

Exam A (Part II)**Name**

Important: Show all your work in the spaces provided. Present your solutions in a well organized, neat, and legible way. Calculators may be used *only* in elementary computational and trig mode, and not in calculus mode (even for exploratory purposes).

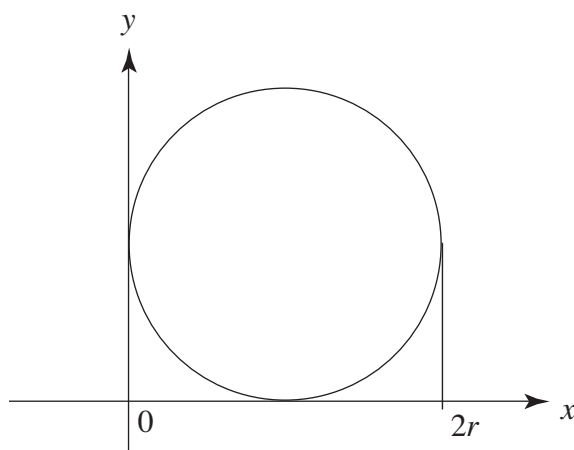
1. Let $y = f(x) = x^3 + 3x^2 - 24x$. Find the critical numbers for this function, determine the intervals over which its graph is increasing or decreasing, find the values of x that give rise to (local) maximum and minimum values.

2. The sum $(4)^{\frac{1}{2}} \cdot \frac{1}{1000} + \left(4 + \frac{1}{1000}\right)^{\frac{1}{2}} \frac{1}{1000} + \left(4 + \frac{2}{1000}\right)^{\frac{1}{2}} \frac{1}{1000} + \cdots + \left(8 + \frac{999}{1000}\right)^{\frac{1}{2}} \frac{1}{1000}$ follows the pattern that the first three terms and the last term establish. Study the sum involved in the definition of $\int_a^b f(x) dx$. For what n , $a = x_0, x_1, \dots, x_{n-1}, x_n = b$, and $y = f(x)$ is the sum above of this form? Find a number C that closely approximates the value of the sum above. (Hint: $8 = 4 + \frac{4000}{1000}$)

$C =$

3. You are given a function $y = f(x)$ that is continuous on the interval $[a, b]$. True or false: Is there always a function $y = F(x)$ such that $\int_a^b f(x) dx = F(b) - F(a)$? If this is false, say why it is false. If it is true, explain how such a function $y = F(x)$ can be defined for the given $y = f(x)$.

4. A circle of radius r is positioned as shown in the figure below.



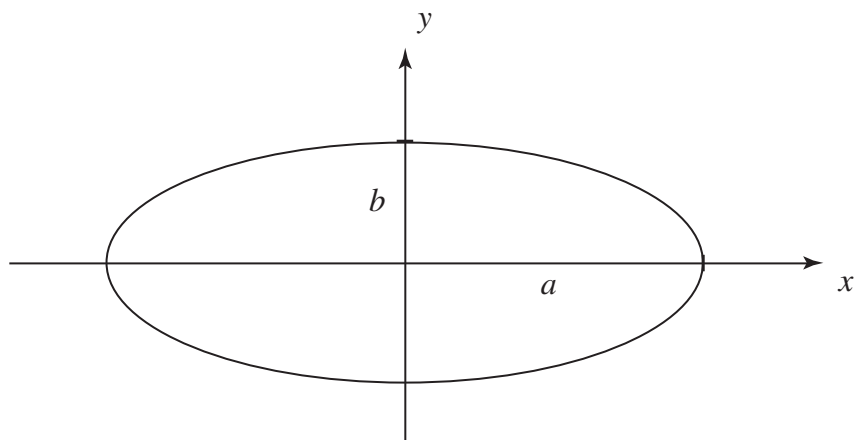
4a. Find the area A under the lower half of the circle and above the interval $[0, 2r]$ by using the geometry of the situation.

$A =$

4b. Express the area A as a definite integral.

$A = \int$

5. The upper half of the ellipse of Figure 3 is the graph of the function $f(x) = \frac{b}{a}\sqrt{a^2 - x^2}$. Revolve the entire right side of the ellipse of the figure one complete revolution around the y -axis.



Express the resulting volume V as a definite integral in two different ways. (One way involves the disc method and the variable y , and the other way the shell method and the variable x .)

$$V = \int$$

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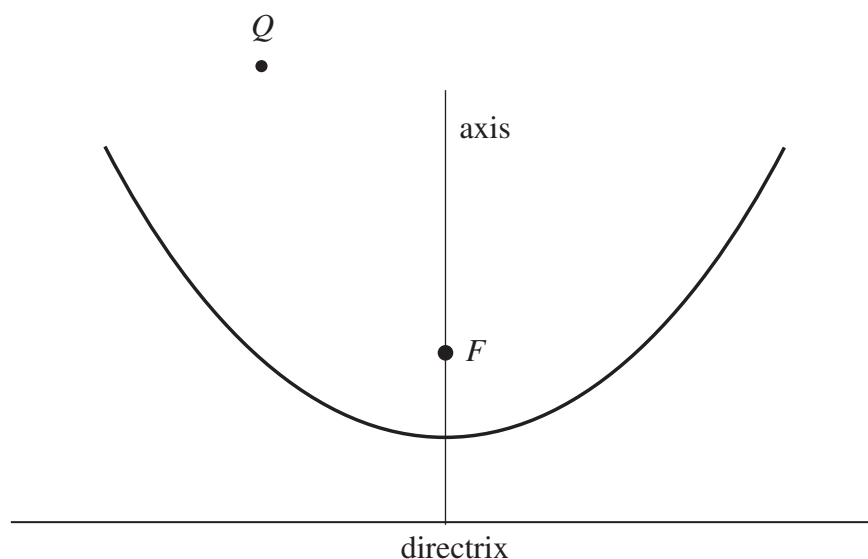
Find the volume that has been described by evaluating one of the two integrals.

$$V =$$

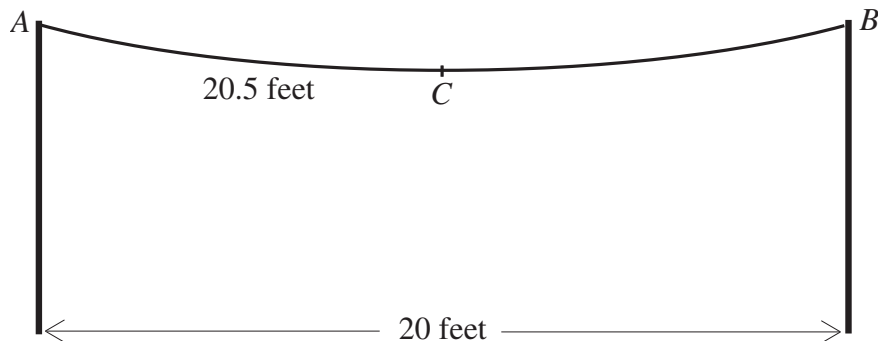
6. One half the circumference of a circle of radius r is πr . Use this fact to evaluate the definite integral $\int_{-r}^r \frac{1}{\sqrt{r^2 - x^2}} dx$.

$$\int_{-r}^r \frac{1}{\sqrt{r^2 - x^2}} dx =$$

7. The figure below depicts a parabolic mirror. A beam of light goes through the point Q , strikes the mirror and is reflected through the focus F . Explain why the path of approach of the beam towards the mirror is parallel to the focal axis of the parabola. [Hint: By Fermat's principle, the path the beam from Q to the mirror to F must be the path of least time. Since the medium is the same throughout, the path needs to be the shortest possible path.]



8. A clothesline 20.5 feet long is attached to two poles at points A and B . The poles are 20 feet apart. The straight line that connects A and B is horizontal. See Figure 6. (The clothesline is completely flexible and inelastic. Compared to the loads that it supports, it is of negligible weight.)



A. A laundry bag with 120 wet socks (60 pairs) weighing a total of 26 pounds has been attached to the clothesline at its midpoint C .

- i. Show that in the situation just described, the point C is 2.25 feet below the line segment AB .
- ii. Let T be the tension in the line and draw a diagram of the forces acting at C . Combine the force diagram and the geometry to show that $T \approx 59.22$ pounds

B. The 120 socks are taken out of the laundry bag and suspended individually on the clothesline at approximately 6 socks per horizontal foot. Since 120 socks weigh 26 pounds, 6 socks weigh 1.3 pounds. Since the distance between the posts is 20 feet, the clothesline supports $w = 1.3$ pounds per foot. Let $d = 10$ be one-half the distance between the posts. The midpoint C of the clothesline has shifted from its previous situation. It is now s feet below the segment AB . Place a coordinate system into the figure so that the origin is at C and the x -axis is parallel to the segment AB .

- i. Let L be the length of the part of the clothesline from C to B . With the coordinate system as specified, we know that the curve of the cable is the graph of the function $f(x) = \frac{s}{d^2}x^2$. Explain why the length L of the cable from C to B is given by

$$L = \int_0^d \sqrt{1 + \left(\frac{2s}{d^2}x\right)^2} dx \quad (\text{nothing cosmic here, just mention a basic fact}).$$

ii. Recall the approximation

$$\int_0^d \sqrt{1 + \left(\frac{2s}{d^2}x\right)^2} dx \approx d + \frac{2}{3} \left(\frac{s}{d}\right)^2 d - \frac{2}{5} \left(\frac{s}{d}\right)^4 d.$$

Use it and the quadratic formula to derive an estimate for s^2 . From this, deduce the approximation $s \approx 1.96$ feet.

iii. Use the conclusion of (ii) to show that the maximum tension $T_{\max}$ and the minimum tension $T_{\min}$ in the clothesline are 35.62 pounds and 33.16 pounds respectively.

Formulas and expressions:

$$\int u \, dv = uv - \int v \, du \quad x = r \cos \theta \quad y = r \sin \theta \quad Ay'' + By' + Cy = 0$$

$$y = D_1 e^{r_1 x} + D_2 e^{r_2 x} \quad y = D_1 e^{2x} + D_2 x e^{2x} \quad y = e^{ax} (D_1 \cos bx + D_2 \sin bx)$$

$$a = \frac{d}{1-\varepsilon^2} \quad b = \frac{d}{\sqrt{1-\varepsilon^2}} \quad a = \frac{d}{\varepsilon^2-1} \quad b = \frac{d}{\sqrt{\varepsilon^2-1}} \quad f'(\theta) = f(\theta) \cdot \tan(\gamma - \frac{\pi}{2})$$

$$f(x) = \frac{s}{d^2} x^2, \quad \tan \alpha = \frac{2s}{d}, \quad T(x) = w \sqrt{\left(\frac{d^2}{2s}\right)^2 + x^2}, \quad T_d = wd \sqrt{\left(\frac{d}{2s}\right)^2 + 1}, \quad T_0 = \frac{wd^2}{2s}.$$

$$ab\pi \quad a^2 = b^2 + c^2 \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \kappa = \frac{ab\pi}{T} \quad F = ma \quad f(t) \cdot r = I \cdot \alpha(t) \quad \frac{d}{dx} \sin x = \cos x$$

$$\int_a^b \sqrt{1+f'(x)^2} \, dx \quad \int_a^b \pi f(x)^2 \, dx \quad 2\pi \int_a^b f(x) \sqrt{1+f'(x)^2} \, dx$$