partial\theta} \end{aligned} \tag{2}$$

and

$$\begin{aligned} \frac{\partial W^{*c}}{\partial\theta} = &\left\{CS'\left(q_1^{*c} + q_2^{*c}\right)\cdot\frac{\partial(q_1+q_2)}{\partial x_1} + 2q_2^{*c}\frac{\partial q_2}{\partial x_1}\right\}\frac{\partial x_1^{*c}}{\partial\theta} \\ &+\left\{CS'\left(q_1^{*c} + q_2^{*c}\right)\cdot\frac{\partial(q_1+q_2)}{\partial x_2} + 2q_1^{*c}\frac{\partial q_1}{\partial x_2}\right\}\frac{\partial x_2^{*c}}{\partial\theta} \\ &+CS'\left(q_1^{*c} + q_2^{*c}\right)\cdot\frac{\partial(q_1+q_2)}{\partial\theta} + 2q_1^{*c}\frac{\partial q_1}{\partial\theta} + 2q_2^{*c}\frac{\partial q_2}{\partial\theta}. \end{aligned} \tag{3}$$

The first two terms in (2) and (3) account for the indirect welfare effect of θ through R&D investment decisions. The third line in both equations accounts for the indirect effect of θ through changes in consumer surplus and firm profits caused by changes in output. Writing

[10] All derivatives are evaluated at $\left(q_i^*, q_j^*, x_i^*, x_j^*, \theta, \varepsilon\right)$ or $\left(q_i^{*c}, q_j^{*c}, x_i^{*c}, x_j^{*c}, \theta, \varepsilon\right)$

the output effect of θ on W^* and W^{*c} in more detail yields

$$\left(\frac{q_2^*}{3}-\frac{5q_1^*}{3}\right)\cdot\frac{\partial c_1}{\partial\theta}+\left(\frac{q_1^*}{3}-\frac{5q_2^*}{3}\right)\cdot\frac{\partial c_2}{\partial\theta} \tag{4}$$

and

$$\left(\frac{q_2^{*c}}{3}-\frac{5q_1^{*c}}{3}\right)\cdot\frac{\partial c_1}{\partial\theta}+\left(\frac{q_1^{*c}}{3}-\frac{5q_2^{*c}}{3}\right)\cdot\frac{\partial c_2}{\partial\theta} \tag{5}$$

where $\partial c_i/\partial\theta = -f'(z_i)\gamma(x_i)x_j < 0$. Expression (4) is the welfare effect under NC from a change in θ through its effect on firm output, which is generated by a change in both firms' marginal cost of production. As θ increases, holding x_1 and x_2 fixed, each firm experiences a reduction in its marginal cost of production. With lower marginal production costs, both firms increase their second-stage output decisions, which increases consumer surplus and firm profits. An analogous interpretation holds for (5).[11] Substituting (4) and (5) into (2) and (3) respectively shows that a change in θ affects welfare through two channels: R&D investment and marginal cost. Proposition 1 shows how equilibrium welfare changes with respect to the exogenous spillover parameter. For the NC case these welfare changes depend on the elasticity of R&D investment with respect to θ which we define as
$\eta_{x_i,\theta} = -\left(\theta/x_i\left(\theta,0\right)\right)\cdot\partial x_i\left(\theta,0\right)/\partial\theta$.

Proposition 1 *There exists* $\overline{\varepsilon}>0$*, such that for all* $0\leq\varepsilon<\overline{\varepsilon}$

$$\frac{\partial W^{*c}\left(\theta,\varepsilon\right)}{\partial\theta}>0 \tag{6}$$

and if $2\gamma'\left(x_i\right)x_i-\gamma\left(x_i\right)<0$ *for all* x_i*, there exists* $\overline{a}>0$*, such that for all* $a>\overline{a}$

$$\frac{\partial W^{*}\left(\theta,\varepsilon\right)}{\partial\theta}\geq 0 \quad if \quad \eta_{x_i,\theta}\leq\frac{2}{3} \tag{7}$$

[11]It is easy to show that there exists $\overline{\varepsilon}$ such that (4) and (5) are strictly positive for all $\varepsilon<\overline{\varepsilon}$.

and

$$\frac{\partial W^*(\theta,\varepsilon)}{\partial \theta} < 0 \quad if \quad \eta_{x_i,\theta} > \frac{2}{3}. \tag{8}$$

Proposition 1 establishes the sign of the change in total welfare with respect to a change in θ for the RJV and NC cases when the initial cost asymmetry is small. Proposition 1 shows that equilibrium welfare is monotonically increasing under a RJV for all θ, and increasing for small θ under NC. This result formalizes and extends Leahy and Neary's (2007) numeric example for the symmetric case. Equilibrium welfare under a RJV is monotonically increasing because joint profit maximization implies that R&D investment increases as θ increases. Therefore the welfare effect through R&D investment of an increase in θ reinforces the positive welfare effect through marginal cost reduction. The technical condition on the absorptive capacity function in the second half of the proposition ensures that absorptive capacity is not built too quickly. This upper bound on the rate at which absorptive capacity can be built, guarantees that each dollar of R&D investment benefits the competitor enough to reduce the incentive to invest in R&D under NC. This investment disincentive induces a countervailing welfare effect through the R&D channel that offsets the welfare effects through the marginal cost channel. Moreover, if R&D investment is sufficiently elastic, the increase in θ reduces R&D investment enough to offset its welfare-increasing effect through the reduction in marginal costs.

If the technical condition on $\gamma(\cdot)$ is violated for some x_i, the method of proof used suggests that it may be more likely that $\partial W^*(\theta,\varepsilon)/\partial\theta \geq 0$ for all θ as in (7). Consider however the limiting case in which $\lim_{x_i\to\infty} \gamma'(x_i)\, x_i = \infty$. This limiting case approximates the d'Apremont and Jacquemin (1988) model. Since the sign of $\partial W^*(\theta,\varepsilon)/\partial\theta$ still depends on the elasticity implicit in (7) and (8), we conjecture that the technical condition on $\gamma(\cdot)$ is not necessary.

3.2 Marginal welfare effect of the cost advantage ε

Direct calculation of $\partial W^*/\partial\varepsilon$ and $\partial W^{*c}/\partial\varepsilon$ implies

$$\begin{aligned}\frac{\partial W^*}{\partial\varepsilon} = & \left\{CS'\left(q_1^*+q_2^*\right)\cdot\frac{\partial(q_1+q_2)}{\partial x_1}+2q_2^*\frac{\partial q_2}{\partial x_1}\right\}\frac{\partial x_1^*}{\partial\varepsilon} \\ & +\left\{CS'\left(q_1^*+q_2^*\right)\cdot\frac{\partial(q_1+q_2)}{\partial x_2}+2q_1^*\frac{\partial q_1}{\partial x_2}\right\}\frac{\partial x_2^*}{\partial\varepsilon} \\ & +CS'\left(q_1^*+q_2^*\right)\cdot\frac{\partial(q_1+q_2)}{\partial\varepsilon}+2q_1^*\frac{\partial q_1}{\partial\varepsilon}+2q_2^*\frac{\partial q_2}{\partial\varepsilon}\end{aligned} \tag{9}$$

and

$$\begin{aligned}\frac{\partial W^{*c}}{\partial\varepsilon} = & \left\{CS'\left(q_1^{*c}+q_2^{*c}\right)\cdot\frac{\partial(q_1+q_2)}{\partial x_1}+2q_2^{*c}\frac{\partial q_2}{\partial x_1}\right\}\frac{\partial x_1^{*c}}{\partial\varepsilon} \\ & +\left\{CS'\left(q_1^{*c}+q_2^{*c}\right)\cdot\frac{\partial(q_1+q_2)}{\partial x_2}+2q_1^{*c}\frac{\partial q_1}{\partial x_2}\right\}\frac{\partial x_2^{*cc}}{\partial\varepsilon} \\ & +CS'\left(q_1^{*c}+q_2^{*c}\right)\cdot\frac{\partial(q_1+q_2)}{\partial\varepsilon}+2q_1^{*c}\frac{\partial q_1}{\partial\varepsilon}+2q_2^{*c}\frac{\partial q_2}{\partial\varepsilon}.\end{aligned} \tag{10}$$

The first two terms in (9) and (10) account for the indirect welfare effect of ε through R&D investment decisions. The third line in both equations accounts for the direct effect through changes in consumer surplus and firm profits caused by changes in output. Performing an analogous decomposition of the output effect to that in (4) and (5) implies that the output effect under NC equals $q_1^*/3-5q_2^*$ and under a RJV equals $q_1^{*c}/3-5q_2^{*c}$. For ε close to zero, both terms are strictly negative. An increase in ε also reduces welfare through the *R&D* effect. Proposition 2 thus shows that welfare decreases since an increase in ε increases firm 2's initial cost disadvantage.

Proposition 2 *There exists $\bar{\varepsilon}>0$, such that for all $0\leq\varepsilon<\bar{\varepsilon}$, $\partial W^*\left(\theta,\varepsilon\right)/\partial\varepsilon<0$ and $\partial W^{*c}\left(\theta,\varepsilon\right)/\partial\varepsilon<0$ for all θ.*

Proposition 2 follows from the fact that the welfare effects through R&D investment and marginal costs are all decreasing. This occurs for two reasons. First, fixing firm 1's unit cost of production, as firm 2's unit cost of production increases, R&D investment decreases, yielding less cost reduction and welfare. Second, an increase in ε directly reduces firm 2's production by more than firm 1's production increase in the second stage also inducing a decrease of welfare.

3.3 The welfare ranking of RJV and non-cooperative R&D investment

In this section we study how the size of the initial cost asymmetry affects the difference in welfare between a RJV and NC when the initial cost asymmetry is small. Proposition 3 provides sufficient conditions for the critical spillover value to be unique when it exists. Then, we establish how a change in ε affects the critical spillover value when the initial cost asymmetry ε is small and demand is sufficiently large. Finally, using Hammerschmidt's (2009) absorptive capacity function, we show how the rate at which absorptive capacity is built plays a key role in determining how a change in the cost asymmetry affects the critical spillover value.

Proposition 3 *Suppose* $\Theta^*(\varepsilon) \neq \emptyset$, *and* $2\gamma'(x_i)\, x_i - \gamma(x_i) < 0$ *for all* x_i. *Then there exists* $\overline{a}, \overline{\varepsilon} > 0$ *such that for all* $0 \leq \varepsilon < \overline{\varepsilon}$ *and* $a > \overline{a}$, $\theta^*(\varepsilon)$ *is the unique critical spillover value. Moreover, for* $0 \leq \theta < \theta^*(\varepsilon)$, $W^*(\theta, \varepsilon) > W^{*c}(\theta, \varepsilon)$, *and for* $\theta^*(\varepsilon) \leq \theta \leq 1$, $W^*(\theta, \varepsilon) \leq W^{*c}(\theta, \varepsilon)$.

Define $\Lambda_\theta(\theta, \varepsilon) = \partial W^*(\theta, \varepsilon)/\partial\theta - \partial W^{*c}(\theta, \varepsilon)/\partial\theta$ where $\Lambda_\theta(\theta, \varepsilon)$ measures the effect of a change in θ on the welfare difference under NC and a RJV. The proof of Proposition 3 shows, if, when the initial cost asymmetry is small there exists a critical spillover value $\theta^*(\varepsilon)$, then equilibrium welfare under NC crosses equilibrium welfare under a RJV from above, i.e. $\Lambda_\theta(\theta^*(\varepsilon), \varepsilon) < 0$. Then Proposition 1 directly implies that W^* and W^{*c} cross at most once. Suppose that for some ε , $W^{*c}(0, \varepsilon) > W^*(0, \varepsilon)$. Under the conditions in Proposition 3, there cannot exist a critical spillover value since $W^{*c}(\theta, \varepsilon) > W^*(\theta, \varepsilon)$ for all θ. Therefore a necessary condition for $\Theta^*(\varepsilon) \neq \emptyset$ is $W^*(0, \varepsilon) > W^{*c}(0, \varepsilon)$. As a result $W^*(\theta, \varepsilon) > W^{*c}(\theta, \varepsilon)$

for $\theta < \theta^*(\varepsilon)$ and $W^*(\theta, \varepsilon) \leq W^{*c}(\theta, \varepsilon)$ for $\theta \geq \theta^*(\varepsilon)$. Figure 1 presents a graphical representation of Propositions 1 and 3 based on Hammerschimdt's (2009) example, in which firm i 's cost reduction function is $f(z_i) = s(z_i/1 + z_i)^\rho$, its absorptive capacity function is $\gamma(x_i) = (x_i/1 + x_i)^\lambda$, $a = 1.5$, $c = 0.25$, $s = 0.125$, $\rho = 0.47$, and $\lambda = 0.005$. $\gamma(\cdot)$ satisfies the technical condition of Proposition 1 for all $\lambda \in \left[0, \frac{1}{2}\right)$.[12] Consistent with Proposition 1, the example shows that equilibrium welfare under a RJV (W^{*c}) is monotonically increasing in θ and that equilibrium welfare under NC (W^*) is increasing for low values of θ.

Figure 1 also confirms that W^* can be decreasing for larger values of θ. Consistent with Proposition 3, this example shows that for $\theta > \theta^*(\varepsilon)$, a RJV provides higher welfare than under NC, and when $\theta < \theta^*(\varepsilon)$ the reverse is true.

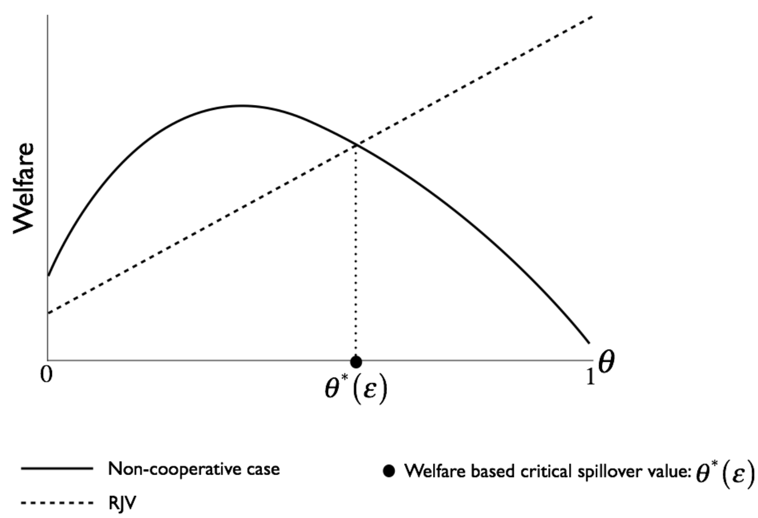

Figure 1: Welfare based critical spillover.

To understand how a change in ε affects the critical spillover value, define $\Lambda_\varepsilon(\theta, \varepsilon) = \partial W^*(\theta, \varepsilon)/\partial\varepsilon - \partial W^{*c}(\theta, \varepsilon)/\partial\varepsilon$. Then direct calculation yields $\partial\theta^*(\varepsilon)/\partial\varepsilon = -\Lambda_\varepsilon(\theta^*(\varepsilon), \varepsilon)/\Lambda_\theta(\theta^*(\varepsilon), \varepsilon)$. Under the conditions of Proposition 3, $sign(\partial\theta^*(\varepsilon)/\partial\varepsilon) = sign(\Lambda_\varepsilon(\theta^*(0), 0))$ for ε close to zero. That is, when the cost asymmetry

[12]The functional forms for f and γ imply that for $\varepsilon = 0$ the firms' R&D investments are symmetric under a RJV.

is small, the change in the welfare difference with respect to ε between a RJV and NC determines the direction of the change of the welfare-based critical spillover as ε changes. By Proposition 2, equilibrium welfare in both the NC and RJV cases (the solid and the dashed lines, as function of θ, in Figure 1 respectively) shift down as ε increases. Thus, the magnitude of the reduction in equilibrium welfare under a RJV with respect to an increase in the initial cost asymmetry, relative to the non-cooperative R&D investment case determines whether the critical spillover value $\theta^*(\varepsilon)$ decreases, stays at the initial level, or increases. If $\theta^*(\varepsilon)$ is decreasing, i.e. the welfare loss due to an increase in ε is less under a RJV than under non-cooperative R&D investment, and a RJV will generate higher welfare than non-cooperative R&D investment for a larger set of exogenous spillovers. This result is contrary to the claim in Röller, Siebert, and Tombak (2007) that an increase in an initial cost asymmetry must discourage RJV formation. If $\theta^*(\varepsilon)$ is increasing, welfare will be larger under a RJV for a smaller set of exogenous spillovers. Direct calculation of $\Lambda_\varepsilon(\theta^*(0), 0)$ shows that both outcomes are possible.

Figure 2 shows how the rate at which absorptive capacity is built can influence the direction of the change of $\theta^*(\varepsilon)$ with respect to ε.[13] Increasing ε reduces $\theta^*(\varepsilon)$ when $\lambda < \lambda_s$ and increases $\theta^*(\varepsilon)$ when $\lambda > \lambda_s$. When firms can build absorptive capacity at a high rate ($\lambda < \lambda_S$), introducing an initial cost asymmetry by giving firm 1 a cost advantage, reduces the welfare under a RJV less than under NC. Under NC a higher rate of absorptive capacity acquisition reduces a firm's incentive to invest in R&D since its competitor can build absorptive capacity with very little R&D investment. This reduced incentive results in lower equilibrium R&D investment and, therefore, less equilibrium welfare. However, under a RJV a higher rate of absorptive capacity acquisition (lower λ) allows the firms to internalize the spillover benefits at a lower cost, thereby reducing the welfare losses from an increase in the initial cost asymmetry. These effects work in the opposite direction when λ increases and absorptive capacity is built more slowly.

[13]Decreases in λ make $\gamma(\cdot)$ more concave since $-\gamma''(x)/\gamma'(x)$ is decreasing in λ.

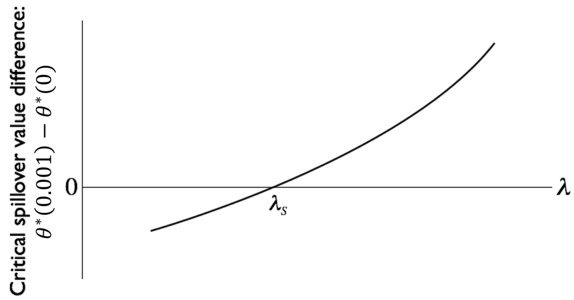

Figure 2: Difference between the welfare-based critical spillover values for the cases with cost asymmetry $\theta^*(0.001)$ and without a cost asymmetry $\theta^*(0)$ and the ability to build absorptive capacity λ.

3.4 Large initial cost asymmetries

Allowing for large initial cost asymmetries does not appear to be amenable to analytic evaluation. We can however consider the effect of ε in the context of the above example. If the initial cost asymmetry is above 0.2, the cost difference is drastic since firm 2 will not operate and firm 1 will be the monopolist. With such a drastic initial cost difference, the issue of comparing RJV and NC becomes moot. Without an active spillover mechanism, welfare falls due to lower aggregate R&D investment, less aggregate cost reduction (firm 2's cost are unaffected), and lower production due to firm 1's ability to exploit its market power. For values of ε below 0.2, the cost difference is non-drastic. We find that the qualitative features and implications of Propositions 1 through 3 still hold. Also, the qualitative behavior of aggregate R&D investment with respect to ε and θ in the RJV and NC cases is the same as in the small asymmetry case. Individual and aggregate R&D investment continue to be decreasing in ε. However, we also find that each firm's R&D investment can be non-monotonic in θ, as opposed to the increasing behavior they exhibit in the small asymmetry case. For large enough ε firm 1's R&D investment decision can be decreasing for large θ, and firm 2's R&D investment can be decreasing for small values of θ. Finally, our simulations suggest that moving from symmetry to a large ε reduces welfare under a RJV by less than welfare under NC, regardless of the rate at which absorptive capacity is built. That is, when the initial cost asymmetry is large (but non-drastic), assuming symmetry in production costs underestimates the range of values of θ for which a RJV generates higher equilibrium welfare than under NC.

3.5 Alternative cost asymmetry formulations

In this section we consider a more general cost asymmetry formulation in which $c_1 = c - \alpha\varepsilon - f(z_1)$ and $c_2 = c + (1-\alpha)\varepsilon - f(z_2)$ for $0 \leq \alpha \leq 0.5$. Our main model corresponds to $\alpha = 0$ while $\alpha = 0.5$ implies that changes in ε do not change the initial average industry marginal production cost. One can also consider initial cost asymmetries that imply lower average initial costs with this model by allowing for negative values of ε. With this more general formulation, the welfare changes in Propositions 1 and 3 are qualitatively the same as in the main model because these propositions describe welfare properties for ε close to zero.

With regard to Proposition 2, for the case in which $\alpha = 0$ and $\varepsilon < 0$, it remains the case that $\partial W^*/\partial\varepsilon$ and $\partial W^{*c}/\partial\varepsilon$ are both negative for ε close to zero. For the case in which $\alpha = 0.5$ (and $\varepsilon > 0$), the signs of $\partial W^*/\partial\varepsilon$ and $\partial W^{*c}/\partial\varepsilon$ are indeterminate. This indeterminancy arises because the effect of ε on welfare through aggregate R&D investment and through the marginal cost of production is zero in the limit as $\varepsilon \to 0$.[14] Thus, we can no longer use these channels to sign the marginal welfare changes. However, numerical analysis indicates that if the initial cost asymmetry is smaller than 0.005, then both W^* and W^{*c} are increasing in ε when $\alpha = 0.5$.

With regard to the effect of λ on $\theta^*(\varepsilon)$, note that the positive slope of the curve in Figure 2 implies that $d\theta^*(\varepsilon)/d\varepsilon$ is increasing in λ. This continues to be true in our more general model for $\alpha \in [0, 0.5]$ although the implications of an initial negative cost shock ($\varepsilon > 0$) and an

[14] $\partial W^*(\theta,\varepsilon)/\partial\varepsilon = \Gamma(\theta,\varepsilon) + 2(q_1^* - q_2^*)$ and $\partial W^{*c}(\theta,\varepsilon)/\partial\varepsilon = \Gamma^c(\theta,\varepsilon) + 2(q_1^{*c} - q_2^{*c})$. The functions $\Gamma(\theta,\varepsilon)$ and $\Gamma^c(\theta,\varepsilon)$ are the welfare effects of R&D investment under NC and RJV in (9) and (10) respectively. The term $2(q_1^* - q_2^*)$ is the welfare effect of a small change of ε through the marginal cost channel under NC. Analogous analysis and interpretation hold for $2(q_1^{*c} - q_2^{*c})$ in the RJV case.

initial positive cost shock ($\varepsilon < 0$) are somewhat different. With an initial negative cost shock (as would occur when firms in an industry are affected differentially by increased regulatory costs), the relative welfare benefits of RJVs increase when firms can build absorptive capacity at a higher rate (low λ). Under NC, the lower-cost firm is discouraged from investing in R&D as doing so works to reduce its competitor's cost disadvantage. As λ increases, aggregate R&D investment increases under NC and decreases under RJV, thus moderating the relative welfare benefits of an RJV and causing $\theta^*(\varepsilon)$ to increase.

With an initial positive cost shock (as would occur when firms in an industry are affected differentially by decreased regulatory costs), an increase in λ causes $\theta^*(\varepsilon)$ to fall faster or increase less slowly. While an increase in $\varepsilon > 0$ causes $W^{*c}(\theta, \varepsilon)$ to shift down along $W^*(\theta, \varepsilon)$, a decrease in $\varepsilon < 0$ causes $W^{*c}(\theta, \varepsilon)$ to shift up along $W^*(\theta, \varepsilon)$. The latter shift will generate a smaller change in NC welfare due to the concavity of W^* with respect to θ (Proposition 1), and thus necessitate a smaller value of $\theta^*(\varepsilon)$. This result again favors the formation of RJVs.

4 Conclusion

The literature on strategic R&D investment following the seminal work of d'Aspremont and Jacquemin (1988), and advanced by Kamien and Zang (2000) to include endogenous spillover effects, has looked at the effect of spillovers on aggregate R&D investment in a symmetric framework. The predominant focus has been to compare non-cooperative R&D investment with RJVs based on aggregate R&D investment. However, the empirical literature supports the existence of production cost asymmetries among firms, and comparisons based on aggregate R&D investment are equivalent to welfare-based comparisons only under symmetry. In this paper, we extend this work by introducing a cost asymmetry and by evaluating the effect of the magnitude of this cost asymmetry and the exogenous spillover in terms of welfare.

This paper presents four key theoretical results for a small initial cost asymmetry. First, we show that in the presence of an initial cost asymmetry it is no longer sufficient to compare RJVs and NC solely in terms of aggregate R&D investment. With an initial cost asymmetry, the critical spillover value is determined by direct marginal cost differences in addition to differences in aggregate R&D investment, whereas with symmetric costs the critical spillover value can be determined exclusively by comparing aggregate R&D investment levels. Second, the effect of a change in the size of the initial cost asymmetry on the critical spillover value is ambiguous because the change in the critical spillover value is shown to depend on the rate at which a firm can build absorptive capacity. When an increase of the initial cost asymmetry reduces welfare under NC and RJV, then for firms with a high rate of building absorptive capacity, an increase of the cost asymmetry reinforces the welfare reduction under NC and weakens the welfare reduction under a RJV. Contrary to the analysis in Röller, Siebert, and Tombak (2007), an increase of the initial cost asymmetry can promote RJV formation. These effects reverse for firms with low rate of building absorptive capacity. Third, with small initial cost asymmetries, welfare is monotonically increasing in the exogenous spillover parameter under a RJV, whereas under NC welfare can be non-monotonic in the size of the exogenous spillover parameter increases. This occurs because, while an increase of the exogenous spillover parameter directly reduces marginal costs thereby increasing aggregate production and welfare under a RJV and under NC, the effect through the R&D investment channel increases welfare under a RJV and reduces welfare under NC. Finally, fixing the unit cost of production of the low-cost firm, as the high-cost firm becomes less productive, equilibrium welfare decreases because of lower R&D investment, higher production costs, and less aggregate production.

These results expand our understanding about when RJVs provide higher welfare than non-cooperative R&D investment in a model that can incorporate the empirical reality that firms in an industry exhibit heterogeneous costs. In particular, our model provides an insight about how the interaction between each firms' ability to learn from its competitor's R&D and firm heterogeneity affect welfare differences between a RJV and NC. Moreover, our analysis shows that introducing an initial cost asymmetry generates richer welfare differences between a RJV and NC. Therefore the criteria commonly used to compare RJVs and NC, which solely focuses on aggregate R&D investment, and relies on the absence of initial cost asymmetries can be misleading for policy purposes.

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5 Appendix

5.1 Proof of Lemma 1

First, we show that $\varepsilon = 0$, implies that R&D investments are equal among firms. Firm i's FOC in G^{RNC} is $2q_i(x_i, x_j, \theta, \varepsilon) \cdot \partial q_i(x_i, x_j, \theta, \varepsilon)/\partial x_i = 1$, where $i \neq j$. Then the subgame perfect investment levels are described by

$$2q_1(x_1, x_2, \theta, \varepsilon)\frac{\partial q_1}{\partial x_1}(x_1, x_2, \theta, \varepsilon) = 1 \tag{11}$$

and

$$2q_2(x_2, x_1, \theta, \varepsilon)\frac{\partial q_2}{\partial x_2}(x_2, x_1, \theta, \varepsilon) = 1. \tag{12}$$

Given the demand and costs assumptions,

$$q_1(x_1, x_2, \theta, \varepsilon) = \frac{1}{3}(a - c + \varepsilon + 2f(z_1) - f(z_2))$$

and

$$q_2(x_2, x_1, \theta, \varepsilon) = \frac{1}{3}(a - c - 2\varepsilon + 2f(z_2) - f(z_1)).$$

When $\varepsilon = 0$, (11) and (12) are symmetric. Therefore $x_1^*(\theta, 0) = x_2^*(\theta, 0) = x^*(\theta, 0)$. Analogously $x_1^{*c}(\theta, 0) = x_2^{*c}(\theta, 0) = x^{*c}(\theta, 0)$. Now we show that if $\theta \in \Theta'(0)$, then $\theta \in \Theta^*(0)$. Suppose $\theta \in \Theta'(0)$, then $X^*(\theta, 0) = 2x^*(\theta, 0)$ and $X^{*c}(\theta, 0) = 2x^{*c}(\theta, 0)$. By the definition of $\Theta'(\varepsilon)$, $\theta \in \Theta'(0)$ implies $x^*(\theta, 0) = x^{*c}(\theta, 0)$. Substituting $x^*(\theta, 0)$, $x^{*c}(\theta, 0)$, and θ into (11) and (12) and into the definitions of CS, Π_i, and W yields $W^*(\theta, 0) = W^{*c}(\theta, 0)$. Next we prove that if $\theta \in \Theta^*(0)$, then $\theta \in \Theta'(0)$. By the definition of $\Theta^*(\varepsilon)$, $\theta \in \Theta^*(0)$ implies $W^*(\theta, 0) = W^{*c}(\theta, 0)$. By the definition of W^* and W^{*c}, $W^*(\theta, 0) - W^{*c}(\theta, 0) = W(x^*, x^*, \theta, 0) - W(x^{*c}, x^{*c}, \theta, 0)$ where $x^* = x^*(\theta, 0)$ and $x^{*c} = x^{*c}(\theta, 0)$. Define $\hat{W}(x, \theta, 0) = W(x, x, \theta, 0)$ and $q(x, \theta, 0) = q_i(x, x, \theta, 0)$. Without loss of generality assume $x^* \geq x^{*c}$ then $W^*(\theta, 0) - W^{*c}(\theta, 0) = \int_{t=x^{*c}(\theta,0)}^{x^*(\theta,0)} \partial\hat{W}(t, \theta, 0)/\partial x dt$.

If $\partial\hat{W}(t,\theta^*(0),0)/\partial x>0$ for all t, then $W^*(\theta,0)=W^{*c}(\theta,0)$ if, and only if, $x^*(\theta,0)=x^{*c}(\theta,0)$. To show this is true for A sufficiently large, note that $\partial\hat{W}/\partial x=8q(x,\theta,0)\,\partial q(x,\theta,0)/\partial x-2$. Direct calculation yields $\partial q(x,\theta,0)/\partial x=f'(z)(1+\gamma'(x)\theta x+\gamma(x)\theta)>0$ and $q(x,\theta,0)=\frac{1}{3}(a-c-f(z))$. Because $q(x,\theta,0)$ is linear and increasing in a, then there exists $\overline{a}>0$ such that $\frac{\partial\hat{W}}{\partial x}(t,\theta^*(0)^*,0)>0$ for all t. Then $x^*(\theta,0)=x^{*c}(\theta,0)$ and $\theta\in\Theta'(0)$. □

5.2 Proof of Proposition 1

Lemmas 2, 3, and 4 provide the key steps for the proof of (7) and (8). Lemma 2 states that, in the limit as $\varepsilon\to 0$, the firms' strategies and equilibrium decisions converge in both the non-cooperative R&D investment and RJV cases. Lemma 3 states that when the initial cost asymmetry is small, θ affects the equilibrium R&D investment level under NC in the same direction as it affects the incentives to invest in the symmetric case. Lemma 4 states that, for large enough demand and an absorptive capacity function that does not build absorptive capacity too quickly, an increase in θ necessarily reduces the incentives to invest under NC. The rest of the proof refers to the change in equilibrium welfare under a RJV and only uses implications from the maximization of joint profits.

Lemma 2 *For all i and j,*

1. $\lim_{\varepsilon\to 0}(x_i^*(\theta,\varepsilon),q_i(x_i,x_j,\theta,\varepsilon))=(x^*(\theta,0),q_i(x_i,x_j,\theta,0))$,
2. $\lim_{\varepsilon\to 0}(x_i^{*c}(\theta,\varepsilon),q_i(x_i,x_j,\theta,\varepsilon))=(x^{*c}(\theta,0),q_i(x_i,x_j,\theta,0))$,
3. $\lim_{\varepsilon\to 0}x_i^*(\theta,\varepsilon)=x^*(\theta,0)$ *and* $\lim_{\varepsilon\to 0}q_i^*\left(x_i^*(\theta,\varepsilon),x_j^*(\theta,\varepsilon),\theta,\varepsilon\right)=q^*(\theta)$, *and*
4. $\lim_{\varepsilon\to 0}x_i^{*c}(\theta,\varepsilon)=x^{*c}(\theta,0)$ *and* $\lim_{\varepsilon\to 0}q_i^{*c}\left(x_i^{*c}(\theta,\varepsilon),x_j^{*c}(\theta,\varepsilon),\theta,\varepsilon\right)=q^{*c}(\theta)$.

Proof: The proof follows directly from the fact that $q_i(x_i,x_j,\theta,\varepsilon)$ and $\partial q_i(x_i,x_j,\theta,\varepsilon)/\partial x_i$ are continuous in x_i, x_j, θ and ε, that $x_1^*(\theta,0)=x_2^*(\theta,0)$, and that $x_1^{*c}(\theta,0)=x_2^{*c}(\theta,0)$. □

Lemma 3 *For any stable equilibrium of G^{RNC}, for all $i\neq j\in\{1,2\}$, there exists $\overline{\varepsilon}$ such that for all $0\leq\varepsilon<\overline{\varepsilon}$*

$$sign\left(\frac{\partial x_i^*\left(\theta,\varepsilon\right)}{\partial\theta}\right)=sign\left(\frac{\partial^2\Pi_i\left(x_i^*,x_j^*,\theta,0\right)}{\partial x_i\partial\theta}\right) \tag{13}$$

and

$$sign\left(\frac{\partial x_i^{*c}\left(\theta,\varepsilon\right)}{\partial\theta}\right)=sign\left(\frac{\partial^2\Pi_i\left(x_i^{*c},x_j^{*c},\theta,0\right)}{\partial x_i\partial\theta}+\frac{\partial^2\Pi_j\left(x_j^{*c},x_i^{*c},\theta,0\right)}{\partial x_i\partial\theta}\right). \tag{14}$$

Proof: To prove (13), totally differentiating (11) and (12) yields

$$\frac{\partial x_i^*}{\partial\theta}=\frac{\begin{vmatrix} -\left(\frac{\partial q_i}{\partial\theta}\frac{\partial q_i}{\partial x_i}+q_i^*\frac{\partial^2 q_i}{\partial x_i\partial\theta}\right) & \frac{\partial q_i}{\partial x_j}\frac{\partial q_i}{\partial x_i}+q_i^*\frac{\partial^2 q_i}{\partial x_i\partial x_j} \\ -\left(\frac{\partial q_j}{\partial\theta}\frac{\partial q_j}{\partial x_j}+q_j^*\frac{\partial^2 q_j}{\partial x_j\partial\theta}\right) & \left(\frac{\partial q_j}{\partial x_j}\right)^2+q_j^*\frac{\partial^2 q_j}{\partial x_j^2}\end{vmatrix}}{\begin{vmatrix} \left(\frac{\partial q_1}{\partial x_1}\right)^2+q_1^*\frac{\partial^2 q_1}{\partial x_1^2} & \frac{\partial q_i}{\partial x_j}\frac{\partial q_i}{\partial x_i}+q_i^*\frac{\partial^2 q_i}{\partial x_i\partial x_j} \\ \frac{\partial q_j}{\partial x_j}\frac{\partial q_j}{\partial x_i}+q_j^*\frac{\partial^2 q_j}{\partial x_i\partial x_j} & \left(\frac{\partial q_j}{\partial x_j}\right)^2+q_j^*\frac{\partial^2 q_j}{\partial x_j^2}\end{vmatrix}}. \tag{15}$$

The stability condition for the equilibria of G^{RNC} at each (θ,ε) implies the denominator of (15) is strictly positive. Thus,

$$\begin{aligned} sign\left(\frac{\partial x_i^*\left(\theta,\varepsilon\right)}{\partial\theta}\right)= \quad & sign\Bigg[-\left(\frac{\partial q_i}{\partial\theta}\frac{\partial q_i}{\partial x_i}+q_i^*\frac{\partial^2 q_i}{\partial x_i\partial\theta}\right)\left(\left(\frac{\partial q_j}{\partial x_j}\right)^2+q_j^*\frac{\partial^2 q_j}{\partial x_j^2}\right) \\ & +\left(\frac{\partial q_j}{\partial\theta}\frac{\partial q_j}{\partial x_j}+q_j^*\frac{\partial^2 q_j}{\partial x_j\partial\theta}\right)\left(\frac{\partial q_i}{\partial x_j}\frac{\partial q_i}{\partial x_i}+q_i^*\frac{\partial^2 q_i}{\partial x_i\partial x_j}\right)\Bigg] \end{aligned} \tag{16}$$

where the right-hand side of (16) is the sign of the numerator of (15). The continuity of $q_i\left(x_i,x_j,\theta,\varepsilon\right)$, $\partial q_i\left(x_i,x_j,\theta,\varepsilon\right)/\partial x_j$, $\partial q_i\left(x_i,x_j,\theta,\varepsilon\right)/\partial\theta$, $\partial^2 q_i\left(x_i,x_j,\theta,\varepsilon\right)/\partial x_i\partial x_j$ and $\partial^2 q_i\left(x_i,x_j,\theta,\varepsilon\right)/\partial x_i\partial\theta$ in $\left(x_i,x_j,\theta,\varepsilon\right)$ for all i and j implies, that the numerator (15) is continuous. Taking the limit as $\varepsilon\to 0$ of the right-hand side of (16), Lemma 2 and continuity of the numerator of (15) imply that there exists $\overline{\varepsilon}>0$ such that for $0<\varepsilon<\overline{\varepsilon}$,

$$sign\left(\frac{\partial x_i^*(\theta,\varepsilon)}{\partial\theta}\right) = sign\left[\left(-\left(\left(\frac{\partial q_j}{\partial x_j}\right)^2 + q_j^*\frac{\partial^2 q_j}{\partial x_j^2}\right) + \left(\frac{\partial q_i}{\partial x_j}\frac{\partial q_i}{\partial x_i} + q_i^*\frac{\partial^2 q_i}{\partial x_i \partial x_j}\right)\right) \cdot \left(\frac{\partial q_i}{\partial \theta}\frac{\partial q_i}{\partial x_i} + q_i^*\frac{\partial^2 q_i}{\partial x_i \partial\theta}\right)\right] \quad (17)$$

where each term on the right-hand side of (17) is evaluated at $(\theta, 0)$. The stability condition for the equilibrium of G^{RNC} at $(\theta, 0)$ has the form $A^2 - B^2 > 0$, where

$$A = \left(\left(\frac{\partial q_i\left(x_i^*(\theta,0), x_j^*(\theta,0), \theta, 0\right)}{\partial x_i}\right)^2 + q_i^*\frac{\partial^2 q_i\left(x_i^*(\theta,0), x_j^*(\theta,0), \theta, 0\right)}{\partial x_i^2}\right),$$

$$B = \left(\frac{\partial q_i\left(x_i^*(\theta,0), x_j^*(\theta,0), \theta, 0\right)}{\partial x_j}\frac{\partial q_i\left(x_i^*(\theta,0), x_j^*(\theta,0), \theta, 0\right)}{\partial x_i} + q_i^*\frac{\partial^2 q_i\left(x_i^*(\theta,0), x_j^*(\theta,0), \theta, 0\right)}{\partial x_i \partial x_j}\right),$$

and $-A + B$ is the term in the square brackets on the right-hand side of (17). The SONC of firm i 's profit maximization problem for $(\theta, 0)$ is equivalent to $A < 0$. If $B > 0$, then $-A > B$. If $B < 0$, then $-A > B$. Therefore $-A + B > 0$ and the right-hand side of (17) is equal to $sign\left(\partial q_i/\partial\theta \cdot \partial q_i/\partial x_i + q_i^* \cdot \partial^2 q_i/\partial x_i \partial\theta\right)$ evaluated at $(\theta, 0)$, which can also be written as $sign\left(\partial\Pi_i\left(x_i^*, x_j^*, \theta, 0\right)/\partial x_i \partial\theta\right)$ for all $i \neq j$. To prove (14), totally differentiating the FOC of (1) yields

$$\frac{\partial x_i^{*c}}{\partial\theta} = \frac{M_{11}M_{22} - M_{12}M_{21}}{\Delta_{11}\Delta_{22} - \Delta_{12}\Delta_{21}} \quad (18)$$

where

$$M_{11} = -\left(\frac{\partial q_i}{\partial\theta}\frac{\partial q_i}{\partial x_i} + q_i^{*c}\frac{\partial^2 q_i}{\partial x_i \partial\theta} + \frac{\partial q_j}{\partial\theta}\frac{\partial q_j}{\partial x_i} + q_j^{*c}\frac{\partial^2 q_j}{\partial x_i \partial\theta}\right),$$

$$M_{12} = \Delta_{12} = \frac{\partial q_i}{\partial x_i}\frac{\partial q_i}{\partial x_j} + q_i^{*c}\frac{\partial^2 q_i}{\partial x_i \partial x_j} + \frac{\partial q_j}{\partial x_i}\frac{\partial q_j}{\partial x_j} + q_j^{*c}\frac{\partial^2 q_j}{\partial x_i \partial x_j},$$

$$M_{21} = -\left(\frac{\partial q_i}{\partial \theta}\frac{\partial q_i}{\partial x_j} + q_i^{*c}\frac{\partial^2 q_i}{\partial x_j \partial \theta} + \frac{\partial q_j}{\partial \theta}\frac{\partial q_j}{\partial x_j} + q_j^{*c}\frac{\partial^2 q_j}{\partial x_j \partial \theta}\right),$$

$$M_{22} = \Delta_{22} = \left(\frac{\partial q_i}{\partial x_j}\right)^2 + q_i^{*c}\frac{\partial^2 q_i}{\partial x_j^2} + \left(\frac{\partial q_j}{\partial x_j}\right)^2 + q_j^{*c}\frac{\partial^2 q_j}{\partial x_j^2},$$

$$\Delta_{11} = \left(\frac{\partial q_i}{\partial x_i}\right)^2 + q_i^{*c}\frac{\partial^2 q_i}{\partial x_i^2} + \left(\frac{\partial q_j}{\partial x_i}\right)^2 + q_j^{*c}\frac{\partial^2 q_j}{\partial x_i^2},$$

and

$$\Delta_{21} = \frac{\partial q_i}{\partial x_i}\frac{\partial q_i}{\partial x_j} + q_i^{*c}\frac{\partial^2 q_i}{\partial x_i \partial x_j} + \frac{\partial q_j}{\partial x_i}\frac{\partial q_j}{\partial x_j} + q_j^{*c}\frac{\partial^2 q_j}{\partial x_i \partial x_j}.$$

Here the SONC of the reduced form cooperative problem in (1) at each (θ, ε) implies that the denominator of (18) is non-negative. Moreover the continuity of $q_i(x_i, x_j, \theta, \varepsilon)$, $\partial q_i(x_i, x_j, \theta, \varepsilon)/\partial x_j$, $\partial q_i(x_i, x_j, \theta, \varepsilon)/\partial \theta$, $\partial^2 q_i(x_i, x_j, \theta, \varepsilon)/\partial x_i \partial x_j$ and $\partial^2 q_i(x_i, x_j, \theta, \varepsilon)/\partial x_i \partial \theta$ in $(x_i, x_j, \theta, \varepsilon)$ for all i and j implies that the numerator of (18) is continuous. By Lemma (2) and continuity of the numerator of (18) there exists $\overline{\varepsilon} > 0$ such that for $0 \leq \varepsilon < \overline{\varepsilon}$

$$\begin{aligned} sign\left(\frac{\partial x_i^{*c}(\theta, \varepsilon)}{\partial \theta}\right) &= sign\Bigg(\Bigg[-\Bigg(\Bigg[\left(\frac{\partial q_i}{\partial x_j}\right)^2 + q_i^{*c}\frac{\partial^2 q_i}{\partial x_j^2}\Bigg] + \Bigg[\left(\frac{\partial q_j}{\partial x_j}\right)^2 + q_j^{*c}\frac{\partial^2 q_j}{\partial x_j^2}\Bigg]\Bigg) \\ &\quad +2\left(\frac{\partial q_i}{\partial x_i}\frac{\partial q_i}{\partial x_j} + q_i^{*c}\frac{\partial^2 q_i}{\partial x_i \partial x_j}\right)\Bigg] \cdot \\ &\quad \left(\left[\frac{\partial q_i}{\partial \theta}\frac{\partial q_i}{\partial x_i} + q_i^{*c}\frac{\partial^2 q_i}{\partial x_i \partial \theta}\right] + \left[\frac{\partial q_j}{\partial \theta}\frac{\partial q_j}{\partial x_i} + q_j^{*c}\frac{\partial^2 q_j}{\partial x_i \partial \theta}\right]\right)\Bigg) \end{aligned} \tag{19}$$

where the right hand side of (19) is evaluated at $(\theta, 0)$. Finally the SONCs for a local maximum of the reduced form cooperative problem (1) at $(\theta, 0)$ take the form $C < 0$ and where

$$C = \left(\frac{\partial q_i}{\partial x_j}\right)^2 + q_i^{*c}\frac{\partial^2 q_i}{\partial x_j^2} + \left(\frac{\partial q_j}{\partial x_j}\right)^2 + q_j^{*c}\frac{\partial^2 q_j}{\partial x_j^2},$$

$$D = 2\left(\frac{\partial q_i}{\partial x_i}\frac{\partial q_i}{\partial x_j} + q_i^{*c}\frac{\partial^2 q_i}{\partial x_i \partial x_j}\right)^2,$$

and $-C + D$ is the sum of the two first lines in (19). Therefore the SONC of problem (1) for $(\theta, 0)$ implies that if $D > 0$, then $-C > D$, and if $D < 0$, then $-C > D$. Therefore

$-C+D>0$ and the right-hand side of (19) is equal to

$$sign\left(\left(\frac{\partial q_i}{\partial\theta}\frac{\partial q_i}{\partial x_i}+q_i^{*c}\frac{\partial^2 q_i}{\partial x_i\partial\theta}+\frac{\partial q_j}{\partial\theta}\frac{\partial q_j}{\partial x_i}+q_j^{*c}\frac{\partial^2 q_j}{\partial x_i\partial\theta}\right)\right)$$

evaluated at $(\theta,0)$, which we can write as

$$sign\left(\frac{\partial\Pi_i\left(x_i^{*c},x_j^{*c},\theta,0\right)}{\partial x_i\partial\theta}+\frac{\partial\Pi_j\left(x_j^{*c},x_i^{*c},\theta,0\right)}{\partial x_i\partial\theta}\right)$$

for all $i\neq j$, completing the proof of Lemma 3. $\square$

Lemma 4 *Suppose* $2\gamma'(x_i)x_i-\gamma(x_i)<0$. *Then for* $\varepsilon=0$, *there exist* $\overline{a}>0$ *such that for all* $a>\overline{a}$, $\partial\Pi_i\left(x_i^*,x_j^*,\theta,0\right)/\partial x_i\partial\theta<0$.

Proof: By definition $\gamma(\cdot)$ is bounded between 0 and 1, strictly increasing and strictly concave. Therefore, if $2\gamma'(x_i)x_i-\gamma(x_i)<0$, then $-1\leq 2\gamma'(x)x-\gamma(x)\leq 0$.

Under symmetry $z_i=z$, $x_i=x$, and $q_i(x,x,\theta,0)=q(x,\theta)$ for all i, where $q(x,\theta)=\phi(z)=\frac{1}{3}(a-c+f(z))$ and $z=x+\gamma(x)\theta x$. Therefore

$$\begin{aligned}\frac{\partial^2\Pi_i(x,x,\theta,0)}{\partial x_i\partial\theta} &= \frac{2}{9}\left(f'(z)\gamma(x)x\right)f'(z)\left[2+\theta\left(2\gamma'(x)x-\gamma(x)\right)\right]+q\frac{2}{3}\left(f''(z)\gamma(x)x\right.\\ &\quad\left.\cdot\left[2+\theta\left[2\gamma'(x)x-\gamma(x)\right]\right]+f'(z)\left[2\gamma'(x)x-\gamma(x)\right]\right). \qquad (20)\end{aligned}$$

Then $-1\leq 2\gamma'(x)x-\gamma(x)\leq 0$ implies that $2+\theta\left[2\gamma'(x)x-\gamma(x)\right]>0$. Because $f(z)$ is concave in z and because $q(x,\theta)$ is continuous and linear in a for all (x,θ), there exists $\overline{a}>0$ such that for all $a>\overline{a},q(x,\theta)$ is large enough so that (20) will be negative for all (x,θ). This completes the proof of Lemma 4. $\square$

To complete the proof 7 of and 8 in Proposition 1, substituting the equilibrium R&D

investment and production decisions into (2) implies that

$$
\begin{aligned}
\frac{\partial W^*}{\partial \theta} = & \left\{ (q_1^* + q_2^*) \left[\left(\frac{\partial q_1}{\partial c_1} + \frac{\partial q_2}{\partial c_1} \right) \frac{\partial c_1}{\partial x_1} + \left(\frac{\partial q_1}{\partial c_2} + \frac{\partial q_2}{\partial c_2} \right) \frac{\partial c_2}{\partial x_1} \right] \right. \\
& \left. - q_2^* \left[\left(\frac{\partial q_1}{\partial c_1} \frac{\partial c_1}{\partial x_1} + \frac{\partial q_1}{\partial c_2} \frac{\partial c_2}{\partial x_1} \right) - \frac{\partial c_2}{\partial x_1} \right] \right\} \frac{\partial x_1^*}{\partial \theta} \\
& + \left\{ (q_1^* + q_2^*) \left[\left(\frac{\partial q_1}{\partial c_1} + \frac{\partial q_2}{\partial c_1} \right) \frac{\partial c_1}{\partial x_2} + \left(\frac{\partial q_1}{\partial c_2} + \frac{\partial q_2}{\partial c_2} \right) \frac{\partial c_2}{\partial x_2} \right] \right. \\
& \left. - q_1^* \left[\left(\frac{\partial q_2}{\partial c_1} \frac{\partial c_1}{\partial x_2} + \frac{\partial q_2}{\partial c_2} \frac{\partial c_2}{\partial x_2} \right) - \frac{\partial c_1}{\partial x_2} \right] \right\} \frac{\partial x_2^*}{\partial \theta} \\
& + \left[\left(q_1^* \frac{\partial q_1}{\partial c_1} + q_2^* \frac{\partial q_2}{\partial c_1} \right) \frac{\partial c_1}{\partial \theta} + \left(q_1^* \frac{\partial q_1}{\partial c_2} + q_2^* \frac{\partial q_2}{\partial c_2} \right) \frac{\partial c_2}{\partial \theta} \right] \\
& - \left(q_1^* \frac{\partial c_1}{\partial \theta} + q_2^* \frac{\partial c_2}{\partial \theta} \right). \qquad (21)
\end{aligned}
$$

The continuity of $q_i(x_i, x_j, \theta, \varepsilon)$, $\partial q_i(x_i, x_j \theta, \varepsilon)/\partial x_j$, $\partial q_i(x_i, x_j, \theta, \varepsilon)/\partial \theta$, $\partial^2 q_i(x_i, x_j, \theta, \varepsilon)/\partial x_i \partial x_j$ and $\partial^2 q_i(x_i, x_j, \theta, \varepsilon)/\partial x_i \partial \theta$ in $(x_i, x_j, \theta, \varepsilon)$ for all i and j implies the continuity of $\partial W^*/\partial \theta$. Rearranging terms, substituting equilibrium decisions, and taking the limit of (21) as $\varepsilon \to 0$, it follows from Lemma 2 that $\lim_{\varepsilon \to 0} \partial W^*(\theta, \varepsilon)/\partial \theta = 4 q_i^* f'(z_i^*) \gamma(x_i^*) \left[\frac{2}{3} - \eta_{x_i,\theta}\right]$. By Lemma 3 and the assumptions on $f(\cdot)$ and $\gamma(\cdot)$, there exists $\bar{\varepsilon} > 0$ such that for $0 \leq \varepsilon < \bar{\varepsilon}$, if $\partial^2 \Pi_i(x_i^*, x_j^*, \theta, 0)/\partial x_i \partial \theta > 0$, then $\partial x_i^*(\theta, \varepsilon)/\partial \theta > 0$, and if $\partial^2 \Pi_i(x_i^*, x_j^*, \theta, 0)/\partial x_i \partial \theta < 0$, then $\partial x_i^*/\partial \theta < 0$. By Lemma 4 if $2\gamma'(x_i) x_i - \gamma(x_i) < 0$, there exists $a > \bar{a}$ such that $\partial^2 \Pi_i(x_i^*, x_j^*, \theta, 0)/\partial x_i \partial \theta < 0$. Therefore if $2\gamma'(x_i) x_i - \gamma(x_i) < 0$, by the continuity of $\partial W^*(\theta, \varepsilon)/\partial \theta$ and Lemmas 3 and 4, then there exists $\bar{a}, \bar{\varepsilon} > 0$ such that for $0 \leq \varepsilon < \bar{\varepsilon}$, and $a > \bar{a}$, (7) and (8) in Proposition 1 hold.

Now we turn to prove (6) in Proposition 1. We want to show that there exists $\bar{\varepsilon} > 0$ such that for $0 \leq \varepsilon < \bar{\varepsilon}$, $\partial W^{*c}(\theta', \varepsilon)/\partial \theta > 0$. Since $W^{*c} = CS^{*c} + \Pi_1^{*c} + \Pi_2^{*c}$, first we show that there exist $\bar{\varepsilon}_1 > 0$ such that for $0 \leq \varepsilon < \bar{\varepsilon}_1$, $\Pi_1^{*c} + \Pi_2^{*c}$ increases in θ, and then we show that there exist $\bar{\varepsilon}_2 > 0$ such that for $0 \leq \varepsilon < \bar{\varepsilon}_2$, CS^* increases in θ. We can then define $\bar{\varepsilon} = \min\{\bar{\varepsilon}_1, \bar{\varepsilon}_2\}$

Let $(x_i^{*c}(\theta, 0), x_j^{*c}(\theta, 0))$ be the unique solution to (1) when $\varepsilon = 0$. In this case $x_i^{*c}(\theta, 0) = x_j^{*c}(\theta, 0) = x^{*c}(\theta)$. Define the indirect joint profit function under symmetry by $\Pi\left(x^{*c}\left(\theta\right),\theta\right) = 2\left(q\left(x^{*c}\left(\theta\right),\theta\right)\right)^2 - 2x^{*c}\left(\theta\right)$. Let $\theta' > \theta$. Then by profit maximization $\Pi\left(x^{*c}\left(\theta'\right),\theta'\right) > \Pi\left(x^{*c}\left(\theta\right),\theta'\right)$. Since $q\left(x,\theta\right) = \frac{1}{3}\left(a - c + f\left(x + \gamma\left(x\right)\theta x\right)\right)$, $f\left(\cdot\right)$ increasing implies that $\Pi\left(x^{*c}\left(\theta\right),\theta'\right) > \Pi\left(x^{*c}\left(\theta\right),\theta\right)$.

Therefore $\Pi\left(x^{*c}\left(\theta'\right),\theta'\right) > \Pi\left(x^{*c}\left(\theta\right),\theta\right)$. By Lemma 5 and continuity of the joint profits $\Pi_i\left(x_i,x_j,\theta,\varepsilon\right) + \Pi_j\left(x_j,x_i,\theta,\varepsilon\right)$ in x_i,x_j,θ and ε, there exists $\overline{\varepsilon} > 0$ such that for $0 < \varepsilon < \overline{\varepsilon}$, $\Pi\left(x^{*c}\left(\theta'\right),\theta'\right) > \Pi\left(x^{*c}\left(\theta\right),\theta\right)$ implies

$$\begin{aligned}&\Pi_i\left(x_i^{*c}\left(\theta',\varepsilon\right),x_j^{*c}\left(\theta',\varepsilon\right),\theta',\varepsilon\right) + \Pi_j\left(x_j^{*c}\left(\theta',\varepsilon\right),x_i^{*c}\left(\theta',\varepsilon\right),\theta',\varepsilon\right)\\ &> \Pi_i\left(x_i^{*c}\left(\theta,\varepsilon\right),x_j^{*c}\left(\theta,\varepsilon\right),\theta,\varepsilon\right) + \Pi_j\left(x_j^{*c}\left(\theta,\varepsilon\right),x_i^{*c}\left(\theta,\varepsilon\right),\theta,\varepsilon\right).\end{aligned}$$

In the symmetric case the cooperative problem is

$$\max_{x_1,x_2\geq 0}\Pi_1 + \Pi_2 = q_1^2\left(x_1,x_2,\theta,0\right) + q_2^2\left(x_2,x_1,\theta,0\right) - \left(x_1 + x_2\right). \tag{22}$$

The FOC's of (22) are

$$2q_1\left(x_1,x_2,\theta,0\right)\frac{\partial q_1\left(x_1,x_2,\theta,0\right)}{\partial x_1} + 2q_2\left(x_2,x_1,\theta,0\right)\frac{\partial q_2\left(x_1,x_2,\theta,0\right)}{\partial x_1} - 1 = 0 \tag{23}$$

and

$$2q_1\left(x_1,x_2,\theta,0\right)\frac{\partial q_1\left(x_1,x_2,\theta,0\right)}{\partial x_2} + 2q_2\left(x_2,x_1,\theta,0\right)\frac{\partial q_2\left(x_1,x_2,\theta,0\right)}{\partial x_2} - 1 = 0. \tag{24}$$

By the definition of $q_1\left(x_1,x_2,\theta,\varepsilon\right)$ and $q_2\left(x_2,x_1,\theta,\varepsilon\right)$, $q_1\left(x_1,x_2,\theta,0\right) = q_2\left(x_2,x_1,\theta,0\right)$. Therefore (23) and (24) form a symmetric system with solution $x_1 = x_2$. Therefore solving (22) is equivalent to solving

$$\max_{x\geq 0}\Pi\left(x,\theta\right) = 2q^2\left(x,\theta\right) - 2x \tag{25}$$

which has the unique solution $x\left(\theta\right)$. Monotonicity of $\gamma\left(\cdot\right)$ implies that there exists an inverse function $\chi\left(\omega,\theta\right): R_+ \to R_+$ that maps the effective and symmetric R&D investment ω into

R&D investment x, such that $\chi(\omega,\theta)=x$. Therefore (25) can be written as

$$\max_{\omega\geq 0}\tilde{\Pi}(\omega,\theta)=2\phi^2(\omega)-2\chi(\omega,\theta). \tag{26}$$

The FOC of (26) implies that

$$2\phi(\omega)\frac{\partial\phi(\omega)}{\partial\omega}-\frac{\partial\chi(\omega,\theta)}{\partial\omega}=0.$$

Hence

$$\frac{\partial\omega(\theta)}{\partial\theta}=\frac{\frac{\partial\chi(\omega,\theta)}{\partial\omega\partial\theta}}{2\left[\left(\frac{\partial\phi(\omega)}{\partial\omega}\right)^2+\frac{\partial^2\phi(\omega)}{\partial\omega^2}\right]-\frac{\partial^2\chi(\omega,\theta)}{\partial\omega^2}}. \tag{27}$$

The SONC of (26) implies that the denominator of (27) is non-positive and direct calculation yields $\partial^2\chi(\omega,\theta)/\partial\omega\partial\theta<0$, hence $\partial\omega(\theta)/\partial\theta>0$. Moreover, by the definition of $\chi(\omega,\theta)$, in equilibrium $\partial\chi(\omega(\theta),\theta)/\partial\theta>0$. By the definition of $\phi(\omega)$, $\partial\omega(\theta)/\partial\theta>0$ implies $\partial\phi(\omega(\theta))/\partial\theta>0$ which is equivalent to $\partial q^{*c}(\theta,0)/\partial\theta>0$. By the definition of $CS(Q)$, $\partial q^{*c}(\theta,0)/\partial\theta>0$ is equivalent to $\partial CS(Q^{*c})/\partial\theta>0$. By Lemma 2 and the continuity of $CS(Q)$ in x_i,x_j,θ and ε, there exists $\bar{\varepsilon}>0$ such that for $0<\varepsilon<\bar{\varepsilon}$, if $\partial CS(Q^*)/\partial\theta>0$, then $\partial CS(Q^{*c})/\partial\theta>0$, completing the proof. □

5.3 Proof of Proposition 2

Lemmas 2 and 5 provide the key steps of the proof. Lemma 5 states that for a small initial cost asymmetry, aggregate investment is decreasing in ε under a RJV and under NC.

Define $q(x,\theta)\equiv q_i(x,x,\theta,0)$, $q^*(\theta)\equiv q(x^*(\theta,0),\theta)$, and $q^{*c}(\theta)\equiv q(x^{*c}(\theta,0),\theta)$.

Lemma 5 *Given the assumptions on demand, the unit cost of production, $f(\cdot)$, and $\gamma(\cdot)$,* *there exists $\overline{\varepsilon} > 0$ such that for $0 \leq \varepsilon < \overline{\varepsilon}$, if the equilibrium of G^{RNC} is locally stable,*

$$\frac{\partial x_1^* (\theta, \varepsilon)}{\partial \varepsilon} + \frac{\partial x_2^* (\theta, \varepsilon)}{\partial \varepsilon} < 0 \tag{28}$$

and

$$\frac{\partial x_1^{*c} (\theta, \varepsilon)}{\partial \varepsilon} + \frac{\partial x_2^{*c} (\theta, \varepsilon)}{\partial \varepsilon} < 0. \tag{29}$$

Proof: Given the assumptions on the demand and the unit cost of production direct calculation yields, in the NC case for every $i \neq j$,

$$\frac{\partial x_i^* (\theta, \varepsilon)}{\partial \varepsilon} = \frac{\begin{vmatrix} -\frac{\partial q_i}{\partial \varepsilon}\frac{\partial q_i}{\partial x_i} & \frac{\partial q_i}{\partial x_j}\frac{\partial q_i}{\partial x_i} + q_i^* \frac{\partial^2 q_i}{\partial x_i \partial x_j} \\ -\frac{\partial q_j}{\partial \varepsilon}\frac{\partial q_j}{\partial x_j} & \left(\frac{\partial q_j}{\partial x_j}\right)^2 + q_j^* \frac{\partial^2 q_j}{\partial x_j^2} \end{vmatrix}}{\begin{vmatrix} \left(\frac{\partial q_i}{\partial x_i}\right)^2 + q_i^* \frac{\partial^2 q_i}{\partial x_i^2} & \frac{\partial q_i}{\partial x_j}\frac{\partial q_i}{\partial x_i} + q_i^* \frac{\partial^2 q_i}{\partial x_i \partial x_j} \\ \frac{\partial q_j}{\partial x_j}\frac{\partial q_j}{\partial x_i} + q_j^* \frac{\partial^2 q_j}{\partial x_i \partial x_j} & \left(\frac{\partial q_j}{\partial x_j}\right)^2 + q_j^* \frac{\partial^2 q_j}{\partial x_j^2} \end{vmatrix}} \tag{30}$$

and in the RJV case,

$$\frac{\partial x_i^{*c} (\theta, \varepsilon)}{\partial \varepsilon} = \frac{N_{11}N_{22} - N_{12}N_{21}}{D_{11}D_{22} - D_{12}D_{21}} \tag{31}$$

where

$N_{11} = -\left(\frac{\partial q_i}{\partial \varepsilon}\frac{\partial q_i}{\partial x_i} + \frac{\partial q_j}{\partial \varepsilon}\frac{\partial q_j}{\partial x_i}\right)$,
$N_{12} = D_{12} = \frac{\partial q_i}{\partial x_i}\frac{\partial q_i}{\partial x_j} + q_i^{*c}\frac{\partial^2 q_i}{\partial x_i \partial x_j} + \frac{\partial q_j}{\partial x_i}\frac{\partial q_j}{\partial x_j} + q_j^{*c}\frac{\partial^2 q_j}{\partial x_i \partial x_j}$,
$N_{21} = -\left(\frac{\partial q_i}{\partial \varepsilon}\frac{\partial q_i}{\partial x_j} + \frac{\partial q_j}{\partial \varepsilon}\frac{\partial q_j}{\partial x_j}\right)$,
$N_{22} = D_{22} = \left(\frac{\partial q_i}{\partial x_j}\right)^2 + q_i^{*c}\frac{\partial^2 q_i}{\partial x_j^2} + \left(\frac{\partial q_j}{\partial x_j}\right)^2 + q_j^{*c}\frac{\partial^2 q_j}{\partial x_j^2}$,
$D_{11} = \left(\frac{\partial q_i}{\partial x_i}\right)^2 + q_i^{*c}\frac{\partial^2 q_i}{\partial x_i^2} + \left(\frac{\partial q_j}{\partial x_i}\right)^2 + q_j^{*c}\frac{\partial^2 q_j}{\partial x_i^2}$, and
$D_{21} = \frac{\partial q_i}{\partial x_i}\frac{\partial q_i}{\partial x_j} + q_i^{*c}\frac{\partial^2 q_i}{\partial x_i \partial x_j} + \frac{\partial q_j}{\partial x_i}\frac{\partial q_j}{\partial x_j} + q_j^{*c}\frac{\partial^2 q_j}{\partial x_i \partial x_j}$.

Local stability of an equilibrium of G^{RNC} at (θ, ε) implies that the denominator of (30) is positive. The SONC for a local maximum of the reduced form cooperative problem implies that the denominator of (31) is positive. Define

$$S_i^j(\theta,\varepsilon) \equiv \frac{\partial q_i\left(x_i^*, x_j^*, \theta, \varepsilon\right)}{\partial x_j}\frac{\partial q_i^*\left(x_i^*, x_j^*, \theta, \varepsilon\right)}{\partial x_i} + q_i^*\frac{\partial^2 q_i\left(x_i^*, x_j^*\theta, \varepsilon\right)}{\partial x_i \partial x_j}$$

and

$$\begin{aligned} L_i^j(\theta,\varepsilon) \equiv\ & \frac{\partial q_i\left(x_i^{*c}, x_j^{*c}, \theta, \varepsilon\right)}{\partial x_j}\frac{\partial q_i\left(x_i^{*c}, x_j^{*c}, \theta, \varepsilon\right)}{\partial x_i} + q_i^{*c}\frac{\partial^2 q_i\left(x_i^{*c}, x_j^{*c}, \theta, \varepsilon\right)}{\partial x_i \partial x_j} \\ & + \frac{\partial q_j\left(x_j^{*c}, x_i^{*c}, \theta, \varepsilon\right)}{\partial x_j}\frac{\partial q_j\left(x_j^{*c}, x_i^{*c}, \theta, \varepsilon\right)}{\partial x_i} + q_j^{*c}\frac{\partial^2 q_j\left(x_j^{*c}, x_i^{*c}, \theta, \varepsilon\right)}{\partial x_i \partial x_j} \end{aligned}$$

for all $i, j \in \{1,2\}$. Then

$$\begin{aligned} sign\left(\frac{\partial x_i^*(\theta,\varepsilon)}{\partial \varepsilon}\right) = sign\Bigg(& -\frac{\partial q_i\left(x_i^*, x_j^*, \theta, \varepsilon\right)}{\partial \varepsilon}\frac{\partial q_i\left(x_i^*, x_j^*, \theta, \varepsilon\right)}{\partial x_i} S_j^j(\theta,\varepsilon) \\ & + \frac{\partial q_j\left(x_j^*, x_i^*, \theta, \varepsilon\right)}{\partial \varepsilon}\frac{\partial q_j\left(x_j^*, x_i^*, \theta, \varepsilon\right)}{\partial x_j} S_i^j(\theta,\varepsilon)\Bigg). \end{aligned}$$

Given the SONC of the cooperative problem,

$$\begin{aligned} sign\left(\frac{\partial x_i^{*c}(\theta,\varepsilon)}{\partial \varepsilon}\right) = sign\Bigg(& -\Bigg[\frac{\partial q_i\left(x_i^{*c}, x_j^{*c}, \theta, \varepsilon\right)}{\partial \varepsilon}\frac{\partial q_i\left(x_i^{*c}, x_j^{*c}, \theta, \varepsilon\right)}{\partial x_i} \\ & + \frac{\partial q_j\left(x_j^{*c}, x_i^{*c}, \theta, \varepsilon\right)}{\partial \varepsilon}\frac{\partial q_j\left(x_j^{*c}, x_i^{*c}, \theta, \varepsilon\right)}{\partial x_i}\Bigg] L_j^j(\theta,\varepsilon) \\ & + \Bigg[\frac{\partial q_i\left(x_i^{*c}, x_j^{*c}, \theta, \varepsilon\right)}{\partial \varepsilon}\frac{\partial q_i\left(x_i^{*c}, x_j^{*c}, \theta, \varepsilon\right)}{\partial x_j} \\ & + \frac{\partial q_j\left(x_j^{*c}, x_i^{*c}, \theta, \varepsilon\right)}{\partial \varepsilon}\frac{\partial q_j\left(x_j^{*c}, x_i^{*c}, \theta, \varepsilon\right)}{\partial x_j}\Bigg] L_i^j(\theta,\varepsilon)\Bigg). \end{aligned}$$

By Lemma 2, $\lim_{\varepsilon\to 0} S_i^i = \lim_{\varepsilon\to 0} S_j^j$, $\lim_{\varepsilon\to 0} S_i^j = \lim_{\varepsilon\to 0} S_j^i$, and $\lim_{\varepsilon\to 0} L_i^i = \lim_{\varepsilon\to 0} L_j^j$. The continuity of $q_i(x_i, x_j, \theta, \varepsilon)$, $\partial q_i(x_i, x_j, \theta, \varepsilon)/\partial x_j$, and $\partial^2 q_i(x_i, x_j, \theta, \varepsilon)/\partial x_i \partial x_j$ in $(x_i, x_j, \theta, \varepsilon)$ for all i and j, implies that S_i^j and L_i^j are continuous in $(x_i, x_j, \theta, \varepsilon)$ for all i and j. By continuity of S_i^i,

S_i^j, L_i^i, and L_i^j there exists $\overline{\varepsilon} > 0$ such that for all $0 \leq \varepsilon < \overline{\varepsilon}$,

$$sign\left(\frac{\partial x_1^*(\theta,\varepsilon)}{\partial\varepsilon} + \frac{\partial x_2^*(\theta,\varepsilon)}{\partial\varepsilon}\right) = sign\left(\left(\frac{\partial q_1(\theta,0)}{\partial\varepsilon} + \frac{\partial q_2(\theta,0)}{\partial\varepsilon}\right)\frac{\partial q_1(\theta,0)}{\partial x_1}\left(-S_1^1 + S_1^2\right)\right) \tag{32}$$

and

$$\begin{aligned} sign\left(\frac{\partial x_1^{*c}(\theta,\varepsilon)}{\partial\varepsilon} + \frac{\partial x_2^{*c}(\theta,\varepsilon)}{\partial\varepsilon}\right) &= sign\left(\left[-L_1^1 + L_1^2\right]\left[\frac{\partial q_1(\theta,0)}{\partial\varepsilon} + \frac{\partial q_2(\theta,0)}{\partial\varepsilon}\right]\right. \\ &\quad \left.\cdot\left[\frac{\partial q_1(\theta,0)}{\partial x_2} + \frac{\partial q_1(\theta,0)}{\partial x_1}\right]\right). \end{aligned} \tag{33}$$

The stability condition of G^{RNC} at each (θ,ε) implies that $-S_1^1(\theta,\varepsilon) + S_1^2(\theta,\varepsilon) > 0$ and the SONC at each (θ,ε) of the reduced form cooperative problem implies that $-L_1^1(\theta,\varepsilon) + L_1^2(\theta,\varepsilon) > 0$. The FOC of the individual firm problem in G^{RNC} at each (θ,ε) implies that $\partial q_1(x_1^*, x_2^*, \theta, \varepsilon)/\partial x_1 > 0$. The FOCs of (1) at each (θ,ε) imply that

$$\frac{\partial q_1\left(x_1^{*c}(\theta,0), x_2^{*c}(\theta,0), \theta, 0\right)}{\partial x_2} + \frac{\partial q_1\left(x_1^{*c}(\theta,0), x_2^{*c}(\theta,0), \theta, 0\right)}{\partial x_1} > 0.$$

Finally the demand and costs assumptions imply that

$$\frac{\partial q_1(x_1^*, x_2^*, \theta, \varepsilon)}{\partial\varepsilon} + \frac{\partial q_2(x_2^*, x_1^*, \theta, \varepsilon)}{\partial\varepsilon} = \frac{\partial q_1(x_1^{*c}, x_2^{*c}, \theta, \varepsilon)}{\partial\varepsilon} + \frac{\partial q_2(x_2^{*c}, x_1^{*c}, \theta, \varepsilon)}{\partial\varepsilon} = -\frac{1}{3}$$

for all θ and ε. Hence the sign of the left-hand side of equations (32) and (33) are negative, which completes the proof of Lemma 5. □

Returning now to the proof of Proposition 2, substituting equilibrium R&D investment and production decisions into W, direct calculation of $\partial W^*/\partial\varepsilon$ and $\partial W^{*c}/\partial\varepsilon$ allows one to rewrite (9) and (10) as

$$
\begin{aligned}
\frac{\partial W^*}{\partial \varepsilon} = & \left\{ (q_1^* + q_2^*) \left[\left(\frac{\partial q_1}{\partial c_1} + \frac{\partial q_2}{\partial c_1} \right) \frac{\partial c_1}{\partial x_1} + \left(\frac{\partial q_1}{\partial c_2} + \frac{\partial q_2}{\partial c_2} \right) \frac{\partial c_2}{\partial x_1} \right] \right. \\
& \left. - q_2^* \left[\left(\frac{\partial q_1}{\partial c_1} \frac{\partial c_1}{\partial x_1} + \frac{\partial q_1}{\partial c_2} \frac{\partial c_2}{\partial x_1} \right) - \frac{\partial c_2}{\partial x_1} \right] \right\} \frac{\partial x_1^*}{\partial \varepsilon} \\
& + \left\{ (q_1^* + q_2^*) \left[\left(\frac{\partial q_1}{\partial c_1} + \frac{\partial q_2}{\partial c_1} \right) \frac{\partial c_1}{\partial x_2} + \left(\frac{\partial q_1}{\partial c_2} + \frac{\partial q_2}{\partial c_2} \right) \frac{\partial c_2}{\partial x_2} \right] \right. \\
& \left. - q_1^* \left[\left(\frac{\partial q_2}{\partial c_1} \frac{\partial c_1}{\partial x_2} + \frac{\partial q_2}{\partial c_2} \frac{\partial c_2}{\partial x_2} \right) - \frac{\partial c_1}{\partial x_2} \right] \right\} \frac{\partial x_2^*}{\partial \varepsilon} \\
& + \left(q_1^* \frac{\partial q_1^*}{\partial c_2} + q_2^* \frac{\partial q_2^*}{\partial c_2} \right) \frac{\partial c_2}{\partial \varepsilon} - q_2^* \frac{\partial c_2}{\partial \varepsilon} \qquad (34)
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial W^{*c}}{\partial \varepsilon} = & \left\{ (q_1^{*c} + q_2^{*c}) \left[\left(\frac{\partial q_1}{\partial c_1} + \frac{\partial q_2}{\partial c_1} \right) \frac{\partial c_1}{\partial x_1} + \left(\frac{\partial q_1}{\partial c_2} + \frac{\partial q_2}{\partial c_2} \right) \frac{\partial c_2}{\partial x_1} \right] \right\} \frac{\partial x_1^{*c}}{\partial \varepsilon} \\
& + \left\{ (q_1^{*c} + q_2^{*c}) \left[\left(\frac{\partial q_1}{\partial c_1} + \frac{\partial q_2}{\partial c_1} \right) \frac{\partial c_1}{\partial x_2} + \left(\frac{\partial q_1}{\partial c_2} + \frac{\partial q_2}{\partial c_2} \right) \frac{\partial c_2}{\partial x_2} \right] \right\} \frac{\partial x_2^{*c}}{\partial \varepsilon} \\
& + \left(q_1^{*c} \frac{\partial q_1}{\partial c_2} + q_2^{*c} \frac{\partial q_2}{\partial c_2} \right) \frac{\partial c_2}{\partial \varepsilon} - q_2^{*c} \frac{\partial c_2}{\partial \varepsilon} \qquad (35)
\end{aligned}
$$

where (34) and (35) are evaluated at (θ, ε). Define

$$
\begin{aligned}
T_i(\theta, \varepsilon) \equiv & \left\{ (q_i^* + q_j^*) \left[\left(\frac{\partial q_i}{\partial c_i} + \frac{\partial q_j}{\partial c_i} \right) \frac{\partial c_i}{\partial x_i} + \left(\frac{\partial q_i}{\partial c_j} + \frac{\partial q_j}{\partial c_j} \right) \frac{\partial c_j}{\partial x_i} \right] \right. \\
& \left. - q_j^* \left[\left(\frac{\partial q_i}{\partial c_i} \frac{\partial c_i}{\partial x_i} + \frac{\partial q_i}{\partial c_j} \frac{\partial c_j}{\partial x_i} \right) - \frac{\partial c_j}{\partial x_i} \right] \right\}
\end{aligned}
$$

and

$$
T_i^c(\theta, \varepsilon) \equiv \left\{ (q_i^{*c} + q_j^{*c}) \left[\left(\frac{\partial q_i}{\partial c_i} + \frac{\partial q_j}{\partial c_i} \right) \frac{\partial c_i}{\partial x_i} + \left(\frac{\partial q_i}{\partial c_j} + \frac{\partial q_j}{\partial c_j} \right) \frac{\partial c_j}{\partial x_i} \right] \right\}
$$

for $i \neq j$. Define

$$
\begin{aligned}
T(\theta) \equiv & \left\{ (q^* + q^*) \left[\left(\frac{\partial q_i}{\partial c_i} + \frac{\partial q_j}{\partial c_i} \right) \frac{\partial c_i}{\partial x_i} + \left(\frac{\partial q_i}{\partial c_j} + \frac{\partial q_j}{\partial c_j} \right) \frac{\partial c_j}{\partial x_i} \right] \right. \\
& \left. \left. - q^* \left[\left(\frac{\partial q_i}{\partial c_i} \frac{\partial c_i}{\partial x_i} + \frac{\partial q_i}{\partial c_j} \frac{\partial c_j}{\partial x_i} \right) - \frac{\partial c_j}{\partial x_i} \right] \right\} \right|_{\varepsilon=0}
\end{aligned}
$$

and

$$T^c(\theta) \equiv \left\{ (q^{*c} + q^{*c}) \left[\left(\frac{\partial q_i^{*c}}{\partial c_i} + \frac{\partial q_j^{*c}}{\partial c_i} \right) \frac{\partial c_i}{\partial x_i} + \left(\frac{\partial q_i^{*c}}{\partial c_j} + \frac{\partial q_j^{*c}}{\partial c_j} \right) \frac{\partial c_j}{\partial x_i} \right] \right\} \Bigg|_{\varepsilon=0}$$

for $i \neq j$, where $q^* = q^*(\theta, 0)$ and $q^{*c} = q^{*c}(\theta, 0)$. Then we can rewrite expressions (34) and (35) as

$$\begin{aligned} \frac{\partial W^*(\theta,\varepsilon)}{\partial \varepsilon} = {} & T_1(\theta,\varepsilon) \frac{\partial x_1^*(\theta,\varepsilon)}{\partial \varepsilon} + T_2(\theta,\varepsilon) \frac{\partial x_2^*(\theta,\varepsilon)}{\partial \varepsilon} \\ & \left(q_1^*(\theta,\varepsilon) \frac{\partial q_1(x_1^*, x_2^*, \theta, \varepsilon)}{\partial c_2} + q_2^*(\theta,\varepsilon) \frac{\partial q_2(x_1^*, x_2^*, \theta, \varepsilon)}{\partial c_2} \right) \frac{\partial c_2}{\partial \varepsilon} \\ & -q_2^*(\theta,\varepsilon) \frac{\partial c_2}{\partial \varepsilon} \end{aligned}$$

and

$$\begin{aligned} \frac{\partial W^{*c}(\theta,\varepsilon)}{\partial \varepsilon} = {} & T_1^c(\theta,\varepsilon) \frac{\partial x_1^{*c}(\theta,\varepsilon)}{\partial \varepsilon} + T_2^c(\theta,\varepsilon) \frac{\partial x_2^{*c}(\theta,\varepsilon)}{\partial \varepsilon} \\ & \left(q_1^{*c}(\theta,\varepsilon) \frac{\partial q_1(x_1^*, x_2^*, \theta, \varepsilon)}{\partial c_2} + q_2^{*c}(\theta,\varepsilon) \frac{\partial q_2(x_1^*, x_2^*, \theta, \varepsilon)}{\partial c_2} \right) \frac{\partial c_2}{\partial \varepsilon} \\ & -q_2^{*c}(\theta,\varepsilon) \frac{\partial c_2}{\partial \varepsilon}. \end{aligned}$$

By the definitions of q_i and c_2 it follows that $\partial q_i(x_i, x_j, \theta, \varepsilon)/\partial c_j = 1/3$, $\partial q_i(x_i, x_j, \theta, \varepsilon)/\partial c_i = -2/3$, and $\partial c_2/\partial \varepsilon = 1$ for all $(x_i, x_j, \theta, \varepsilon)$.
By Lemma 2, $\lim_{\varepsilon \to 0} T_i(\theta, \varepsilon) = T(\theta)$ and $\lim_{\varepsilon \to 0} T_i^c(\theta, \varepsilon) = T^c(\theta)$, therefore

$$\lim_{\varepsilon \to 0} \frac{\partial W^*(\theta,\varepsilon)}{\partial \varepsilon} = T(\theta) \left(\frac{\partial x_1^*(\theta,0)}{\partial \varepsilon} + \frac{\partial x_2^*(\theta,0)}{\partial \varepsilon} \right) - \frac{4}{3} q^*(\theta, 0) \tag{36}$$

and

$$\lim_{\varepsilon \to 0} \frac{\partial W^{*c}(\theta,\varepsilon)}{\partial \varepsilon} = T^c(\theta) \left(\frac{\partial x_1^{*c}(\theta,0)}{\partial \varepsilon} + \frac{\partial x_2^{*c}(\theta,0)}{\partial \varepsilon} \right) - \frac{4}{3} q^{*c}(\theta, 0). \tag{37}$$

The demand and cost assumptions directly imply that $T(\theta)$ and $T^c(\theta)$ are strictly positive. By Lemma 5, there exists $\bar{\varepsilon} > 0$ such that for $0 \leq \varepsilon < \bar{\varepsilon}$, $\partial x_1^*/\partial \varepsilon + \partial x_2^*/\partial \varepsilon < 0$ and $\partial x_1^{*c}/\partial \varepsilon + \partial x_2^{*c}/\partial \varepsilon < 0$. Therefore the right hand sides of (36) and (37) are strictly negative.

The continuity of $q_i(x_i, x_j, \theta, \varepsilon)$, $\partial q_i(x_i, x_j, \theta, \varepsilon)/\partial x_j$, $\partial q_i(x_i, x_j, \theta, \varepsilon)/\partial \varepsilon$, and $\partial^2 q_i(x_i, x_j, \theta, \varepsilon)/\partial x_i \partial x_j$ in $(x_i, x_j, \theta, \varepsilon)$ for all i and j implies the continuity of $\partial W^*(\theta, \varepsilon)/\partial \varepsilon$ and $\partial W^{*c}(\theta, \varepsilon)/\partial \varepsilon$. Then by the continuity of $\partial W^*(\theta, \varepsilon)/\partial \varepsilon$ and $\partial W^{*c}(\theta, \varepsilon)/\partial \varepsilon$ there exists $\overline{\varepsilon} > 0$ such that for $0 \leq \varepsilon < \overline{\varepsilon}$, $\partial W^*(\theta, \varepsilon)/\partial \varepsilon < 0$ and $\partial W^{*c}(\theta, \varepsilon)/\partial \varepsilon < 0$. □

5.4 Proof of Proposition 3

First, we show that if the conditions of Proposition 1 hold and if $\Theta^*(\varepsilon) \neq \emptyset$, there exists $\overline{a}, \overline{\varepsilon} > 0$ such that for $0 \leq \varepsilon < \overline{\varepsilon}$ and $a > \overline{a}$, $\Theta^*(\varepsilon)$ is a singleton. Then, we show the welfare ranking for all θ. Suppose there exists $\theta^*(\varepsilon) \in \Theta^*(\varepsilon)$. By Lemma 2 and continuity of $\partial W^*(\theta^*(\varepsilon), \varepsilon)/\partial\theta - \partial W^{*c}(\theta^*(\varepsilon), \varepsilon)/\partial\theta$, there exist $\overline{a}, \overline{\varepsilon} > 0$ such that for $0 \leq \varepsilon < \overline{\varepsilon}$ and $a > \overline{a}$

$$sign\left(\frac{\partial W^*(\theta^*(\varepsilon), \varepsilon)}{\partial\theta} - \frac{\partial W^{*c}(\theta^*(\varepsilon), \varepsilon)}{\partial\theta}\right) = sign\left(\frac{\partial W^*(\theta^*(0), 0)}{\partial\theta} - \frac{\partial W^{*c}(\theta^*(0), 0)}{\partial\theta}\right). \tag{38}$$

By Lemma 1, $\varepsilon = 0$ implies $x_i^*(\theta^*(0), 0) = x_i^{*c}(\theta^*(0), 0)$ for all i. Therefore we can write[15]

$$\begin{aligned}\frac{\partial W^*(\theta^*(0), 0)}{\partial\theta} - \frac{\partial W^{*c}(\theta^*(0), 0)}{\partial\theta} &= 2q_1^*\left(-2\frac{\partial x_1^*(\theta^*(0), 0)}{\partial\theta} + \frac{2}{3}\frac{\partial x_1^{*c}(\theta^*(0), 0)}{\partial\theta}\right)\frac{\partial c_2}{\partial x_1} \\ &\quad + \frac{2}{3}q_1^*\frac{\partial x_1^{*c}(\theta^*(0), 0)}{\partial\theta}\frac{\partial c_2}{\partial x_1}.\end{aligned} \tag{39}$$

By Lemmas 3 and 4, $\partial x_1^*(\theta^*(0), 0)/\partial\theta < 0$. If the conditions of Proposition 1 hold, by Lemma 2, $\partial x_1^{*c}(\theta^*(0), 0)/\partial\theta > 0$. By the definition of c_2, $\partial c_2/\partial x_1 < 0$ for all x_1, x_2, θ, and ε. Therefore the right-hand side of (38) is strictly negative. Therefore if $\Theta^*(\varepsilon) \neq \emptyset$ and the conditions in Proposition 1 hold, there exist $\overline{a}, \overline{\varepsilon} > 0$ such that for $0 < \varepsilon < \overline{\varepsilon}$ and $a > \overline{a}$, the left-hand side of (39) is negative. Moreover, $\theta^*(\varepsilon) \in \Theta^*(\varepsilon)$ so the conditions in Proposition 1 imply $\partial W^*(\theta^*(\varepsilon), \varepsilon)/\partial\theta < 0$ and $\partial W^{*c}(\theta^*(\varepsilon), \varepsilon)/\partial\theta > 0$. Recall that by Proposition 1, $\partial W^*(0, \varepsilon)/\partial\theta > 0$ and $\partial W^{*c}(\theta, \varepsilon)/\partial\theta > 0$ for all θ. Then by the continuity

[15] Notice that $\partial W^*(\theta^*(0), 0)/\partial\theta = -4q_i^{*c}\partial c_j/\partial x_i \partial x_i^*/\partial\theta - (4/3)(q_i^*\partial c_i/\partial\theta + q_i^*\partial c_j/\partial\theta)$ and $\partial W^{*c}(\theta^*(0), 0)/\partial\theta = -(4/3)\, q_i^{*c}(\partial c_i/\partial x_i + \partial c_j/\partial x_i)\, dx_i^{*c}/d\theta - (4/3)(q_i^{*c}\partial c_i/\partial\theta + q_i^{*c}\partial c_j/\partial\theta)$.

of $\partial W^{*}(\theta,\varepsilon)/\partial\theta$ and $\partial W^{*c}(\theta,\varepsilon)/\partial\theta$ with respect to ε there exist $\overline{a},\overline{\varepsilon} > 0$ such that for $0 < \varepsilon < \overline{\varepsilon}$ and $a > \overline{a}$, if $W^{*}(\theta,\varepsilon)$ intersects $W^{*c}(\theta,\varepsilon)$, it can only do it from above at $\theta^{*}(\varepsilon)$. This directly implies that $W^{*}(\theta,\varepsilon) > W^{*c}(\theta,\varepsilon)$ for all $\theta \in [0,\theta^{*}(\varepsilon))$. Suppose there exists $\widetilde{\theta} \in (\theta^{*}(\varepsilon),1]$ such that $W^{*}\left(\widetilde{\theta},\varepsilon\right) = W^{*c}\left(\widetilde{\theta},\varepsilon\right)$. The continuity of $W^{*}(\theta,\varepsilon)$ and $W^{*c}(\theta,\varepsilon)$ in (θ,ε) requires that $\partial W^{*}\left(\widetilde{\theta},\varepsilon\right)/\partial\theta > \partial W^{*c}\left(\widetilde{\theta},\varepsilon\right)/\partial\theta > 0$, but this contradicts $\partial W^{*}(\theta^{*}(\varepsilon),\varepsilon)/\partial\theta < 0$ and $\partial W^{*c}(\theta^{*}(\varepsilon),\varepsilon)/\partial\theta > 0$. Therefore there is no $\widetilde{\theta} \in (\theta^{*}(\varepsilon)^{*},1]$ such that $W^{*}\left(\widetilde{\theta},\varepsilon\right) = W^{*c}\left(\widetilde{\theta},\varepsilon\right)$, which implies that $\theta^{*}(\varepsilon)$ is unique.

The welfare ranking follows from the fact that for $0 < \varepsilon < \overline{\varepsilon}$ and given $a > \overline{a}$, if $\theta^{*}(\varepsilon)$ exists, $W^{*}(\theta,\varepsilon)$ can intersect $W^{*c}(\theta,\varepsilon)$ only from above. □